Examining Lightning Channel Electrical Properties with Time Domain Fractal Lightning Modeling

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Abstract

The electrical properties of the lightning channel are determined by a complex interplay of dielectric breakdown, plasma heating, expansion, and cooling. These details are observable through their effect on the channel current and electromagnetic wave emissions. This suggests that electromagnetic observations of lightning can be used to study channel physics and behavior.

This paper presents the Time Domain Fractal Lightning model (TDFL), a high-level tool that connects channel electrical properties to charge and current flow for branched, tortuous lightning channels. The TDFL extends ideas of fractal discharges to the time domain through the retarded time electric field integral equation. Here we present a preliminary application of the TDFL to stepped leader extension and compare the predicted electromagnetic emissions to observations. The results show crude qualitative agreement, demonstrating that more complex channel behavior should be included and that such behavior can be studied with high-level tools like the TDFL.

1 Introduction

The lightning channel is a hot conductive plasma channel formed and maintained by Joule heating and dissipated by expansion and radiative cooling. The physics involved spans many orders of magnitude from sub-millimeter electron avalanches to 100 km lightning discharges. The details are beyond the scope of this paper (summaries of the physics of dielectric breakdown can be found in [1, 2, 3]). Here it suffices to mention that at the ends of the channel, strong electric fields drive charge outward into a sheath of corona discharge and lead to the extension of the channel. Such processes continue until the driving electric field decays, currents cease, and the channel cools. Excellent reviews of lightning behavior and properties can be found in [4] and [5].

The lightning "flash" or return stroke, a large current pulse carrying up to several 100 kA, has been studied in great detail (see for instance pp. 143-188 of [4] or Chapter 6 of [5]). The extension of the lightning channel is less well-studied as it is a much less intense process. Though the exact mechanism is uncertain and depends on the polarity of the charge on the channel, negative lightning leaders tend to extend in a series of discrete steps. This stepping reflects the gradual heating of the air by electron motion in the high-field region near the leader tip. When the air near the tip is heated sufficiently, ionization, current flow, and subsequent Joule heating produce positive feedback until the dense charges on the existing channel flow into the newly-formed plasma channel segment. The goal of this paper is to describe a tool to study electromagnetic waves emitted by this stepping process and to connect the electromagnetic emissions to the effective properties of the leader channel.

2 The Time Domain Fractal Lightning Model

The Time Domain Fractal Lightning model (TDFL) is a time domain simulation of current and charge flow on a lightning channel. The TDFL is based on a physical model of currents and charges on narrow channels of arbitrary segmented geometry as shown in figure 1. Maxwell's equations are solved via the time

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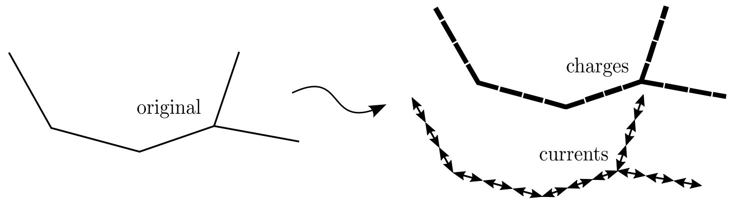

Figure 1: The channel segmentation scheme used in the TDFL.

domain electric field integral equation (EFIE):

$$\vec{E}(\vec{x},t) = \frac{1}{4\pi\epsilon_0}\int d^3x' \left\{ \frac{\hat{R}}{R^2}\left[\rho(\vec{x'},t')\right]_{\text{ret}} + \frac{\hat{R}}{cR}\left[\frac{\partial\rho(\vec{x'},t')}{\partial t'}\right]_{\text{ret}} - \frac{1}{c^2R}\left[\frac{\partial\vec{J}(\vec{x'},t')}{\partial t'}\right]_{\text{ret}} \right\}. \quad (1)$$

where $\vec{E}$ is the electric field produced by the channel, ϵ_0 is the permittivity of free space, ρ is the volume charge density, $\vec{J}$ is the current density, $\vec{x}$ and $\vec{x'}$ are position vectors, $\vec{R} = \vec{x} - \vec{x'}$, $R = |\vec{R}|$, $t' = t - R/c$, $\hat{}$ signifies unit vectors, and the $[\cdots]_{\text{ret}}$ represents evaluation in retarded time (t'). Note that the EFIE directly solves Maxwell's equations; no approximations have been made. See [6, section 6.5] for a derivation. Current and charge are assumed spatially uniform over segments with a simple time interpolation scheme. Evaluation of the EFIE on the current segments with the method of moments, the thin wire approximation, charge conservation, and Ohm's law produces a system of sparse linear equations that can be solved to evolve the system forward in time [7].

Such time marching schemes are often unstable but can be stabilized by a simple averaging scheme described in [8]. The stability of the TDFL has been verified by assuming time harmonic solutions and expressing the resulting constraints as a generalized eigenvalue problem (details available upon request). The TDFL has been validated by comparison to existing simulations (for instance duplicating the results in figure 11 of [9]).

TDFL channels in principle have complex geometry, gradually extend, and have time-varying electrical properties. These features are included by adding new segments to the system and by allowing the channel properties to change over time. The channels that result develop over many milliseconds and produce complex geometries and nonlinear current behavior.

3 Leader Step Model

In the present preliminary work, however, we focus on the stepping involved in leader channel extension. As such, we simply consider a single step on a single channel. We approximate this as unaffected by geometry and so consider an isolated straight channel. As the timescale of a single step is less than 1 μs, we consider a channel 600 m long such that the far end of the channel is not causally connected to a step on the near end of the channel. The step length is taken to be 15 m, corresponding to leader steps at low altitude. The geometry is shown at left in figure 2.

The initial conditions are set by assuming a step initiates after the channel has come to equilibrium in an applied electric field. This equilibrium is established by treating the step segments as non-conductive and running the simulation until equilibrium is reached. The resistance of the step segments is then decreased exponentially until it reaches the target value. The exponential time constant corresponds roughly to the current rise-time.

As the resistance in the step segments falls, current starts to flow into the step. A series of sample current profiles at various times are shown at right in figure 2. The current pulse emits an electromagnetic wave which is measured at a vertical E-field receiver on the ground below the step.

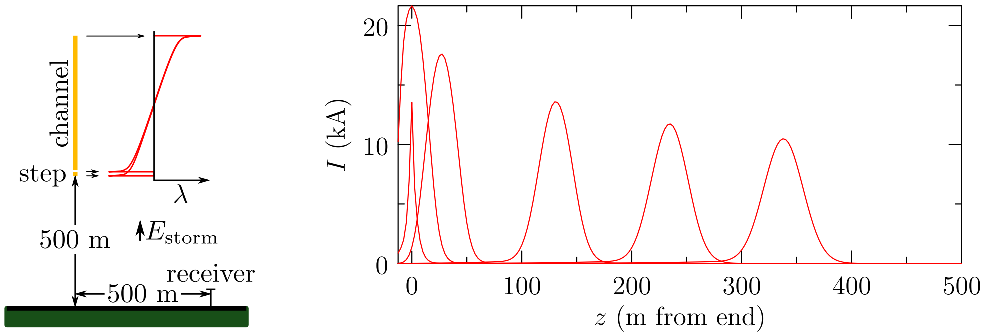

Figure 2: Left: geometry of step simulations and equilibrium charge distributions. Right: sample current pulse shape shown vs position along the channel for several different times. First the current pulse rises up from the step junction, then propagates along the channel.

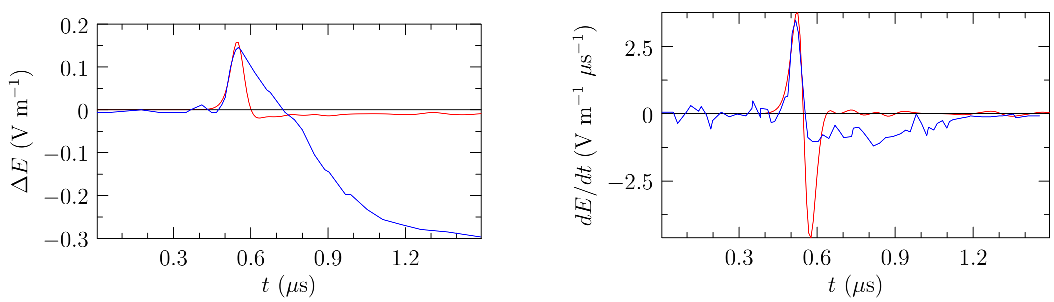

Figure 3: Measurements (blue) compared to best fit simulation result (red, 20 ns rise-time, low resistance). Left: field change vs time. Right: electric field derivative vs time.

4 Comparison to Observation

The preliminary model described above is compared to natural lightning negative stepped leader electric field records in figure 3. The data is taken from figure 4 of [10] as measured under similar circumstances. The simulation parameters that best fit the data are a current pulse rise-time (resistance fall-time) of 20 ns with low channel resistivity. The simulated electric field derivative pulses are bipolar, as expected from a current pulse with a rapid rise and fall. The experimentally observed pulse is also bipolar, but the second inverted component has much smaller peak intensity and longer duration. This indicates that the current pulse associated with natural leader stepping rises rapidly but does not fall immediately, suggesting that the natural lightning channel step continues to draw charge after the step forms. This is likely due to the formation of the corona sheath around the step, a process that can take several microseconds.

The normalization of the simulation is unfortunately not constrained due to uncertainty in the strength of the applied electric field and the channel length. As such, we cannot yet comment on the implications of the size of the observed field change.

5 Discussion

The results given above make a simple connection between the electrical properties of the lightning channel during leader stepping and measured electromagnetic emissions. The main result is an overall qualitative agreement with the shape of the electromagnetic pulses produced by leader steps but a disagreement with the balance between rise and fall in currents and electric fields. The observations show short rise-time and much slower fall-time, while in the simulation short rise-times lead to short fall-times. As mentioned above, this suggests that electromagnetic simulations of the channel must include the effects of the corona sheath. The corona sheath has also been suggested as important in return stroke behavior [11, 12], so this often-ignored effect should be studied further.

We believe that the approach outlined here and demonstrated with preliminary results shows great promise to help understand the electromagnetic properties of the lightning channel. Plasma and gas dynamical models of the channel can be examined in this respect as they may or may not generate electrical properties that reproduce the observed electromagnetic emissions. Finally, though the simulations above were relatively small-scale, large-scale phenomena such as recoil streamers and subsequent return strokes can also be studied. Further development of the TDFL promises to shed light on these and many other questions.

6 Acknowledgments

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