Ionospheric heating and ELF/VLF wave generation by HAARP

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I. EXECUTIVE SUMMARY

This proposal requests support for modelling of the ionospheric changes due to HF (high-frequency) heating by the HAARP (High Frequency Active Auroral Research Program) facility and the subsequent ELF/VLF wave generation by modulation of the polar electrojet current. The virtual antenna that HAARP creates can have many applications in areas where effective emission of ELF/VLF waves is needed. Our goal is to study these phenomena theoretically and to create a general but easy-to-use software tool that can be used by other researchers interested in HF heating and ELF/VLF wave generation. The tool will be very flexible in order to provide straightforward solutions for various ionospheric heating conditions and for the study of ELF/VLF wave generation. In addition, various other 1 ms effects in the D region and upper regions of ionosphere, such as optical emissions (Pedersen et al., 2003), will be resolved.

For illustration of the features of the proposed model, we present preliminary results of a 1D model described in detail in the Appendix.

II. PROGRAM DESCRIPTION AND PLAN

A. Introduction

HAARP is a high power transmitter facility operating in the HF frequency range. After an upgrade to 3.6 MW power scheduled to be completed in 2006 it will have an unprecedented effective radiated power (ERP) of the order of GW in the 2.8-10 MHz range (see Figure 1), only a factor of ten less than that of lightning. Powerful HF facilities are used to study various effects occurring throughout all regions of ionosphere. A review of these studies can be found, e.g., in Rietveld et al. (1993). Our proposed software tool will model the HF heating of the ionosphere, up to F region. The conductivity changes in the D and lower E regions of ionosphere modulate the polar electrojet current and produce electromagnetic radiation at the modulating ELF/VLF frequency. This effect has been observed experimentally (Barr et al., 1999), although the observations do not always match the present theoretical predictions (Rietveld et al., 1993).

The HF heating of the D region has been of interest for communication purposes in the past. The problem has been treated theoretically by various researchers. Below we list some of these models, with the assumptions that were used. In our proposed model, we are going to address the issues that we believe are important for accurate results and were not taken into account in these models. The assumptions of these models are also summarized in Table I.

1. Tomko (1981)

Thermal (Maxwellian) electron distribution function is assumed. HF wave propagation is studied in 1D. The self-absorption is included in an equation for the electromagnetic power flux. However, the effects of wave phase...
changes are not included. The heating is modelled as energy absorption by electrons from the e/m wave, with cooling by collisions with ambient molecules.

2. Papadopoulos et al. (1990)

The kinetic equations are solved and conductivity changes are found for propagation only along the direction of the geomagnetic field. The wave propagation is not modelled, therefore, no self-absorption is calculated.


The ionospheric heating is done in an external work (referred to as APTI Technical report 5007, 1991). According to the brief description in Papadopoulos et al. (2003), the heating is done in 1D, with self-absorption, and a thermal electron distribution is assumed. The magnetization of electrons is not mentioned, but probably is included. The radiated ELF/VLF waves are calculated by integrating over the source current.

4. Rowland et al. (1996)

The measurements at HIPAS (high-power auroral simulation facility) are compared to the computer simulation results. The heating is not modelled, but fixed conductivity changes are used, from Papadopoulos et al. (1990) model. The electrojet current modulation and VLF/ELF emission is modelled in 2D.

5. Rowland (1999)

Same as Rowland et al. (1996), except for a 3D model of VLF/ELF emission.


Only ELF/VLF wave generation is calculated, with a given current source, using electron-magnetohydrodynamic (EMHD) approach in Fourier space.

7. Milikh et al. (1999)

A 1D model of HF wave propagation using analytical expression for temperature modification (Gurevich, 1978, p. 19). The conductivity modifications are used as inputs for Zhou et al. (1996) model to obtain the ELF/VLF wave emission. The conductivity changes for the DC current are described, but it is not completely clear whether HF conductivity changes are taken into account for calculation of self-consistent propagation of the HF wave.

8. Kuo et al. (2002)

Analytical model of HF heating. The kinetic equation is not solved numerically (electron distribution is characterized by $T_e$), but the vibrational $N_2$ energy loss resonance is discussed. Authors predict a dramatic increase of electron temperature modulation, together with conductivity modulation, when the electric field exceeds the $N_2$ vibrational “bump”. The absence of the described thermal instability in previous kinetic equation solution (Papadopoulos et al., 1990) is explained by low levels of electric field. Our model will solve the kinetic equation for all levels of electric field and test this hypothesis.

9. Relevant work at Stanford University

Stanford VLF group has performed extensive studies of transient light emission phenomena associated with lightning. The phenomenon called Elves is very similar to the present task. During an Elves event, heating of the lower ionosphere by a lightning-generated pulse occurs, causing optical emissions (Taranenko et al., 1993). The difference is that the pulse is not modulated, with the frequency content limited to $<10$ kHz. Cylindrical 2D simulations of this phenomenon were performed (Veronis et al., 1999). The heating by VLF waves was also considered in the past (Taranenko et al., 1992).

Our tool will include various aspects that were either ignored or included only approximately in previous models. For example, we use the kinetic equation to find the electron distribution function rather than assuming a Maxwellian distribution. Furthermore, we include all the nonlinear effects due to self-induced absorption of the HF wave, and will perform multidimensional calculations of HF propagation. Below, we look at all parts of the posed problem in detail. To support our statements in the following discussion, we present results from 1D heating model using a non-Maxwellian electron distribution, which is described in detail in the Appendix, together with preliminary (unpublished) results.
### TABLE I

All HF heating models are one-dimensional when self-absorption is included, or zero-dimensional otherwise.

<table>
<thead>
<tr>
<th>Model</th>
<th>Non-Maxwellian</th>
<th>Mag. field</th>
<th>Dimensions</th>
<th>Self-absorption</th>
<th>ELF generation</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Tomko (1981)</em></td>
<td>no</td>
<td>yes</td>
<td>1</td>
<td>yes</td>
<td>estimate</td>
</tr>
<tr>
<td><em>Papadopoulos et al. (1990)</em></td>
<td>yes</td>
<td>yes</td>
<td>0</td>
<td>no</td>
<td>estimate</td>
</tr>
<tr>
<td><em>Papadopoulos et al. (2003)</em></td>
<td>no</td>
<td>yes(?)</td>
<td>1</td>
<td>yes</td>
<td>analytical</td>
</tr>
<tr>
<td><em>Rowland et al. (1996)</em></td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>2D FDTD</td>
</tr>
<tr>
<td><em>Rowland (1999)</em></td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>3D FDTD</td>
</tr>
<tr>
<td><em>Zhou et al. (1996)</em></td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>EMHD</td>
</tr>
<tr>
<td><em>Milikh et al. (1999)</em></td>
<td>no</td>
<td>yes</td>
<td>1</td>
<td>yes(?)</td>
<td>EMHD</td>
</tr>
<tr>
<td><em>Kuo et al. (2002)</em></td>
<td>mixed</td>
<td>no</td>
<td>0</td>
<td>no</td>
<td>analytical</td>
</tr>
<tr>
<td><strong>This project</strong></td>
<td>yes</td>
<td>yes</td>
<td>1-3</td>
<td>yes</td>
<td>analytical/FDTD</td>
</tr>
</tbody>
</table>

**FIG. 2** Electron temperature changes due to steady-state HF heating: (a) calculated by *Tomko (1981)*, assuming Maxwellian electron distribution; (b) present calculations, which are described in Appendix, with non-Maxwellian electron distributions at each altitude.

#### B. Electron distribution modifications

It is believed that *D*-region electron temperature can be changed by more than an order of magnitude through heating by powerful HF waves from ground-based emitters (*Rietveld et al.*, 1993). Calculations of temperature changes which assume a Maxwellian electron distribution were done in the past (*Tomko, 1981*), and are shown in Figure 2(a).

In the Appendix to this proposal we describe our new (unpublished) 1D HF propagation model which calculates the electron distribution at each altitude. The kinetic equation for electron distribution is solved using a modified version of a standard software package ELENDIF (*Morgan and Penetrante*, 1990), expanded to include oscillations of the electric field and ambient magnetic field. The equation being solved is derived and discussed in detail in Appendix A.1. For a non-Maxwellian distribution, the temperature is not defined, but one can calculate the *effective temperature* $T_e$, defined in terms of the average electron energy $\langle E \rangle$ as $T_e = \frac{2}{3k_B} \langle E \rangle$. The calculated changes of $T_e$ are presented in Figure 2(b). Although there are some similarities in the results (such as shifting of the altitude of maximum $\Delta T_e$ upward with increasing HF power), there are also some significant differences, such as higher nonlinear dependence of max $\Delta T_e$ on HF power. These new findings result from a more advanced method of solution in the present model.

Although there are noticeable differences even in the results for $T_e$, it alone cannot describe all the effects associated...
FIG. 3 Dynamic friction function in air, \( F_D(E) = \sum N_i \sigma_i(E) / \Delta E \). It is plotted in terms of the electric field necessary for the forces exerted to be in equilibrium, \( eE = F_D \). The non-relativistic portion is calculated on the basis of cross-section data available online (Phelps, 2005), the relativistic portion is from Bethe and Ashkin (1953, p. 254). The electric breakdown level (see Section A.4) is also shown.

with the change of the electron distribution function in strong variable electric field. In particular, the high-energy “tail” of the actual distribution is very different from that of a Maxwellian. This significantly changes the estimate of the rates of various processes, such as optical emissions (Pedersen et al., 2003; Vlasov et al., 2004), ionization and attachment. The new HAARP power level creates a special need to calculate carefully the modifications of the electron distribution function for the description of these effects. The D-region conductivity changes also can be estimated more accurately if the full electron distribution function is known.

The background free electrons in air are accelerated by the heating HF electric field and slowed down by collisions with molecules. The electron energy losses in air can be described in terms of the dynamic friction function (Gurevich, 1978, p. 50). According to Figure 3, it has a complicated shape determined by a multitude of inelastic electron-molecule processes, which include excitations of rotational, vibrational and electronic levels of molecules. The most prominent feature is a peak due to nitrogen vibrational losses. A theoretical investigation by Kuo et al. (2002) predicts dramatic changes in the bulk properties of the ionospheric plasma, such as \( T_e \) and conductivity, when the force applied by the external electric field exceeds this peak, due to acceleration of electrons to higher energies. It is evident that this effect cannot be studied in terms of a thermal electron distribution. The kinetic calculations of Appendix A do not show any discontinuities in the conductivity (see Figures 4, 18). In Figures 3–4, we introduce a convenient unit, the Townsend (Td), for measuring the reduced electric field \( E/N \), where \( N \) is the molecule density of the atmosphere:

\[
1 \text{Td} = 10^{-21} \text{V-m}^2
\]

(1)

For variable fields, \( E \) is understood as the root-mean-square (RMS) field.

In the Appendix, we present calculations of \( E/N \) using the new 1D model. From Figure 15 one might conclude that the electric breakdown field might be exceeded. However, this is not true. We argue in Appendix A that the oscillatory field has a lesser effect than that of a DC field. The correct parameter to use is an equivalent effective electric field, given by equation (A11):

\[
E_{\text{eff}} = \frac{E}{\sqrt{1 + \left( \frac{\omega_{\text{eff}}}{\nu_{\text{m,eff}}} \right)^2}}
\]

where \( \omega_{\text{eff}} = \omega \pm \omega_H \) for extraordinary and ordinary mode heating, and \( \nu_{\text{m,eff}}/N \approx 2 \times 10^{-13} \text{ s}^{-1} \text{m}^3 \) is an effective (average) collision frequency with a value that follows from an estimate of the electric breakdown field \( E_{\text{br}} \) in equation (A14). To correctly compare the electric field values to features of the dynamic friction function, we must therefore use,
FIG. 4 Steady-state DC conductivity at $h = 80$ km altitude resulting from heating by extraordinary mode HF waves (assuming $f_H = 1$ MHz) at $f = 3$ MHz and 7 MHz. The solid line corresponds to a heating DC field along geomagnetic field. Kuo et al. (2002) predict dramatic change of conductivity at electric field necessary for exceeding the $N_2$ vibrational “bump” in the dynamic friction function (shown). Also shown is the electric breakdown field.

FIG. 5 Used nighttime and daytime electron density profiles (NSSDC, 2001).

for example, $E/E_{br}$. The relevant plots for nighttime and daytime electron density profiles (Figure 5) are presented in Figures 6 and 7, correspondingly. For comparison, we repeat the dynamic friction from Figure 3 in these Figures.

In Appendix A.5, we suggest that at high electron density, which is present in upper $E$ and $F$ ionospheric regions, the electron distribution will have a tendency to acquire a Maxwellian shape due to electron-electron collisions. This effect is not yet included in the model presented in the Appendix, because the current model extends only to $D$ and lower $E$ regions. However, the proposed software tool will be capable of investigating this effect.
C. Self-consistent HF wave propagation

The HF wave propagates through the $D$ region simultaneously modifying its conductivity. The self-absorption is very important for calculations of the electromagnetic energy flux. We expect a nonlinear dependence of the $D$-region electric field values on the transmitted power. The propagation of the HF wave, therefore, has to be modelled self-consistently.

It is possible to solve Maxwell’s equations by the finite-difference time-domain (FDTD) methods with the current $\mathbf{J} = \sigma \mathbf{E}$ using the conductivity tensor calculated in Appendix A.6. However, this would be computationally difficult in the 3D case because of a huge number of spatial cells. The cell size for a full electromagnetic FDTD code is determined by the wavelength $\lambda$ of the carrier wave. For 3 MHz, this corresponds to $\lambda = 100$ m in vacuum, possibly modified by the change in the refraction index. To have good spatial resolution, a cell size of at most $\sim10$ m size
TABLE II Conductivity changes

<table>
<thead>
<tr>
<th>Component</th>
<th>HF Frequency</th>
<th>$\Delta\sigma/\sigma$ Nov 03</th>
<th>$\Delta\sigma/\sigma$ Feb 07</th>
<th>Increase in $\Delta\sigma/\sigma$ after upgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedersen</td>
<td>3 MHz</td>
<td>0.31611</td>
<td>0.10065</td>
<td>-68.2%</td>
</tr>
<tr>
<td>Hall</td>
<td>3 MHz</td>
<td>-0.51829</td>
<td>-0.75078</td>
<td>44.9%</td>
</tr>
<tr>
<td>Parallel</td>
<td>3 MHz</td>
<td>-0.70602</td>
<td>-0.84014</td>
<td>19.0%</td>
</tr>
<tr>
<td>Pedersen</td>
<td>7 MHz</td>
<td>0.28005</td>
<td>-0.081722</td>
<td>-129.2%</td>
</tr>
<tr>
<td>Hall</td>
<td>7 MHz</td>
<td>-0.57227</td>
<td>-0.82585</td>
<td>44.3%</td>
</tr>
<tr>
<td>Parallel</td>
<td>7 MHz</td>
<td>-0.74023</td>
<td>-0.87578</td>
<td>18.3%</td>
</tr>
</tbody>
</table>

is needed. For a region of height of ~40 km and radius of ~20 km, this means ~8 $\times$ 10$^6$ cells for a 2D cylindrical model. This is achievable with modern computer technology, but requires a lot of resources. For a 3D model, this would require ~5 $\times$ 10$^{10}$ cells, which is not easily achievable yet.

To make the calculations more resource efficient, and to achieve the feasibility for full 3D calculations, we are going to develop approximate methods, such as WKB, in which the cell size is determined by the scale of inhomogeneities and/or the wavelength corresponding to the ELF/VLF modulation frequency, instead of the wavelength of the carrier HF wave. This method concentrates on the calculation of the energy transfer and the envelope of the HF wave. This is the method that has been used in all current models even in a 1D case (Milikh et al., 1999; Papadopoulos et al., 2003; Tomko, 1981). The accuracy of the approximate methods can be tested with an FDTD calculation in a 1D model. A further computational enhancement can be achieved by dynamically changing the cell sizes. As an initial step, we performed 1D self-consistent WKB calculations with details described in the Appendix B.

Earlier one-dimensional simulations (such as Tomko, 1981) do not take into account the electromagnetic wave phase changes, and therefore the wave front changes. It is argued in Appendix B that these can be important for the electric field amplitude changes. For example, there are significant differences in $D$ region heating by a plane HF wave and a wave from a point source, as shown in Figure 17 for extraordinary mode heating at $f = 7$ MHz by upgraded HAARP. The curving of ray paths, or lensing of the HF beam would produce changes of similar magnitude. Our proposed 2D and 3D models will include all possible changes in the ray paths that are necessary for accurate calculations. This is even more important when we extend our model into the $F$ region, as the HF ray deflection from straight lines is significant (Rietveld et al., 1993).

Another aspect that requires simulations in more than one dimensions are the effects of the geomagnetic field. Since the HAARP HF beam can be pointed over a wide range of angles by appropriate phasing of the array dipoles, the can have an arbitrary angle with the geomagnetic field. This situation requires a 3D model.

D. ELF/VLF wave generation

The low-frequency (DC) conductivity of the ionosphere is modified by the HF wave, as described in Appendix C. When using the non-Maxwellian electron distribution function, it is important to use an accurate expression for conductivity (C1), instead of an approximate expression (C2) employed in Maxwellian and other approximate models. The relative conductivity changes for maximum HAARP emission before and after the upgrade, at the altitude of 80 km, are shown by the black intervals in Figures 18 and 20. For convenience, the changes of different components are summarized in Table II. The peculiar behavior of the Pedersen component (which increases then decreases with the HF power) is explained by the fact that it is $\sim \nu_m/(\nu_m^2 + \omega_H^2)$, as shown in (C1–C2). Here $\nu_m$ is the collision frequency which always grows with HF power, and $\omega_H = \text{const}$ is the electron gyrofrequency.

The electrojet current will be calculated using the given electrojet field and modified conductivity, $\mathbf{J} = \sigma_{\text{DC}} \mathbf{E}$. The ELF/VLF modulation of the HF wave, as shown in Figure 8, will create periodic changes of conductivity in an illuminated horizontal area of an ionospheric layer between 70 and 120 km. The electrojet current will have a component modulated at the same ELF/VLF frequency, filling this volume. We propose to use both FDTD (Rowland, 1999) and analytical methods (Papadopoulos et al., 2003; Zhou et al., 1996) to calculate the electromagnetic radiation from this current component.

E. Extension of the model into upper $E$ and $F$ regions

The investigation of HF heating effects in upper $E$ and $F$ regions poses an important physics problem. For example, such a study will help find an explanation to the recently discovered phenomenon of optical emission enhancement (Pedersen et al., 2003). The physics of HF heating of the upper $E$ and $F$ regions involves dealing with properties
different from the $D$ region. These include changes in the composition of the atmosphere, increase in free electron and ion density, and further decrease of collision frequencies with neutrals. These properties require investigation of new aspects such as collective plasma phenomena and electron interactions with electrons and ions. These change the electron distribution function, as suggested in Appendix A.5. The HF wave propagation study can be done using the same methods as proposed in Section II.C. Due to high electron density, it will be essential to include self-modification of the conductivity tensor by the HF wave as well as the deflection of HF rays from straight paths.

III. STATEMENT OF WORK (SOW)

The tasks to be undertaken under the proposed program are theoretical investigation and creation of an HF heating software tool.

A. Theoretical tasks

1. Theoretical investigation of modifications of electron distribution function and background conductivity by HF heating of $D$ and lower $E$ regions of ionosphere, using a 1D HF propagation model.

2. Theoretical investigation of multidimensional effects related to the deflection of the HF rays from straight paths and propagation in arbitrary direction in respect to the vertical and the geomagnetic field using 2D and 3D HF propagation models.

3. Theoretical investigation of generation of ELF/VLF electromagnetic emission by modulated electrojet current.

4. Study of HF heating effects up to 300 km (in the $F$ region) by the use of 2D and 3D models.

5. Calculation of optical emissions stimulated by HF heating.

The completion of these tasks will result in scientific publications in refereed journals.

B. Creation of an HF heating software tool

This general and easy-to-use tool will have the following capabilities:
1. Easy-to-use GUI (graphical user interface).

2. Capability of multidimensional calculations, including arbitrary direction of the HF beam in respect to the vertical and geomagnetic field.

3. Inputs
   - background electron density
   - electrojet field
   - heating beam properties: frequency, ERP and radiation pattern, modulation frequency and pattern.

4. Final outputs:
   - ELF/VLF field on ground and in space
   - optical emissions

5. Intermediate outputs, useful for ionosphere diagnostic purposes:
   - ionosphere conductivity modifications, up to $F$ region (0–300 km)
   - ionospheric currents at the harmonics of the modulation frequency (0–300 km)

The completion of this task will result in a software tool facilitating work of other researchers.

IV. MILESTONE SUMMARY

The completion of the theoretical tasks and creation of the HF heating software tool is going to take place in the following stages over three years:

- **Year 1.** Calculate of the electron distribution function by solving the kinetic equation for various electric fields, atmosphere densities and angles between the Poynting vector and the geomagnetic field, and tabulate the obtained conductivity tensor values.

- **Years 1-2.** Use finite-difference time domain (FDTD) method or similar methods to calculate the self-consistent propagation of HF waves; creation of a 1D model for propagation of the HF wave vertically along the geomagnetic field. Study of WKB methods of solution and comparison with FDTD model.

- **Year 2.** 2D and 3D models for propagation of the HF wave. Study of the effects of the geomagnetic field and possible deflection of rays from straight paths. Extension of the model into upper regions of ionosphere. Study of effects in $E$ and $F$ regions, such as optical emissions (Pedersen et al., 2003).

- **Year 3.** Use the obtained conductivity tensor values at each point in space and time to calculate the generation of ELF/VLF waves by the modulation of the polar electrojet current.

V. RESEARCH PERSONNEL AND FACILITIES

The proposed research will be carried out using dedicated computing facilities at Stanford University Electrical Engineering Department by Professor U. S. Inan (PI) and Drs. T. F. Bell and N. G. Lehtinen. Drs. Inan, Bell and Lehtinen have long standing experience in theoretical investigation of ionosphere. Their participation, therefore, is essential to the success of the proposed research program. The investigation of the proposed problem will involve extensive computer modelling with a possible use of supercomputer facilities. Professor Inan’s research group is currently involved in a number of experimental efforts involving wave generation by HAARP.

VI. PRELIMINARY RESULTS

The Appendix discusses the proposed model in detail and presents results of the new 1D non-Maxwellian HF heating.
APPENDIX A: Kinetic equation

1. Modifications to ELENDIF

ELENDIF (Morgan and Penetrante, 1990) calculates the electron distribution function in a mixture of partially ionized gases which may or may not be under the influence of an external quasi-static electric field. This is done by solving the kinetic equation, with an assumption that a two-term spherical harmonic expansion of the electron distribution function is adequate. The code can treat

1. Quasistatic electric field
2. Elastic scattering on neutrals and ions
3. Inelastic and superelastic scattering
4. Electron-electron collisions
5. Attachment and ionization
6. Photon-electron processes
7. External electron sources.

We also propose to use the two-term spherical harmonic expansion for the problem of HF heating. However, to do this we have to modify ELENDIF to include

1. Non-static electric field
2. Geomagnetic field

Below we derive the modifications that have to be made to the original ELENDIF code to include these items.

Let \( f(v,t) \) be the electron distribution function in velocity space, normalized so that \( \int f(v,t) d^3v = N_e \). The kinetic equation for \( f(v,t) \) is

\[
\frac{\partial f}{\partial t} - \frac{e}{m} (E + v \times B_E) \cdot \nabla_v f = \left( \frac{\partial f}{\partial t} \right)_c \tag{A1}
\]

Here \( e = -q_e > 0 \), \( E = E(t) \) is the electric field of the wave and \( B_E = const \) is the local geomagnetic field. We neglect the action of the magnetic field of the wave on the non-relativistic electrons, since this is small compared to the action of electric field.

We use the fact that \( f(v,t) \) is not excessively anisotropic to expand it in a spherical harmonic series:

\[
f(v,t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y_l^m(\Omega) f_l^m(v,t)
\]

where \( v = |v| \) and \( \Omega = (\theta, \phi) \) determines the direction of \( v \), \( \theta \) and \( \phi \) being the polar and azimuthal angle. The spherical harmonics are

\[
Y_l^m(\Omega) = Y_l^m(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos \theta)e^{im\phi}
\]

where the associated Legendre polynomials \( P_l^m \) are given by

\[
P_l^m(\cos \theta) = \frac{(-1)^m}{2^l l!} (\sin \theta)^m \frac{d^{l+m}}{d(\cos \theta)^{l+m}} (\sin \theta)^{2l}
\]

\[
P_l^{-m}(\cos \theta) = (-1)^m \frac{(l-m)!}{(l+m)!} P_l^m(\cos \theta)
\]

where \( m \geq 0 \). The spherical harmonics satisfy the orthonormality relation:

\[
\int_{4\pi} Y_l^m(\Omega)(Y_{l'}^{m'}(\Omega))^* d\Omega = \delta_{mm'} \delta_{ll'}
\]
We neglect all terms except \( l = 0 \) and \( l = 1 \) and use the explicit formulas

\[
Y_0^0(\theta, \phi) = \frac{1}{\sqrt{4\pi}} \\
Y_{-1}^1(\theta, \phi) = \sqrt{\frac{3}{8\pi}} \sin \theta e^{-i\phi} \\
Y_0^1(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos \theta \\
Y_{-1}^1(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}
\]

and therefore look for the solution in the form

\[
f(v, t) = \frac{1}{\sqrt{4\pi}} f_0^0(v, t) + \sqrt{\frac{3}{4\pi}} \left[ f_1^0(v, t) \cos \theta + \left( \frac{e^{-i\phi} f_{-1}^1(v, t) + e^{i\phi} f_1^1(v, t)}{\sqrt{2}} \right) \sin \theta \right]
\]

This can be written as (Gurevich, 1978; Tomko, 1981)

\[
f(v, t) = f_0(v, t) + \hat{v} \cdot \mathbf{f}_1(v, t) \tag{A2}
\]

where \( \hat{v} = v/v = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta \) is the unit vector in the direction of \( v \) and

\[
f_0(v, t) = \frac{1}{\sqrt{4\pi}} f_0^0 \\
f_1(v, t) = \sqrt{\frac{3}{4\pi}} \left( f_{-1}^1 + f_1^1 \right) \frac{1}{\sqrt{2}} \hat{x} + \frac{-i f_{-1}^1 + i f_1^1}{\sqrt{2}} \hat{y} + f_0^1 \hat{z}
\]

We substitute the expansion (A2) into (A1) and use the orthogonality of spherical harmonics to obtain equations for \( f_0 \) and \( f_1 \). We use the following useful relations:

\[
\nabla_v (\hat{v} f_1) = \vec{1}_1 \frac{f_1}{v} + \vec{1}_\parallel \frac{\partial f_1}{\partial v} \\
\langle \vec{1}_\parallel A \rangle = \langle \hat{v} (\hat{v} \cdot A) \rangle = \frac{1}{3} A \\
\langle \vec{1}_\perp A \rangle = A - \frac{1}{3} A = \frac{2}{3} A
\]

where \( A \) is an arbitrary vector independent of \( \Omega \), and \( \vec{1}_1 \) and \( \vec{1}_\parallel \) are operators projecting onto the plane perpendicular to \( \hat{v} \) and on the direction of \( \hat{v} \), so that \( \vec{1}_\perp + \vec{1}_\parallel = \vec{1} \), the unit operator.

Averaging (A1) over \( \Omega \) (equivalent to taking a projection onto \( Y_0^0 \)), we get

\[
\frac{\partial f_0}{\partial t} - \frac{e \mathbf{E}}{m} \cdot \left( \frac{2 f_1}{3v} + \frac{1 \partial f_1}{\partial v} \right) = \left( \frac{\partial f}{\partial t} \right)_c \tag{A3}
\]

Multiplying (A1) by \( \hat{v} \) and averaging over \( \Omega \) is equivalent to taking a projection onto \( Y_{-1}^{1.1.1} \). After this operation, we get

\[
\frac{1}{3} \frac{\partial f_1}{\partial t} - \frac{1}{3} \frac{e}{m} \left( \mathbf{E} \frac{\partial f_0}{\partial v} + \mathbf{B} \times \mathbf{f}_1 \right) = \left( \hat{v} \left( \frac{\partial f}{\partial t} \right) \right)_c \tag{A4}
\]

We treat the collision integral as described in (Raizer, 1997, ch. 5). It consists of elastic and inelastic collisions:

\[
\left( \frac{\partial f}{\partial t} \right)_c = \left( \frac{\partial f}{\partial t} \right)_{ec} + \left( \frac{\partial f}{\partial t} \right)_{ic}
\]

The elastic collisions only change the particle direction (there is a small elastic loss term which will be considered later):

\[
\left( \frac{\partial f}{\partial t} \right)_{ec} = \nu_e(v) \int_{\Omega'} d\Omega' q(\theta) (f(\Omega') - f(\Omega))
\]
FIG. 9 Elastic collision frequency for different electron energies as a function of altitude

where \( \nu_c(v) \) is the collision rate and

\[
q(\theta) = \frac{1}{2\pi\sigma_{el}} \frac{d\sigma_{el}}{d(\cos \theta)}
\]

is the probability of scattering into angle \( \theta \) per unit solid angle.

We can calculate that

\[
\left\langle \frac{\partial f}{\partial t} \right\rangle_{ec} \Omega = \left\langle \left( \frac{\partial f_0}{\partial t} \right)_{ec} \right\rangle_\Omega = 0
\]

\[
\left\langle \dot{\nu} \left( \frac{\partial f}{\partial t} \right) \right\rangle_{ec} \Omega = \left\langle \dot{\nu} \left( \dot{\nu} \cdot \left( \frac{\partial f_1}{\partial t} \right) \right) \right\rangle_\Omega = -\frac{1}{3} \nu_m f_1
\]

where \( \nu_m = \nu_c(1 - \cos \theta) \) is the elastic momentum transfer rate. It is equal to \( \nu_c \) for isotropic scattering and usually is a better measure of elastic collision frequency than \( \nu_c \) for anisotropic scattering. The last equation can include the inelastic scattering, too. In this case, \( \nu_m \) is to be understood as the total momentum transfer rate.

The equation for \( f_1 \) can be rewritten so that it is expressed in terms of \( f_0 \):

\[
\frac{\partial f_1}{\partial t} + \nu_m f_1 - \omega_H \hat{z} \times f_1 = \frac{eE}{m} \frac{\partial f_0}{\partial v}
\]  
(A4)

where \( \omega_H = eB_E/m \) is the electron gyrofrequency, and \( \hat{z} \) is the direction of \( B_E \).

This equation suggests that the use of the original version of ELENDIF is inappropriate, because it assumes

\[
\nu_m f_1 = \frac{eE}{m} \frac{\partial f_0}{\partial v}
\]

which is valid only if \( \nu_m \gg \omega \) and \( \nu_m \gg \omega_H \) where \( \omega \) is the characteristic time scale of change of the electric field. In Figure 9, we plot the elastic collision frequency \( \nu_m \) as a function of altitude. The Figure shows that at altitudes \( \geq 70-80 \) km these assumptions are not valid.

Let us assume \( E = \text{Re}Ee^{-i\omega t} \) and \( f_1 = \text{Re}\tilde{f}_1e^{-i\omega t} \). The isotropic part \( f_0 \) is assumed to change very little during the period of oscillations. Then (A4) is the real part of the following equation:

\[
(\nu_m - i\omega)\tilde{f}_1 - \omega_H \tilde{z} \times \tilde{f}_1 = \tilde{A}(\omega)\tilde{f}_1 = \frac{eE}{m} \frac{\partial f_0}{\partial v}
\]
so that

$$
\tilde{f}_1 = \tilde{A}^{-1}(\omega) \tilde{E} \frac{e}{m} \frac{\partial f_0}{\partial v}
$$

where

$$
\tilde{A}(\omega) = (\nu_m - i\omega) \mathbf{I} - \omega_H \tilde{T}
$$

Here we introduced one more operator $\tilde{T}$ defined by $\tilde{T} \mathbf{A} = \hat{z} \times \mathbf{A}$. This operator has the properties $\tilde{T} \mathbf{I} = 0$, $\tilde{T}^2 = -\mathbf{I}$, $\tilde{T} \mathbf{I} = \mathbf{T}$ (where now $\mathbf{I}$, $\mathbf{I}$ are in respect to $\hat{z}$, the direction of $\mathbf{B}_E$). In the coordinates it is written as

$$
\tilde{T} = \begin{pmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

To find the inverse of $\tilde{A}$, we use the fact that for arbitrary complex numbers $\alpha$, $\beta$ ($\alpha \neq 0$)

$$
(\alpha \mathbf{I} + \beta \tilde{T})^{-1} = \frac{\alpha}{\alpha^2 + \beta^2} \mathbf{I} - \frac{\beta}{\alpha^2 + \beta^2} \tilde{T} + \frac{1}{\alpha} \mathbf{I} = \frac{1}{\alpha} \mathbf{I} + \frac{1}{\alpha - i\beta} \mathbf{I}_+ + \frac{1}{\alpha + i\beta} \mathbf{I}_-
$$

Here we introduced projection operators $\mathbf{I}_+ = (\mathbf{I} + i\tilde{T})/2$ and $\mathbf{I}_- = (\mathbf{I} - i\tilde{T})/2$ on counterclockwise (positive, $\hat{x} + i\hat{y}$) and clockwise ($\hat{x} - i\hat{y}$) directions, correspondingly. We have

$$
\tilde{A}^{-1}(\omega) = \frac{1}{\nu_m - i(\omega - \omega_H)} \mathbf{I}_+ + \frac{1}{\nu_m - i(\omega + \omega_H)} \mathbf{I}_- + \frac{1}{\nu_m - i\omega} \mathbf{I}
$$

(A5)

For propagation along $\mathbf{B}_E$, the counter-clockwise $\tilde{E}_+ = \tilde{I}_+ \mathbf{E}$ and clockwise $\tilde{E}_- = \tilde{I}_- \mathbf{E}$ electric field components are usually referred to as extraordinary and ordinary modes. Note that the extraordinary mode has a resonance at $\omega = \omega_H$.

Let us now turn to the solution of (A3), which becomes

$$
\frac{\partial f_0}{\partial t} - \frac{e}{3mv^2} \frac{\partial (v^2 \mathbf{E} \cdot \mathbf{f}_1)}{\partial v} = Q(f_0)
$$

where $Q(f_0) = \left\langle \left( \frac{\partial \mathbf{f}_1}{\partial \mathbf{E}^*} \right)_{\mathbf{E}_c} \right\rangle$ is the inelastic collision integral.

When substituting $\mathbf{f}_1$, we have to find the average of the product $\mathbf{E} \cdot \mathbf{f}_1$ over a period of oscillations. It is written in terms of complex amplitudes as $\frac{1}{2} \text{Re} (\mathbf{E}^* \cdot \mathbf{f}_1)$:

$$
\frac{\partial f_0}{\partial t} - \frac{e^2}{6mv^2} \frac{\partial}{\partial v} \left( \text{Re} \left( \mathbf{E}^* \cdot \tilde{A}^{-1}(\omega) \mathbf{E} \right) v^2 \frac{\partial f_0}{\partial v} \right) + Q(f_0)
$$

Let us switch from $f_0(v)$ to

$$
n(\mathbf{E}) = 2\pi \left( \frac{2}{m} \right)^{3/2} f_0(\mathbf{E}) \mathbf{E}^{1/2}
$$

which is normalized so that $\int n(\mathbf{E}) \mathbf{E} = N_0$. The kinetic equation for $n$ is (using also $\partial/\partial v = \sqrt{2mE} \partial/\partial \mathbf{E}$):

$$
\frac{\partial n}{\partial t} = \frac{\partial}{\partial \mathbf{E}} \left( \mathbf{D} \mathbf{E}^{3/2} \frac{\partial}{\partial \mathbf{E}} \frac{n}{\mathbf{E}^{1/2}} \right) + Q(n)
$$

(A6)

where the inelastic collision integral is

$$
Q(n) = \sum_s N_s [R_s(\mathbf{E} + \mathbf{E}_s)n(\mathbf{E} + \mathbf{E}_s) - R_s(\mathbf{E})n(\mathbf{E})]
$$

(A7)

The sum is taken over different inelastic processes, with rates $R_s(\mathbf{E}) = v\sigma_s(\mathbf{E})$, where $v$ is the velocity and $\sigma_s$ are the inelastic cross-sections.
An equation similar to (A6) was used, e.g., by Papadopoulos et al. (1990). The "diffusion coefficient" in energy space due to elastic collisions is given by

$$D(\mathcal{E}) = \frac{e^2 \text{Re} \left( \mathbf{E}^* \cdot \mathbf{A}^{-1}(\omega) \mathbf{E} \right)}{3m}$$  \hspace{1cm} (A8)

For constant electric field, equation (A8) becomes

$$D(\mathcal{E}) = \frac{2e^2 \mathbf{E} \cdot \mathbf{A}^{-1}(0) \mathbf{E}}{3m}$$

where the factor of 2 comes from differences in time averaging of oscillating and constant cases.

In electric field components,

$$D(\mathcal{E}) = D_+(\mathcal{E}) + D_-(\mathcal{E}) + D_z(\mathcal{E}) = \frac{e^2 |\mathbf{E}_+|^2}{3m} \frac{\nu_m}{(\omega - \omega_H)^2 + \nu_m^2} + \frac{e^2 |\mathbf{E}_-|^2}{3m} \frac{\nu_m}{(\omega + \omega_H)^2 + \nu_m^2} + \frac{e^2 |\mathbf{E}_z|^2}{3m} \frac{\nu_m}{\omega^2 + \nu_m^2}$$ \hspace{1cm} (A9)

where $\mathbf{E}_{\pm,z} = \mathbf{E}_{\pm,z}$ for each component

$$D_{\pm,z} = \frac{e^2 |\mathbf{E}_{\pm,z}|^2}{3m} \frac{\nu_m}{\omega_{\text{eff}}^2 + \nu_m^2}$$

where $\omega_{\text{eff}} = \omega \mp \omega_H$ for $\mathbf{E}_\pm$ field and $\omega_{\text{eff}} = \omega$ for $\mathbf{E}_z$ field. With elastic energy losses from collisions with molecules of mass $M$ and temperature $T$ the kinetic equation becomes (Morgan and Penetrante, 1990; Raizer, 1997):

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial \mathcal{E}} \left( \left( D + \frac{2m}{M} \nu_m T \right) \mathcal{E}^{3/2} \frac{\partial n}{\partial \mathcal{E}} + \frac{2m}{M} \nu_m E_n \right) + Q(n)$$ \hspace{1cm} (A10)

The collision integral $Q(n)$ for $T \neq 0$ includes superelastic terms, describing the gain of energy by electrons from excited molecules. The original version of ELENDIF (Morgan and Penetrante, 1990) uses the same equation (A10) except for the expression for $D(\mathcal{E})$:

$$D_{\text{ELENDIF}}(\mathcal{E}) = D(\mathcal{E})_{\omega_{\text{eff}} = 0} = \frac{2e^2 |\mathbf{E}|^2}{3m\nu_m}$$

Comparing to (A9), we see that the oscillatory electric field has approximately the same effect as in the DC case, but with a reduced effective value:

$$E_{\text{eff}} = \frac{E}{\sqrt{1 + \left( \frac{\omega_{\text{eff}}}{\nu_m \omega_{\text{eff}}} \right)^2}}$$ \hspace{1cm} (A11)

with some effective (average) value of collision frequency $\nu_{m,\text{eff}}$. When $\nu_m \ll \omega_{\text{eff}}$, the reduction is by a factor of $\nu_{m,\text{eff}} / \omega_{\text{eff}}$.

The results of the modified ELENDIF calculations are presented in Figure 10. One can see that $\omega_{\text{eff}} \neq 0$ effectively corresponds to a lower DC electric field, as suggested by equation (A11). In all presented calculation results for HF field, $E$ denotes the root-mean-square (RMS) value. We take the value of the electron gyrofrequency to be $f_H = 1$ MHz.

2. Scaling of kinetic equation

Equation (A10) has a scaling property; it transforms into itself if we change $\mathbf{E} \rightarrow C \mathbf{E}$, $N \rightarrow CN$, $\omega_{\text{eff}} \rightarrow C \omega_{\text{eff}}$, $t \rightarrow Ct$ for arbitrary scaling factor $C$. To study the properties of electric discharges, researchers usually employ a quantity known as reduced electric field $E/N$ measured in Townsends, units defined in equation (1). For variable fields, we also introduce a reduced frequency

$$\omega_r = \frac{\omega_{\text{eff}} / (s^{-1})}{N/(10^{13} \text{ m}^{-3})}$$ \hspace{1cm} (A12)

The numerical coefficient is chosen for convenience so that the typical microscopic time scales are of the order of 1 (e.g., maximum value of $\nu_m / (N/10^{13})$ is $\approx 4$). The numerical value of $\omega_r$ is convenient in another way because

$$\omega_r = \frac{f}{\text{MHz}} \text{ at } h_0 \approx 91 \text{ km}$$ \hspace{1cm} (A13)
FIG. 10 Comparison of electron distributions for various RMS values of $E/N$, given in Td, for extraordinary mode heating at an altitude of $h_0 = 91$ km by an HF wave at $f = 3$ MHz (dashed lines) and $f = 7$ MHz (dash-dotted lines). The corresponding reduced frequency (A12) is $\omega_{r,\text{eff}} = 2$ and 6. The solid lines show solutions for constant electric field, in the absence of an ambient magnetic field. The sharp drop at $\sim 2.5$ eV is due to electron energy losses to $N_2$ vibrational excitations (Vlasov et al., 2004).

3. Analytical solutions of kinetic equation

Under assumption of a constant free path length $l = v/\nu_m = (N\sigma_m)^{-1} = \text{const}$ and absence of inelastic collisions, an analytical solution of (A10) (for any one particular mode) exists. The electrons for $T = 0$ have a so-called Margenau distribution (Margenau, 1946; Raizer, 1997):

$$f_0(v) = C \exp \left[ -\frac{3m^3}{4Mc^2E^2l^2}(v^4 + 2v^2\omega_{\text{eff}}^2l^2) \right]$$

where $E^2 = \langle |E|^2 \rangle = \langle \tilde{E}_\pm^2 \rangle/2$ is the square of the RMS field and $\omega_{\text{eff}} = \omega \mp \omega_H$. For $\omega_{\text{eff}} = 0$, it turns into a Druyvesteyn distribution (Chapman and Cowling, 1995, p. 387), which is significantly different in shape from a Maxwellian distribution.

4. Electric breakdown

To correctly calculate the electron distribution, the ionization and attachment should enter as terms in the collision integral (A7), as shown by Yoshida and Phelps (1983). The calculated ionization $\nu_i$ and attachment $\nu_a$ rates are presented in Figure 11.

As suggested by equation (A11), the higher electric field frequencies require higher amplitudes to get the same electric distribution, which explains the shifting of the curves for $\nu_i$ and $\nu_a$ to higher fields for higher frequencies. The electric breakdown corresponds to the condition $\nu_i > \nu_a$, which occurs at $E > E_{\text{br}}$. The dependence of $E_{\text{br}}/N$ on the effective frequency is presented in Figure 12. We can use an approximate expression

$$\frac{E_{\text{br}}}{N} \approx (113.7 \text{ Td}) \sqrt{1 + \left( \frac{\omega_r}{2.0} \right)^2} \quad \text{within 6\% error} \quad (A14)$$

with $\omega_r$ given by (A12). This equation, by the way, follows from (A11), if we assume $\nu_{m,\text{eff}}/N \approx 2 \times 10^{-13} \text{ s}^{-1}\text{m}^3$. 
FIG. 11 Attachment and ionization rates for extraordinary mode heating at \( f = 3 \text{ MHz} \) (dashed lines) and \( f = 7 \text{ MHz} \) (dash-dotted lines) at altitude \( h_0 = 91 \text{ km} \) (the corresponding reduced frequency (\( \lambda_{A12} \)) is \( \omega_{r, \text{eff}} = 2 \) and 6). For comparison, the rates for DC field without ambient magnetic field is also shown (solid lines). The red circles show the points at which \( \nu_a = \nu_i \), a condition that determines the onset of electric breakdown.

FIG. 12 RMS electric breakdown field as a function of the effective frequency.

5. Electron-electron and electron-ion collisions

The electron-electron and electron-ion collisions become important when the background ionization level becomes high, which happens in the \( F \) layer of ionosphere. The electron-ion (or electron-electron, which is of the same order) collision rate \( \nu_{ei} \) is given by

\[
\nu_{ei}(E) = 4\pi N \nu b_0^2 \log \frac{\lambda_D}{b_0}, \quad b_0 = \frac{e^2}{2\pi \varepsilon_0 E}
\]
where \( \lambda_D = v_{th}/\omega_p \) is the Debye radius, \( v_{th} = \sqrt{T_{ee}/m} \) is a typical (thermal) electron velocity, \( v = \sqrt{2E/m} \). The electron-neutral collision rate \( \nu_m \) is compared to \( \nu_{ei} \) in Figure 13. We see that in \( D \) region we can safely neglect the electron-electron and electron-ion collisions, which was done in the present calculations. However, these processes will be included in the extension of our model to the \( F \) region.

Our modified ELENDIF code is capable of description of both electron-electron and electron-ion collisions (Morgan and Penetrante, 1990). The electron-ion collisions are described as elastic collisions with a Coulomb cross-section, while for electron-electron collision there is a special additional term in the collision integral. Electron-electron collisions play a role in bringing the electron distribution to a Maxwellian shape (Morgan and Penetrante, 1990).

6. Conductivity

The conductivity tensor for the HF wave is expressed through the anisotropic part of the distribution function:

\[
\mathbf{J} = \vec{\sigma}_{HF} \mathbf{E} = -e \int v(\dot{\mathbf{v}} \cdot \mathbf{f}_1(v)) d^3v
\]

from which we obtain the conductivity tensor

\[
\vec{\sigma}_{HF} = -\frac{4\pi e^2}{3m} \int \vec{A}^{-1}(\omega)v^3 \frac{\partial f_0}{\partial v} dv
\]

where \( \vec{A}^{-1}(\omega) \) is given by (A5). In terms of \( n(E) \),

\[
\vec{\sigma}_{HF} = -\frac{2e^2}{3m} \int \vec{A}^{-1}(\omega)E^{3/2} \frac{\partial}{\partial E} \frac{n}{E^{1/2}} dE
\]

For extraordinary and ordinary waves and the parallel component

\[
\sigma_{HF,\pm} = -\frac{2e^2}{3m} \int \frac{E^{3/2}}{\nu_m - i(\omega \mp \omega_H)} \frac{\partial}{\partial E} \frac{n}{E^{1/2}} dE
\]

(A15)

\[
\sigma_{HF,z} = -\frac{2e^2}{3m} \int \frac{E^{3/2}}{\nu_m - i\omega} \frac{\partial}{\partial E} \frac{n}{E^{1/2}} dE
\]

(A16)
The usual conductivity tensor representation in Pedersen, Hall and parallel components is

$$\tau_{\text{HF}} = \begin{pmatrix} \sigma_{\text{HF},p} & -\sigma_{\text{HF},h} & 0 \\ \sigma_{\text{HF},h} & \sigma_{\text{HF},p} & 0 \\ 0 & 0 & \sigma_{\text{HF},z} \end{pmatrix}$$  \hspace{1cm} (A17)

which can be written in our notation as $\tau_{\text{HF}} = \sigma_{\text{HF},p} \mathbf{I} \perp + \sigma_{\text{HF},h} \mathbf{I}^\perp + \sigma_{\text{HF},z} \mathbf{I} \parallel$, with $\sigma_{\text{HF},p} = \frac{1}{2} (\sigma_{\text{HF},+} + \sigma_{\text{HF},-})$ and $\sigma_{\text{HF},h} = \frac{i}{2} (\sigma_{\text{HF},+} - \sigma_{\text{HF},-})$.

**APPENDIX B: HF wave propagation**

The power flux $S$ in the stationary plane wave in a 1D model is given by the differential equation

$$\frac{dS}{dz} = -\alpha(S, z)S$$  \hspace{1cm} (B1)

where the absorption coefficient $\alpha$ includes self-absorption through dependence on $S$. It is given by $\alpha = 2 \text{Im} k$ where the wave number $k$ has the value

$$k = \frac{\omega}{c} \sqrt{1 + \frac{\omega}{\omega_0}}$$

Instead of $\sigma$, we substitute $\sigma_{\pm}$ from (A15) for extraordinary or ordinary mode. The value of $\sigma_{\pm}$ depends on HF power through distribution $n(E)$.

The effects of the spreading of the beam can be easily included in this simple 1D model, assuming that the propagation rays are straight lines. The stationary solution for the average Poynting vector $S$ satisfies

$$\nabla \cdot S = -(2 \text{Im} k) |S|$$

Assuming $S = |S| \hat{R}$, where $\hat{R}$ is the unit vector in the direction from the source, we have for $S = |S|$ the same equation (B1), except that

$$\alpha = 2 \text{Im} k + \frac{2}{R}$$

with $R$ being the distance from the source.

We performed calculations using a daytime electron density profile (Figure 5). Figures 14 and 15 show the calculated electric field RMS values for the extraordinary mode for maximum HAARP ERP values at the present time and after the upgrade (see Figure 1), for HF emission at 3 MHz and 7 MHz. For the 3 MHz case the decrease in electric field due to (self-)absorption is evident. Figure 15 shows also electric field calculated without the beam spreading taken into account (in green color). The initial energy flux in the case without spreading is chosen to be $S_0 = ERP/(4\pi h^2)$ with $h = 90$ km.

Although comparison of the calculated fields to Figure 3 suggests that fields get close to the electric breakdown threshold, this is not true because the high frequency of the field effectively reduces it. A better measure would be the fraction $E/E_{br}$, plotted in Figures 6 and 7. These figures show that the electric field generated by HAARP in D region is well below the breakdown threshold. This justifies the steady-state assumption made for this calculation, because the electron density practically does not change.

Figures 16 and 17 show the deposited energy and the effective electron temperature as a function of altitude, for the same cases as in Figures 14 and 15, respectively. The effective temperature of a non-Maxwellian distribution can be defined as $T_{\text{eff}} = \frac{T_E}{\tau_E}$. Although including the beam spreading does not seem to produce much effect in Figure 15, the change shown in Figure 17 is more dramatic. Since the power densities are chosen to be the same at $h = 90$ km, the plane wave produces less heating below and more heating above 90 km. The curving of ray paths, or lensing of the HF beam would produce changes of similar magnitude. Our proposed 2D and 3D models will include all possible changes in the ray paths, necessary for accurate calculations of energy deposition and changes in $T_{\text{eff}}$.

**APPENDIX C: Electrojet current modulation**

The electrojet current is modulated because the DC conductivity is modulated. To find the DC conductivity, we must start with equation (A4) and substitute the electric field consisting of both the DC field of the electrojet current
FIG. 14 The $E/N$ ratio at maximum ERP for present and upgraded HAARP, at HF frequency of 3 MHz in the extraordinary mode.

FIG. 15 The $E/N$ ratio at maximum ERP for present and upgraded HAARP, at HF frequency of 7 MHz in the extraordinary mode. Black—with beam spreading (i.e., a wave from a point source), green—without beam spreading (i.e., a plane wave). The point-source and plane wave power densities are chosen to be the same at $h = 90$ km.

and the HF field of the heating wave, i.e. $\mathbf{E} = \mathbf{E}^{(0)} + \text{Re} \mathbf{E} e^{-i\omega t}$. We look for a solution in the form $f_1 = f_1^{(0)} + \text{Re} \tilde{f}_1 e^{-i\omega t}$. The expression (A8) is modified to

$$D(\xi) = \frac{e^2 \text{Re} (\mathbf{E}^* \cdot \mathbf{A}^{-1}(\omega) \mathbf{E})}{3m} + \frac{2e^2 \mathbf{E}^{(0)} \cdot \mathbf{A}^{-1}(0) \mathbf{E}^{(0)}}{3m}$$
FIG. 16 The deposited power and change in effective electron temperature (defined in the text) for steady-state HF heating described in Figure 14.

FIG. 17 Same as Figure 16, for cases of Figure 15 (extraordinary mode, $f = 7$ MHz). Black—with beam spreading (i.e., a wave from a point source), green—without beam spreading (i.e., a plane wave). The point-source and plane wave power densities are chosen to be the same at $h = 90$ km.

If the electroject field is small compared to the HF field, we can actually neglect the change in $D(\mathbf{E})$ and use the same solution for $n(\mathbf{E})$ as for the HF field only. The DC conductivity $\bar{\sigma}_{DC}$ is determined from

$$J^{(0)} = \bar{\sigma}_{DC} \mathbf{E}^{(0)} = -e \int \mathbf{v}(\mathbf{v} \cdot \mathbf{f}_1^{(0)}(\mathbf{v})) d^3 \mathbf{v}$$

from which we find

$$\bar{\sigma}_{DC} = -\frac{2e^2}{3m} \int A^{-1}(0) E^{3/2} \frac{\partial}{\partial E} \frac{n}{E^{1/2}} dE$$  (C1)
FIG. 18 A comparison of conductivity changes experienced by the electrojet current at \( h = 80 \) km altitude from heating by extraordinary mode at \( f = 3 \) MHz and 7 MHz. The solid line shows the conductivity changes created by a DC field in the absence of ambient magnetic field (\( \omega_r = 0 \)). The black intervals connect the conductivities modified by maximum heating by HAARP before and after upgrade (the \( E/N \) values calculated for Figures 14 and 15 are used).

with

\[
\vec{A}^{-1}(0) = \frac{\nu_m}{\nu_m^2 + \omega_H^2} \vec{I}_\perp + \frac{\omega_H}{\nu_m^2 + \omega_H^2} \vec{I}_\parallel + \frac{1}{\nu_m} \vec{I}_\|.
\]

We believe that the expression (C1) is more accurate than the approximate expression used previously, e.g., by Papadopoulos et al. (1990) and Tomko (1981):

\[
\bar{\sigma}_{DC}^a = \frac{e^2}{m} \int \vec{A}^{-1}(0) n d\vec{E}. \tag{C2}
\]

The expressions (C1) and (C2) would be equivalent if \( \vec{A}(0) \) were independent of \( \vec{E} \). However, since this is not true, the results must be different. We present results for both equations (C1) and (C2) in Figures 18 and 19, respectively. The Figures show the conductivity components as functions of electric field (for conditions at \( h = 80 \) km), for different HF heating frequencies. The distribution function used in (C1) and (C2) is the stationary solution of kinetic equation (A10), without the change of electron density due to ionization or attachment. The dramatic change due to thermal instability, predicted by Kuo et al. (2002), should be observed at \( E/N = 80 \) Td, but is not visible from the Figures, even for a DC electric field.

From comparison of Figures 18 and 19, it is evident that equation (C2) used in previous works (Papadopoulos et al., 1990; Tomko, 1981) is inaccurate. The relative change in conductivity \( \sigma(E)/\sigma(E = 0) \) is shown in Figure 20. The calculated steady-state conductivities in the \( D \) region for \( f = 3 \) MHz and 7 MHz, for extraordinary mode heating and daytime electron density, are represented in Figures 21 and 22.

References


FIG. 19 Same as Figure 18, except that approximate formula (C2) is used.

FIG. 20 The relative change in conductivity $\sigma(E)/\sigma(E = 0)$ based on data in Figure 18.

FIG. 21 The conductivity changes for DC conductivity tensor components for maximum ERP of present (−o−) and upgraded (−x−) HAARP for extraordinary mode heating at \( f = 3 \) MHz.

FIG. 22 Same as Figure 21, for 7 MHz.


Phelps, A. V. (2005), JILA collision data compilation, ftp://jila.colorado.edu/collision_data/electronneutral.


Rowland, H. L. (1999), Simulations of ELF radiation generated by heating the high-latitude \( d \) region, *J. Geophys. Res. A*, 104,


With Professor Inan of Stanford University an official Co-Investigator on the French DEMETER mission, there is an outstanding opportunity to complement ground-based ELF/VLF measurements around HAARP (in Alaska) and in the conjugate region (Buoy, Tangaroa) with measurements in these regions at 700-km altitude on DEMETER. Initial coordinated observations above HAARP with DEMETER has been very successful, revealing the first six-component measurements of the ELF/VLF signals generated by HAARP in space, from which the Poynting Vector and the total ELF/VLF power produced by HAARP was derived [Platino et al. 2005, to be submitted to JGR; to be presented at the GEM/CEDAR Workshop in Colorado, June 2005]. Excerpts from this paper are attached.

Professor Inan recently attended the first DEMETER Co-Investigators Workshop held at the French Space Agency (CNE) in Paris. The possibilities that this small spacecraft offer for wave-particle experiments in the context of both HAARP and the broader context of radiation belt remediation were clearly evident. DEMETER is without-question-the-best-equipped spacecraft for comprehensive ELF/VLF measurements at LEO since the old Alouette satellites (built by Canada). The data collected is already allowing the investigators to come up with statistically significant descriptions of the ELF/VLF environment at LEO, including signals from natural whistlers, VLF transmitters (very important for RBR), and all kinds of ELF/VLF emissions. The stated purpose of this spacecraft is to identify ELF/VLF signatures of Earthquakes, and remarkably enough, they in fact have some rather tantalizing results. However, in the context of HAARP and RBR, the important aspect is that DEMETER is geared towards collecting comprehensive ELF/VLF statistics, with which we can determine, for example, the ionospheric illumination region above HAARP, which by the way, is much bigger than one would normally think at first (see attached).

The DEMETER spacecraft is one of the most electromagnetically quiet satellites that have recently been launched. The investigators have taken extreme care to achieve this, and everything on the craft is turned OFF during data acquisition periods. In fact, this particular feature was the reason for DEMETER acquisition to turn-OFF at 65-degree latitude, just when HAARP-signals begin to be prominently observed (see attached excerpts from Platino et al. [2005] describing recent observations). All of the solar-panel-alignment type of motions are implemented at latitudes higher than 65-degrees, so that the on-board software automatically turns-off acquisition at 65-degrees. Most other scientific LEO missions, by the way, are almost the opposite, in that their regions of attention is the auroral regions, so they do not collect data at mid-to-low latitudes. When the solar panels are being rotated, the EMI noise generated by the servos is so high that it renders the ELF reception nearly impossible. In any case, DEMETER Project Scientist (Dr. Michel Parrot) has now agreed to modify their on-board software, so as to extend acquisition beyond 65-degrees just for HAARP. This new software modification has been tested, and the
The next group of HAARP-DEMETER experiments (June 2005) will allow the acquisition of data during the entire pass from horizon to horizon.

The DEMETER opportunity is a truly serendipitous one for HAARP and RBR. Professor Inan has known the DEMETER PI, Michel Parrot for years, and also known his colleague Francois Lefeuvre (who is the Head of the Laboratory that Michel is in) since the late 1970s. They are one of those few truly dedicated ELF/VLF/whistler-mode groups around the world, and their interests (i.e., emissions, lightning, VLF transmitters, wave-particle interactions, chorus, etc.) have always been very close to those pursued by the Stanford VLF group. Stanford maintains an excellent relationship with both of them, and this relationship has now been extremely beneficial as DEMETER can be used to measure HAARP-injected ELF/VLF in every pass, typically once a day. Professor Inan is an officially approved Guest-Investigator on DEMETER and has access (directly) to all of the data from this mission. Our French colleagues have been very cooperative in complete sharing of the data.

At the DEMETER Workshop, it was learned that the mission has CNES support for at least another two years, and (as mentioned above) that they will modify their procedures to acquire data beyond 65-degrees over HAARP. In other words, the HAARP project can count on using this spacecraft for the next two years. Stanford plans to propose to transmit (each pass is 5-10 mins) to DEMETER at least once a day (collecting both burst-mode and survey-mode data), in order to establish a statistically significant data set, with which to "define" the ELF/VLF wave-injection region above HAARP. Based on the data we already have (see attached excerpts from Platino et al. [2005]), it is quite clear that this region has an extent of greater than 400-km in transverse extent. We also propose to conduct similar observations over the geo-magnetically conjugate region, to measure the non-ducted HAARP-ELF/VLF signals that would propagate to the conjugate region, but would not necessarily be observed on the ground (because they are not ducted and are thus outside the ionospheric transmission cone).

In addition, Professor Inan realized at the DEMETER Workshop that the spacecraft is equipped with an extremely sensitive energetic electron detector measuring electrons in the range of 40-keV to 10-MeV, with a geometric factor of 1.1 cm\(^2\)-str. For comparison, the detectors we had on the SEEP/S81-1 mission, which detected transmitter-induced precipitation and lightning-induced precipitation bursts, had geometric factors of 0.21 cm\(^2\)-str. Once again for comparison, the detectors on the DMSP satellites (or any satellite designed for auroral observations) has geometric factors of 0.001 cm\(^2\)-str. The DEMETER electron detectors routinely saturates in the auroral region, but this is perfectly fine, since the science objectives of this mission do not include measurements above 65-degrees anyway. The time resolution of this highly sensitive detector is 1-sec, which should allow us to carry out a host of HAARP and RBR related observations, as itemized below. At the Workshop, Professor Inan met the DEMETER Co-I who built the instrument, Dr. Jean-André Sauvaud, who is well known auroral scientist, but does not really (yet) know
what science to go after with such a sensitive detector on an essentially mid-latitude mission. Dr. Sauvaud is looking forward to our guidance, and we propose to contact him soon with a comprehensive set of particle-detection experiments to be conducted above HAARP and the conjugate region.

With this particle detector, we propose to:

1) Go after the first detection of HAARP-ELF/VLF-induced electron precipitation. The amplified ELF/VLF signals produced by HAARP should be able to precipitate 10-100 electrons/cm^2-str-sec-keV, which would produce greater than tens of counts on this detector, as long as the waves access the relatively low density regions outside the plasmapause, so that the equatorial resonant energies are >40 keV. We will design ON/OFF transmission patterns similar to those used in the SEEP experiments. Note that we can make observations both over HAARP, and also over the conjugate regions.

2) Go after the detection NPM-induced precipitation in connection with TIPER modulated transmissions. The electron energies involved here are definitely in the right range, >100 keV most of the time, so this should be a piece-of-cake (it never is, really). Note that this spacecraft is much better than SAMPEX for us, since SAMPEX only looks at energies greater than 500 keV (too high; too few particles) while the peak of NPM-induced precipitation will be in the range of 150 keV or so. Also, SAMPEX only measures integral counts, while DEMETER instrument can give us spectra. One possible way to seek the signatures we look for may be spectral peaks corresponding to the transmission frequency of NPM.

The portion of our proposed budget that covers the DEMETER component is ~$90K/year, to fund a graduate student research assistance, 5% of time of Dr. Tim Bell, and also once-a-year attendance at DEMETER Workshops. The student on this program will be Denys Piddyachiy, a new student from Ukraine who will be coming to Stanford this summer. He had been working with Yuri Yampolsky in Ukraine, and in fact also knows about the RESONANCE program (which Michail Mogilevsky has been promoting), which Yampolsky group has been involved in. RESONANCE is a Russian mission designed with an orbit specifically targeting repeated observations of the magnetic field lines excited by the HAARP facility.
APPENDIX: HAARP-injected ELF/VLF on DEMETER [from Platino et al., 2005]

Ex, 02/10/2005, 06:52:59.7, L = 4.36,
λ = 60.59°, GMLong = 270.82°, Alt. = 725.6 km

Ey, 02/10/2005, 06:52:59.7

Bx, 02/10/2005, 06:52:59.7, L = 4.36,
λ = 60.59°, GMLong = 270.81°, Alt. = 725.6 km

Bz, 02/10/2005, 06:52:59.7

By, 02/10/2005, 06:52:59.7
Demeter is a low altitude spacecraft equipped with magnetic and electric field antennas, being able to resolve the three components of both fields in the ELF frequency range, and one component of each field in the VLF frequency range. In the previous figure we see a 6 spectrogram of both fields, as observed by Demeter during a pass over the HAARP ionospheric heater in Gakona Alaska. The observed highlighted pulses are generated by HAARP by amplitude modulation of the electrojet currents within the auroral region. The x and y axis represent time and frequency respectively, and the color scale represents signal amplitude in dB, with respect to $1 \text{ pT/Hz}$ and $1 \text{ µV/m/Hz}$ respectively. For this experiment, HAARP transmitted a series of short pulses of 2, 5 and 10 seconds of duration at the frequencies highlighted in the spectrograms shown in the previous figure. The values of magnetic latitude, longitude L-Shell and altitude displayed in the title of both figures describe position of the spacecraft at 06:53:00 UT, i.e. at the beginning of the 6 spectrograms.

In the previous figure we show a typical Demeter pass at high latitudes, observed on December, 04 2004. The spacecraft turns off all its receivers after crossing the 65° of magnetic latitude to perform calibration and orbit correction tasks. The upper left panel displays a spectrogram indicating the observation of one of the electric field in the ELF range. Highlighted is the HAARP transmitted pattern and the pulses observed on Demeter. Point [A] indicates the instant in time when Demeter stops recording. [B] indicates the instant in time when the spacecraft’s orbit projection on the ground is closer to the HAARP heater. The distance to HAARP is indicated in the upper right panel, where points [A] and [B] are highlighted. This fact represents a problem for HAARP-Demeter conjunctions, since the spacecraft’s instruments seem to stop recording when it becomes closer to the heater. Nevertheless this issue has been brought to attention to the PI’s of Demeter and in all future passes, the instruments are now being shut off after crossing over HAARP.

The two bottom panels, display the ground measurements of the HAARP generated ELF pulses, as observed in Chistochina, 36 km away from Gakona. The left spectrogram shows measurements of the
electric field observed on the ground before the Demeter pass, while the right spectrogram shows measurements of the electric field observed on the ground after the Demeter pass.

Measurements of electric and magnetic field in the ELF frequency band for the three spatial components of each field, can be used to estimate the power radiated by the HAARP heater. For this we will refer to the following figure, which shows a scheme of the geometry of the problem.

Here we see a heated region, called C, and a illuminated area in the ionosphere, called B. The lower left panel in the figure shows the process of radiation, where rays emitted from the heated region reflect in the ground back to the ionosphere. By raytracing we can estimate the approximate size of B, form the size of A, which represents the region in space illuminated by the ELF waves emanating from the ionosphere. This region (A) is estimated by observing the position of the spacecraft when the HAARP signals are observed onboard. Then a circular symmetry around the L-Shell corresponding to HAARP ground’s location is assumed to determine the size of A. The lower right panel on the previous figure displays the top view of the region in space illuminated by HAARP, denoted here as A. Highlighted is the point of observation of the signal onboard Demeter spacecraft, as well as the distance to HAARP at the moment of detection of the signal.

Once established the size of the illumination region in space as observed by Demeter, we can integrate the Poynting vector, calculated from the onboard observations of E and B fields at different frequencies. In order to do the integration we must assume a spatial distribution of the Poynting vector over B.
The previous figure shows a scheme of this distribution, indicated in blue line. For the case shown, we see a Gaussian distribution over B. In our calculations of power, we are using three different models, the Gaussian, the $r^{-2}$ and the constant distributions. All three models are estimated from the measurements of Poynting vector obtained onboard the Demeter spacecraft.

The result is shown in the following figure. Here we have the distribution in space of the Poynting vector, as a function of $r$, the radial distance from the HAARP transmitter, assuming as said before, a circular symmetry around the corresponding L-Shell of the heater. The two set of curves shown here correspond to the Gaussian and the $r^{-2}$ distributions, for the 5 different frequencies transmitted from HAARP and observed on Demeter. For each frequency, the value of the Poynting vector is averaged in time over the length of the pulse, obtaining a single observed value for each frequency at the moment of observation. This value is then extrapolated in space the rest of the B region using each of the three mentioned models. The two top right panels indicate the distribution over B of the Poynting vector, taken directly from one of the curves for each model (the one at 798 Hz). The color scale is not the same on both panels, and corresponds to the values of the magnitude of the Poynting vector.

Highlighted is the Demeter orbit and the point of observation of the HAARP signal, as well as the location of the heater.
The power can be directly obtained from integration of the Poynting vector calculated in the previous figure over the area of illumination. We repeat this calculation for each of the 5 frequencies observed, and obtain a corresponding value of power. The results are summarized in the following table, where we have summarized the values of the Poynting vector magnitude \( (S_D) \) and the power \( (P_D) \) for every frequency \( (f) \).

Here we can see that the constant distribution is giving the most reasonable result, while the Gaussian distribution is giving unreal results, being the \( r^{-2} \) an intermediate value, even though a bit higher than it was expected, from comparison with previous similar experiments.

Even though these results are very encouraging, the fact that Demeter stops recording as it approaches to HAARP, is their main drawback. This problem is going to be corrected in the future, and we will be able to have measurements over the heater, so we can estimate the full extent of the A region, without having to assume a given symmetry or radiation pattern.

For **Gaussian distribution**:  

<table>
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<tr>
<th>( f ) [Hz]</th>
<th>( S_D ) [pW/m(^2)]</th>
<th>( P_D ) [W]</th>
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<tr>
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<td>31.7</td>
<td>645</td>
</tr>
<tr>
<td>254</td>
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<td>707</td>
<td>76</td>
<td>1550</td>
</tr>
<tr>
<td>798</td>
<td>150.4</td>
<td>3060</td>
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</table>

For **\( r^{-2} \) distribution**:  

<table>
<thead>
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<th>( f ) [Hz]</th>
<th>( S_D ) [pW/m(^2)]</th>
<th>( P_D ) [W]</th>
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</thead>
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<td>39.26</td>
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<tr>
<td>367</td>
<td>2</td>
<td>2.2</td>
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<tr>
<td>707</td>
<td>76</td>
<td>83.82</td>
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<tr>
<td>798</td>
<td>150.4</td>
<td>165.9</td>
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For **constant distribution**:  

<table>
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<td>798</td>
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</table>
Multi-anode photometric measurements of Optical Emissions associated with Heater-Induced Particle Precipitation

An important component of the overall effort to quantify wave-induced precipitation of energetic electrons from the radiation belts is controlled experiments, in which specific bursts of electrons are precipitated by wave packets of known properties. The HAARP facility offers great potential in this regard, especially now that there is demonstrated evidence of ELF/VLF waves injected by HAARP to be amplified, trigger emissions, and exist in the magnetosphere during multiple bounces between hemispheres [Inan et al., 2004]. The next generation of ELF/VLF wave-injection experiments with HAARP should thus take on as a goal to demonstrate repeated precipitation of energetic electrons, to allow for controlled wave-induced particle precipitation experiments.

The characteristics of ELF/VLF wave-induced electron precipitation depend on altitude and particle energy distributions, and also highly dependent on whether or not the waves injected into the magnetosphere are ducted. In either case, the effects of particle precipitation can be measured via three possible mechanisms: secondary ionization evident in conductivity changes in the ionosphere; optical emissions; and gamma-ray emissions (see Lehtinen et al. [1999]). Currently, effects are measured through the first of these options, by monitoring the amplitude and phase of VLF transmitter signals propagating in the Earth-Ionosphere (E-I) waveguide, using VLF receivers at Chistochina, AK and other sites. Secondary ionization effects reveal themselves as perturbations on the amplitude and phase of these signals – these are similar to the well-known and commonly observed subionospheric VLF signatures of lightning-induced electron precipitation (LEP) events. The subionospheric VLF method is nevertheless limited to the detection of electrons with energy >50 keV, since it is only those electrons which penetrate to the ionospheric layers below the nighttime VLF reflection height of ~85 km.

We propose to design and construct a portable optical system that can be deployed at HAARP and remotely operated, for use in conducting measurements of the optical emissions associated with wave-induced particle precipitation, both by injected HAARP ELF/VLF waves but also by natural emissions, such as chorus. Such a system would provide the means of observing and quantifying the effects of particle precipitation, including an accurate measurement of the energy distribution of precipitating particles, for electrons ranging in energy from a few keV to hundreds of keV.

Quantifying the optical emissions due to energetic electron precipitation depends on the energy distribution of precipitating particles, as mentioned above. Precipitating particles expected to be induced by HAARP-generated ELF/VLF waves are expected to fall in the range 1-150 keV, with fluxes on the order of $10^{-3}$ to $10^{-2}$ erg/cm$^2$/s. Such particles deposit their energies in a wide range of altitudes from 90 – 150 km, producing observable optical emissions in atomic oxygen and molecular nitrogen lines similar to those found in artificial airglow and aurora.
The proposed system will use four Hamamatsu multianode photometer arrays, shown in Figure 1. Four arrays are used to provide increased spatial resolution and spectral information, as explained in more detail in the next section. Each of these photometers require a socket/bleeder assembly, which interfaces the photometer to a printed circuit board and divides the supply voltage over each of the 9 dynodes. 900-volt power is supplied using a PCB-mounted Hamamatsu C4900 power supply. Each of the four photometers, the sockets, and the power supply will be enclosed in a Pelco housing, and mounted on a Pelco pan/tilt unit with controller, all shown in Figure 2. The pan/tilt unit will be remotely operable from any computer with Internet access.

The four photometers will each be mounted with a lens, an interference filter, and an electronic shutter system. Edmund optics 75 mm focal length, 50 mm diameter lenses are chosen to provide a field-of-view of 20 degrees. This field-of-view matches 80 km altitude resolution, from 80 to 160 km, when viewing a section of the ionosphere towards the horizon at a range of 300 km. When viewing the ionosphere closer to the instrument (i.e., above the instrument), the spatial resolution is higher, though the total area viewed is smaller. Optical interference filters will be provided to cover the atomic oxygen forbidden lines at 630.0 nm, 557.7 nm, and 844.6 nm, and the first-negative line of ionized Nitrogen (N\(_2^+ \cdot\) 1N) at 427.8 nm; two of each filter are required, as explained below. Melles Griot low-profile electronic shutter systems will be mounted on each photometer system and programmed to open and close at dusk and dawn respectively; for added redundancy and safety, a standard dawn-to-dusk sensor will be provided, which will cut power to any connected instruments during daylight hours.

Data will be acquired using a National Instruments PCI-6254 board, shown in Figure 1, capable of up to 30 kS/s on 32 channels with 16-bit resolution. This board will be controlled using National Instruments’ Labview software, using a standard PC computer. The computer will require 2 GB of memory, a 400 GB hard drive and a fast processor in order to process and store large quantities of data. Because the expected optical emissions may be very faint, triggering will not be possible.

The photometer anode outputs on each individual photometer will be combined in pairs so that each is used as a linear 8-anode array; Hamamatsu does not offer an eight-anode array in a square package. This yields the optimal resolution of 10 km for each effective anode, and saves the data handling costs by half. Furthermore, the four photometers will
be used in pairs, with each pair recording a particular wavelength. A pair of photometers will be rotated 90 degrees with respect to one another, in order to artificially introduce two-dimensional spatial resolution; this is depicted in Figure 3. Using this arrangement, we can determine altitude variation of optical emissions, as well as the lateral width of the precipitating region. Each pair of photometers will be filtered identically, and two wavelengths will be chosen from the four listed above during a given experiment, with the capability of changing filters. This overall arrangement will provide time resolution and sensitivity far beyond the capabilities of any CCD camera, as well as some spatial and spectral information. Using the relative intensities of two wavelengths, we can estimate the energy distribution of the secondary electrons in the precipitation region [Bernhardt et al, 1989].

![Figure 3: L: individual fields-of-view of two photometers. R: effective combined FOV after post-processing.](image)

In order to locate precipitation ducts in the sky, three possible modes of operation will be introduced.

1 – When no known precipitation signatures are found, the instrument will scan the sky in a programmed sequence during HAARP dedicated transmissions. The 20-degree FOV can cover 90% of the sky in 36 scan locations; the system will record at one scan location for a period of time to match the transmission periods, and then step through all locations in the sky until optical signatures are found.

2 – The instrument will be coordinated with the Stanford VLF receiver at Chistochina, which can use radio direction-finding techniques in real-time to locate the direction from which magnetospheric waves that may cause precipitation (e.g., two-hop echoes of injected HAARP signals or natural chorus emissions) reach the receiver. Once the direction is determined, the optical system will reorient itself to point in that direction.

3 – When the HAARP heater is positioned away from zenith, the optical system will be aligned with the heater, so that precipitation signatures that may be produced with direct overhead injection can be detected.

4 – The optical system will also be coordinated with satellite observations. For example, when HAARP is transmitting for a particular satellite pass (e.g., DEMETER), the optical system will be aimed in the direction so as to track the ionospheric ends of the field lines traversed by the satellite.
Support of Other Experiments

This optical instrument proposed here specifically for detection of energetic electron precipitation will also be available for use in other optical experiments at HAARP, and particularly in dedicated campaigns for measurements of heater-induced artificial airglow. The proposed instrument can provide the first measurements at high time resolution with some (i.e., 8x8) spatial resolution, and enables measurements of the time-evolution of the airglow spot size and shape during a single heater cycle. This type of measurement is essential information for both F-layer airglow measurements, especially for gyroharmonic measurements as in [Kosch et al, 2004] as well as E-layer artificial aurora experiments [Pedersen and Gerken, 2004].

Equipment Budget

<table>
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<tr>
<th>Item</th>
<th>Qty</th>
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<td>Hamamatsu C4900 HV power supply</td>
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<td>Pelco EH5729 Enclosure with heater/blower</td>
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