Longitudinal dependence of lightning-induced electron precipitation

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[1] Observations of lightning–induced electron precipitation (LEP) events at three geographic regions show characteristics which systematically vary with both longitude and hemisphere. These observations are quantitatively interpreted using a novel atmospheric interaction model designed to predict the characteristics of LEP events at any longitude and midlatitude L-shell by accounting for the effects of precipitating electrons which are backscattered from the atmosphere. The model of atmospheric backscatter (ABS) calculates atmospheric backscatter responses for individual monoenergetic electron beams with a single incident pitch angle using a Monte Carlo model of atmospheric interactions. The ABS model also includes an asymmetric (non-ideal dipole) geomagnetic field model in calculations of the pitch angle of backscattered electrons entering the conjugate hemisphere. Using a realistic distribution of precipitating electrons, the results of this backscatter calculation at three separate longitudes are compared with VLF remote sensing data collected on nearly north–south great circle paths (GCPs). Results predicted by the model and confirmed by data indicate that all four primary LEP characteristics exhibit longitudinal and hemispheric dependence which can be explained in terms of precipitating electrons backscattered from the atmosphere. By combining these effects with previously calculated radiation belt electron loss rates due to lightning at a single location it is possible to estimate the global loss of radiation belt electrons due to lightning.


1. Introduction

[2] The magnetosphere is permeated by high-energy electrons in what are known as the Van Allen radiation belts. Discovered by James Van Allen in 1958, the radiation belts consist of an inner and an outer radiation belt separated by a region of depleted electron fluxes known as the slot region. The study of the radiation belts has received significant attention in the past due to the fact that the electrons on-board orbiting satellites are damaged over time by these high energy electron fluxes and are particularly vulnerable during periods of extreme geomagnetic activity caused by coronal mass ejections and other solar events [Baker et al., 2004]. Understanding the physical mechanisms involved in the removal of these electrons is therefore of utmost importance in order to accurately predict the lifetimes of many orbiting satellites.

[3] The motion of radiation belt electrons is governed by the Lorentz force equation [e.g., Walt, 2005, p. 10] which describes three distinct magnetospheric motions. Electrons gyrate around the magnetic field with a gyrofrequency determined by the strength of the magnetic field. While doing so they traverse the magnetosphere along a single magnetic field line to an altitude at which they mirror and bounce back along the same field line. Finally, the electrons drift around the Earth. Most important for the current work is the altitude to which mirroring electrons penetrate as they traverse the magnetic field line. This altitude is determined by the projection of the electron velocity onto the local magnetic field line with a parameter known as the pitch angle \( \alpha = \tan^{-1} \left( \frac{v_y}{v_z} \right) \), where \( v_y \) is the electron velocity component parallel to the magnetic field and \( v_z \) is the component perpendicular to the magnetic field where the pitch angle must always be referenced to some altitude. Knowledge of the altitude (or equivalently the geomagnetic latitude) is necessary because the pitch angle is linked to the magnetic field strength, \( B \), through the first adiabatic invariant of magnetospheric particle motion [e.g., Walt, 2005, p. 39].

[4] Due to magnetospheric interactions, the pitch angle of an electron can be decreased, causing it to mirror at an altitude low enough such that it is removed from the magnetosphere due to Coulomb collisions with neutral atmospheric constituents. The altitude at which this removal occurs is determined by the atmospheric neutral density. The neutral density increases exponentially with decreasing altitude and the altitude at which an electron is typically considered to be lost is below 100 km (though the altitude...
profile of energy deposition depends strongly on the energy of the precipitating electron as shown in section 2). The specific equatorial pitch angle corresponding to a mirror point at this 100 km altitude is known as the equatorial loss cone pitch angle ($\alpha_{eq}^{lc}$). When an electron’s pitch angle is lowered below the loss cone, the electron is said to be precipitated out and lost from the radiation belts.

[5] In this paper electron pitch angles are quoted at various altitudes: 100 km, 150 km, and 200 km as well as at the geomagnetic equator. To differentiate between these locations the pitch angle symbol, $\alpha$, indicates the location (altitude) as a subscript, and the pitch angle (if referenced specifically) as a superscript. For example an electron with an equatorial pitch angle of 13° is written $\alpha_{eq}^{13°}$, while (at some longitude) at a 200 km altitude the same electron may be represented as $\alpha_{200km}^{13°}$, or at 100 km as $\alpha_{100km}^{90°}$. If no altitude is indicated or if the pitch angle is written $\alpha_{0}$ (for the initial pitch angle), then it is assumed that the altitude is 200 km. Furthermore, the quoted pitch angles at each of the above altitudes are always defined to be $<90°$ for downward traveling (precipitating) electrons and $>90°$ for upward traveling (backscattered) electrons. This is in contrast to the conventional definition of the pitch angle in which the definition is with respect to the direction of the local magnetic field. However, the definition herein makes the comparison of pitch angle distributions in conjugate hemispheres easier and is thus more informative for this paper.

[6] One of the important natural loss process for electrons in the range $2 < L < 3$ is lightning-induced electron precipitation (LEP) [Abel and Thorne, 1998a, 1998b], and fully understanding LEP will help quantify the global effects of lightning on the radiation belts. The entirety of the LEP process is shown in Figure 1 where it is important to note that VLF remote sensing of LEP events is most sensitive to $D$-region ionization caused by precipitating electrons in the 0.1 MeV to 0.3 MeV energy range [e.g., Peter and Inan, 2007]. The process begins when a lightning discharge occurs, emitting a large electromagnetic pulse which is guided very efficiently in the earth-ionosphere waveguide. In the near-field region around the lightning flash the wave energy can be thought of in terms of rays which reflect back and forth between the earth and the ionosphere. At each reflection from the ionosphere, a small portion of this energy leaks through the ionosphere and enters the magnetosphere as a whistler mode wave, and is shown propagating up the magnetic field line in Figure 1a. Upon entering the magnetosphere the whistler mode wave propagates out to the equatorial region and interacts with the high-energy radiation belt electrons in a wave-particle interaction.

[7] This interaction occurs when the Doppler-shifted frequency of the whistler mode wave equals the gyrofrequency of a trapped radiation-belt particle [Helliwell et al., 1973]. In this situation a cyclotron resonance occurs whereby the momentum of the electron is redirected through interaction (over several gyroperiods) with the wave magnetic field. The result of this interaction can change the electron pitch angle sufficiently to reduce it below the loss cone, causing the electron to impact the ionosphere and precipitate as shown at the base of the field line in Figure 1a.

[8] Precipitating electrons of sufficient energy produce large amounts of secondary ionization as they interact with the neutral atmosphere resulting in a large disturbance region in the ionosphere as shown at the base of the field line in Figure 1a. When the secondary ionization produced
by these precipitating electrons is sufficient to become an appreciable percentage of the ambient D-region conductivity, the disturbance can be observed remotely using sub-ionospherically propagating VLF transmitter signals [e.g., Inan and Carpenter, 1987].

[5] In the same way that the VLF energy from a lightning strike is efficiently guided between the earth and the D-region ionosphere, anthropogenic VLF transmitter signals operated by navies around the world provide a reliable coherent signal with which to probe the D-region ionosphere. By monitoring the amplitude and phase of the signal for characteristic sudden perturbations with relatively long recoveries, these subionospherically propagating VLF signals can be used as a proxy for the occurrence of LEP. An example disturbance region is shown in Figure 1b over the Central United States, with great circle paths (GCPs) from VLF transmitters (originating in Maine and Puerto Rico) observed in Wyoming and Colorado. When an LEP event occurs, the recorded VLF signal amplitude exhibits a sudden change followed by a relatively slow recovery back to the ambient level. Figure 1c shows the characteristic signature of an LEP event and highlights the four defining characteristics. The onset delay ($\Delta t$) is the time from when the causative lightning flash occurs to when the amplitude of the signal begins to change. The onset delay time is determined by identifying the electromagnetic signature of the lightning flash in the VLF data which is known as a radio atmospheric or ‘sferic’ (as shown in Figure 1c). The onset duration ($t_o$) is the time over which the amplitude of the signal continues to change, the magnitude of amplitude change ($\Delta A$) is the total amount of change observed in the transmitter signal, and the recovery time ($t_r$) is the time it takes for the amplitude of the transmitter signal to return to the level it would have exhibited in the absence of a disturbance. Finally, by observing multiple transmitter signals at multiple locations, as shown by the map in Figure 1b, it is possible to determine the spatial extent of the precipitation region [Johnson et al., 1999; Peter and Inan, 2004].

[6] More important in terms of quantifying radiation belt electron loss is the fact that the observed event characteristics are related to the number, energy and temporal signatures of precipitating electrons. Understanding the complexity of this relationship is thus a major goal of VLF remote sensing. D-region remote sensing using subionospherically propagating VLF signals has long been used as a tool in studying and quantifying the loss of radiation belt electrons due to LEP [e.g., Helliswell et al., 1973; Inan et al., 1985; Dowden and Adams, 1988; Lauben et al., 1999; Peter and Inan, 2004].

[7] The physical interpretation of the characteristic parameters shown in Figure 1c is as follows. The onset delay is the time required for the lightning–generated EMP to propagate in the earth–ionosphere waveguide, couple through the ionosphere into the magnetosphere, interact with a distribution of trapped radiation–belt electrons (at or near the magnetic equator), and for those electrons to finish traversing the magnetic field line and impact the ionosphere where they cause secondary ionization. The onset duration is the time over which the electron distribution continues to impact the ionosphere, causing secondary ionization. The amplitude change is determined both by the amount of secondary ionization created and also the location of the ionization relative to the transmitter-receiver GCP [e.g., Poul森 et al., 1993a]. Finally, the recovery time is determined by the time it takes for the ionospheric chemical equilibrium to be re-established in the disturbed region [e.g., Gluhov et al., 1992].

[12] Due to the indirect connection between these VLF measurements and the parameters of interest (i.e., number, energy and duration of electrons removed from the radiation belts), quantification of the process is by nature a forward-modeling inversion problem. Previous work using this methodology has produced good agreement between the observed amplitude change of LEP events and a comprehensive model of the LEP process [Peter and Inan, 2007]. However, one major discrepancy between previous modeling work and observation is a consistently shorter predicted onset delay (by ∼300 ms) than that observed in data [Inan et al., 1988a; Lauben et al., 1999; Peter and Inan, 2007].

[13] It has been suggested [Foss et al., 1998; Peter and Inan, 2007] that this discrepancy can be attributed to the effects of electrons which are incident upon the atmosphere at ‘grazing incidence’ i.e., the electrons mirror at an altitude very near to 100 km. Such electrons are only slightly inside the loss cone and only briefly interact with the atmosphere before returning to the magnetosphere with slightly less energy and slightly changed pitch angles. These electrons are considered to be ‘backsattered’ from the atmosphere. An electron can backscatter from the atmosphere if in the course of the random scattering process the local pitch angle is altered such that it becomes $\geq 90^\circ$. When this occurs the magnetic mirroring force resulting from the convergence of magnetic field lines causes the electron to return to the magnetosphere before depositing all of its energy into the atmosphere.

[14] The process of atmospheric backscatter can explain the discrepancy in modeled versus observed onset delay as follows. Suppose a distribution of precipitating electrons is incident upon the atmosphere (in the Northern Hemisphere) at an altitude of 100 km and with a pitch angle at grazing incidence (i.e., $\alpha_{100km} = 90^\circ$). A small percentage of these electrons deposit all their energy into the atmosphere with many atmospheric interactions while the majority undergo only a few (or zero) atmospheric interactions losing only a small amount of energy before backscattering. The net effect in the atmosphere is that there has been little energy lost from the electrons and hence a negligible ionospheric disturbance. For the majority of electrons, the effect is that the pitch angle distribution of the backscattered electrons has been broadened, e.g., with pitch angles in the range $\alpha_{100km} \sim 80^\circ$–90°. In the magnetic field of an ideal-dipole these electrons then return to the magnetosphere, traversing the magnetic field line to the conjugate point in the Southern Hemisphere where they are again incident upon the atmosphere with the same range of pitch angles (80°–90°). The same atmospheric backscattering process occurs except that because some of the incident electrons are now deeper inside the loss cone (i.e., penetrate to lower altitudes) a larger portion of the total energy is deposited and the electron distribution further isotropizes in pitch angle (e.g., with electron pitch angles now ranging from $\alpha_{100km} \sim 70^\circ$–90°). The remaining electrons again traverse the magnetic field line back to the Northern Hemisphere and are incident once again on the atmosphere. The incident electrons are now deeper inside the loss cone and deposit more of their energy.
into the atmosphere. If this deposition results in the production of sufficient secondary ionization to significantly scatter the subionospheric VLF transmitter signal then the effect is observed as an LEP event with an onset delay time which is increased by one bounce period (~300–450 ms for 0.1–0.3 MeV electrons) from the initial precipitation.

[15] Under this scenario, it is also important to recognize that a significant percentage of electrons which are initially backscattered from the Northern Hemisphere and then incident upon the Southern Hemisphere deposit their energy in the southern atmosphere. To determine the location (i.e., hemisphere), time (i.e., atmospheric interaction number), and amount of deposited energy from a particular precipitating electron distribution requires knowledge of both the incident energy ($E_0$) and pitch angle ($\alpha_0$) of the precipitating electrons. In the present research, these distributions have been determined by creating a model of the atmospheric backscatter response using a Monte Carlo model of atmospheric interactions with energetic particles based upon the work of Lehtinen et al. [1999].

2. The Model of Atmospheric Backscatter

[16] The atmospheric interaction of precipitating electrons with the neutral atmosphere is inherently a stochastic process and so is well suited to modeling with a Monte Carlo simulation. The Atmospheric Backscatter Model (ABS) is based upon the Monte Carlo model developed by Lehtinen et al. [1999] and tracks the full gyration of each individual precipitating electron around the magnetic field line as it enters the atmosphere, accounting for inhomogeneity of the magnetic field (responsible for mirroring), the dynamic friction force and angular diffusion as well as the production of new electrons via ionization. Additionally the model accounts for the effects of a geomagnetic field of arbitrary orientation making it suitable for calculations at all midlatitude L-shells and longitudes. The ABS model begins simulation of atmospheric interaction at an altitude of 200 km, above which the interaction with the atmosphere is negligible, so that the atmospheric effects on electrons above (but still near) the loss cone can also be evaluated. The neutral density profile was taken from the Mass Spectrometer Incoherent Scatter (MSIS) project [Hedin, 1991].

[17] The ABS model is comprised of an atmospheric backscatter response for any electron distribution defined by a grid of ~16,000 initial pairs of ($E_0, \alpha_0$) in the range $E_0 \in [0.05, 30]$ MeV (98 energies spaced roughly logarithmically) and $\alpha_0 \in [0^\circ, 90^\circ]$ (55 pitch angles spaced every $5^\circ$ from $0^\circ$ to $25^\circ$, every $2.5^\circ$ from $25.5^\circ$ to $75^\circ$, and every $0.5^\circ$ from $76^\circ$ to $90^\circ$) and with three separate magnetic dip angles (the angle of the magnetic field relative to the horizontal plane) of 63°, 68° and 72°. For reference, these dip angles are roughly the mean value of the dip angle over $L = 2.0$, $L = 2.5$, and $L = 3.0$, respectively and correspond to a geographic longitude of 270°E/N. The chosen sampling in pitch angle maximizes the resolution of atmospheric effects on electrons with pitch angles near the loss cone where the greatest variation in backscatter distributions occurs (as shown in sections 2.1 and 2.2). The backscatter distribution for any initial pair ($E_0, \alpha_0$) not specifically calculated can be easily and accurately interpolated using the backscatter distribution for surrounding pairs.

[18] A major improvement of the current ABS model approach over previous calculations of atmospheric backscatter is in the way in which it treats the backscattered population of electrons. The ABS model uses a large number of simulated electrons (≥100,000) and then normalizes the response to that of a single precipitating electron (per unit area). The backscatter distribution can therefore be thought of as the characteristic response of the atmosphere to a specific ($E_0, \alpha_0$) input. In the case of atmospheric backscatter this result can then be used in subsequent calculations, eliminating the need for re-simulation of a new distribution of precipitating electrons. This approach is viable because the interaction of electrons with the atmosphere (i.e., loss of energy or diffusion in pitch angle resulting from interaction with the atmosphere) is determined by the atmospheric neutral density [Jackson, 1998, chapter 13] which remains relatively unchanged by even the largest conceivable flux of electrons precipitated through wave-particle interactions. This property of linearity between the different ($E_0, \alpha_0$) pairs allows the response for any arbitrary input distribution of electrons to be calculated by a combination of the individually calculated atmospheric backscatter responses.

[19] To fully understand the effect of atmospheric backscatter on the observation of LEP events it is necessary to quantify both the resultant energy deposition altitude profile and the atmospheric backscatter $E$ and $\alpha$ distribution. The deposition profile is necessary because it is the newly introduced atmospheric ionization which is detectable using VLF remote sensing, and the atmospheric backscatter distribution (which can be detected by a particle–detector on-board a satellite in low–earth orbit [Voss et al., 1984]) is necessary because it determines the temporal (after the lightning flash) and spatial (i.e., which hemisphere) evolution of the backscattered electrons.

2.1. Dependence of Electron Deposition on Energy and Pitch Angle

[20] In order to fully understand the overall contribution of the ABS model, it is important to first look at the dependencies of the input parameters. The Monte Carlo simulation calculates the amount of energy loss (as a function of altitude) for each simulated electron, resulting in an energy deposition calculated in eV/m. The amount of secondary ionization is then calculated by the often-quoted factor of one electron–ion pair created for every 35 eV of electron energy [Rees, 1963]. Previous studies [e.g., Peter and Inan, 2007; Chevalier et al., 2007] have used a similar approach to determine the altitude profile of secondary ionization created by precipitating electrons. However, these studies emphasized the energy of the particle, generally ignoring the equally important contribution of incident pitch angle, or accounting for it only in a generalized sense.

[21] In the following sections unless otherwise specified, the pitch angle and energy pairs ($E_0, \alpha_0$) are assumed to be in units of MeV and at an altitude of 200 km (i.e., $\alpha_{200km}$). For reference, $\alpha_{100km} \leftrightarrow \alpha_{200km}$, meaning that electrons $\approx 78^\circ$ at an altitude of 200 km are inside the conventionally-defined loss cone while those electrons $\geq 78^\circ$ at 200 km are outside the loss cone.

[22] Figure 2 shows the electron energy deposition profile (or equivalently secondary ionization) as a function of both incident energy and pitch angle of precipitating electrons.
Each panel shows the deposition profile for a single incident energy and pitch angle (per unit area). The abscissae are in units electrons/cm$^2$ and the ordinates show the altitude in km. Each panel shows a different incident pitch angle ($\alpha_{200km}$ indicated at the bottom of the panel). Within each panel the color of each line indicates the incident energy of the precipitating electron with values ranging from 0.05 MeV (dark blue) to 1.0 MeV (dark red) as shown by the color bar, displaying the log$_{10}$ of the incident energy.

![Graph showing electron deposition profiles](image)

**Figure 2.** (a–l) Electron deposition profiles as a function of both ($E_0$, $\alpha_0$) normalized to a single precipitating electron of the specified energy and pitch angle (per unit area). The abscissae are in units electrons/cm$^2$ and the ordinates show the altitude in km. Each panel shows a different incident pitch angle ($\alpha_{200km}$ indicated at the bottom of the panel). Within each panel the color of each line indicates the incident energy of the precipitating electron with values ranging from 0.05 MeV (dark blue) to 1.0 MeV (dark red) as shown by the color bar, displaying the log$_{10}$ of the incident energy.

The dependence of the deposition profile on incident pitch angle is more subtle but can be seen by comparing the different panels to one another. Notice first that the two energy-dependent features described above hold true only for precipitating electrons with pitch angles below the loss cone angle as can be seen in Figures 2a–2g. However, there are some interesting pitch angle dependent variations evident in the deposition profiles. Comparing 2a and 2c it is evident that while the deep-incidence ($\alpha_{200km}$) deposition profile is narrowly peaked (with a 3 dB peak altitude range of 58–67 km) the altitude profile of the $\alpha_{200km}^{70^0}$ has a much broader peak (with a 3 dB altitude range of 60–77 km), and a peak deposition which is less by a factor of about 3. This fact has implications for both the magnitude [Peter and Inan, 2007] and recovery time [Glukhov et al., 1992; Pasko and Inan, 1994] of observed LEP events. Finally, notice that the above loss cone angle of $\alpha_{200km}^{70^0}$ is the deposition profiles alter significantly in two ways (i) the percentage of energy deposited decreases dramatically, and (ii) the energy-dependent variation of the deposition no longer exhibits the same characteristics as described above.

While the higher energy electrons still penetrate to lower altitudes, they deposit less energy (absolute deposition, not just relative) than the lower energy electrons, i.e., a larger percentage of the energy is backscattered. In particular, at a pitch angle just above the loss cone ($\alpha_{200km}$) as shown in Figure 2h, incident electrons of all energies deposit roughly the same amount of energy (per km) at the peak of their respective profiles. In fact the largest peak in deposition shown in Figure 2h occurs for precipitating electrons of ~0.2 MeV and the peak deposition for 1 MeV electrons is approximately equal to the peak deposition for electrons of only 0.05 MeV (at altitudes of 67, and 89 km, respectively). This result is due to the energy-dependent dynamic friction force an electron experiences in the atmosphere which has a minimum at ~1.2 MeV [e.g., Lehtinen et al., 1999]. This fact is especially important for precipitating electrons associated with LEP events of 0.1 MeV–0.3 MeV [Voss et al., 1998; Peter and Inan, 2007] because it means that while proportionally more energy is initially deposited at these energies, there is also significantly more diffusion in pitch angle for those electrons which are backscattered from the atmosphere, as discussed in section 2.2.

### 2.2. Backscatter Response of Precipitating Electrons

The previous section describes the characteristics of electrons lost through interaction with the atmosphere. In this section the focus is on the electrons which remain after an atmospheric interaction i.e., those which have been backscattered. Similar to the deposition profiles, the variations shown in the backscattered distribution as a function of ($E_0$, $\alpha_0$) reveal insight into the dependence of atmospheric backscatter on electron input energy and pitch angle. While the deposition profiles show the altitude-dependent deposition of energy, the backscatter distributions show that the energy and pitch angle of the backscattered electrons vary significantly as a function of both input parameters ($E_0$, $\alpha_0$).

In this section, discussion of percentages deposited or backscattered refer to the percentage of total input energy (i.e., total electrons times individual electron energy) deposited to the atmosphere. Indeed for electrons well above the loss cone, $\alpha_{200km}^{70^0}$ the ABS model predicts that there are more backscattered electrons than input electrons due to the production of new electrons during atmospheric collisions. However, due to conservation of energy these backscattered electrons have only 99.9998% of the total input energy (the remaining small amount of energy being deposited in the rare atmospheric collision at 200 km).

To illustrate these dependencies, Figure 3 shows the backscatter distribution for several different ($E_0$, $\alpha_0$) input
Figure 3. Electron backscatter distributions as a function of both ($E_0$, $\alpha_0$). Each panel has the same axes. The abscissae show normalized energy ($E/E_0$) of the backscattered electrons, ranging from 0–1 and the ordinates show the pitch angle of backscattered electrons (upgoing, at an altitude of 200 km) ranging from 0°–90°. The color bar indicates the backscattered fluence (normalized to a single precipitating electron per unit area) in units of cm$^{-2}$ str$^{-1}$ keV$^{-1}$. Each row shows a specific incident energy ($E_0$), indicated at the left of each row, and each column shows a specific incident pitch angle ($\alpha_{200km}$), indicated at the top of each column. Note that the color bar label corresponds to the log$_{10}$ of the fluence.
pairs. Each row corresponds to a different input energy ($E_0$) and each column to a different input pitch angle ($\alpha_{200km}$). In each individual panel the abscissa corresponds to the energy of backscattered electrons normalized to the input energy (i.e., $E/E_0$) while the ordinate shows the pitch angle of the backscattered electrons at an altitude of 200 km ($\alpha_{200km}$, upgoing). The color bar shows the fluence of backscattered electrons (per unit energy interval and unit solid angle, and normalized to a single precipitating electron per unit area) with all panels displaying the same color axis for easy reference. Finally, the percentage of total incident energy backscattered is shown in the top left corner of each panel.

[27] As described above, backscatter can be considered to have occurred when the result of any individual atmospheric interaction leaves an electron with a local pitch angle of $\geq 90^\circ$. The first column of Figure 3 shows that only a very small percentage of electrons at deep incidence are backscattered and those which are backscattered have lost much of their initial energy, leaving the atmosphere with an isotropized pitch angle distribution which is completely uncorrelated to the incident pitch angle. This result is due to the fact that each atmospheric elastic scattering interaction results in only a small pitch angle deflection [Lehtinen et al., 1999] so that it takes many interactions before the electron has a high enough pitch angle to mirror and return to the magnetosphere. Furthermore, numerous elastic scatterings also imply numerous inelastic scatterings. Although the inelastic scatterings are less frequent and the fractional energy loss in each inelastic scattering is small, they may (but not always, see example below) accumulate and thus lead to a significant total energy loss.

[28] Comparing any column in Figure 3 it appears that for the same initial pitch angle the backscattered fluence is significantly less for higher energy than lower energy electrons. This result is due to the fact that each panel is normalized to a single precipitating electron with the specified energy and pitch angle (per unit area), not to a single unit of fluence (i.e., cm$^{-2}$ str$^{-1}$ keV$^{-1}$). However, it is clear from the percentages shown in each panel that the percentage of total input energy backscattered is roughly constant with $E_0$ below the loss cone (columns 1–5), and increases with $E_0$ for electrons incident with pitch angles at or above the loss cone (columns 4–5).

[29] Next, it is important to note that as the incident pitch angle increases, the number of atmospheric scatterings required to change the local pitch angle to $\geq 90^\circ$ decreases (on average) and therefore the backscattered distribution begins clustering nearer to the initial input distribution. This effect can best be seen by looking at the third row of Figure 3, corresponding to $E_0 = 0.3$ MeV, where the backscatter for higher incident pitch angles show less diffusion in pitch angle and significantly less energy loss. For example, for ($E_0 = 0.3$ MeV, $\alpha_0 = 78^\circ$)—right at the conventionally-defined loss cone angle (i.e., $\alpha_{\text{up}}(100km)$)$\sim 78%$ of the total incident energy is backscattered. In addition, many of the electrons which are backscattered have retained much of their initial energy ($\approx 290$ keV) but have experienced a relatively large diffusion in pitch angle to between $72^\circ$–$82^\circ$ due to the fact that inelastic scatterings are less frequent than elastic. Above the loss cone angle at, for example ($E_0 = 0.3$ MeV, $\alpha_0 = 80^\circ$, almost 96% of the total initial energy is backscattered with the majority of backscattered electrons having experienced a pitch angle diffuson to between $74^\circ$–$84^\circ$. Note that due to the definition of the loss cone angle, the first scattering for such electrons has to occur above 100 km.

[30] Both of these observations are important with respect to the characteristics of observed LEP events. First, as only $\sim 22%$ of the energy incident at ($E_0 = 0.3$ MeV, $\alpha_0 = 78^\circ$) is initially deposited into the atmosphere, the initial precipitation may not be detectable using subionospheric VLF remote sensing. In addition, since after the first backscatter the pitch angle distribution has broadened ($72^\circ$–$82^\circ$), more energy is deposited into the conjugate hemisphere. Assuming a dipole magnetic field, the pitch angle distribution of electrons leaving one hemisphere is the same entering the conjugate hemisphere at the same altitude (the fact that such is not physically the case for the true magnetic field has a significant effect on the backscatter distribution as discussed in section 3.1). In the case of a dipole field, all of the backscattered electrons—containing 78% of the initial energy—reach the conjugate hemisphere, of which more than 36% of remaining energy is deposited (30% of the initial energy, which is 30% more than in the first deposition of 22%), with only $\sim 50%$ of the total initial energy remaining in the magnetosphere to return to the initial hemisphere. This result suggests that for grazing-incident electrons at this longitude the ionospheric disturbance in the conjugate hemisphere is larger than in the initial hemisphere. For the case of ($E_0 = 0.3$ MeV, $\alpha_0 = 80^\circ$) the results are even more pronounced. The broadening of pitch angles (input into the conjugate hemisphere) to $74^\circ$–$84^\circ$ leads to 13% of the energy which reaches the conjugate hemisphere being deposited (12% of the initial energy, or 200% more than the initial deposition of 6%), leaving roughly 83% of the initial energy un-deposited.

[31] The dependence of the backscatter distribution on both ($E_0$, $\alpha_0$) is of critical importance for the evolution of an LEP event because a typical wave-particle interaction results in an equatorial pitch angle change of only a fraction of a degree [Chang and Inan, 1983; Inan et al., 1999; Bortnik et al., 2006a, 2006b]. This result indicates that a typical precipitating electron distribution is incident onto the atmosphere at a grazing angle of incidence and $\pm 70%$ of the energy is backscattered (as shown in the fourth column of Figure 3) and is subsequently incident on the conjugate hemisphere in its next atmospheric interaction.

3. Geographic Dependence of Electron Deposition and Backscatter

[32] The previous sections have discussed the dependence and importance of energy and pitch angle on the amount of ionization created (i.e., deposited into the atmosphere) as well as the energy and pitch angle distribution of electrons backscattered from the atmosphere. This section focuses on modeling the expected characteristics of precipitating electrons at different longitudes, and shows how observation of an LEP event in one hemisphere can also provide information about precipitation in the conjugate hemisphere whether or not the conjugate event is detected in measurements.

3.1. Geographic Variation of the Loss Cone Angle

[33] As discussed above, in a perfect dipole field the observed characteristics of LEP events at the same latitude
and different longitudes would be expected to be the same, with any (slight) differences attributed to systematic variations in the source lightning spectrum with different locations or to differences in the GCP between the respective transmitters and receivers. However, due to the fact that the Earth’s magnetic field is not a perfectly centered dipole (rather it may be roughly approximated as an off-center, and tilted dipole) there are significant differences in the magnetic field strength as a function of longitude, the most well-known of which is the South Atlantic Anomaly [e.g., Blake et al., 2001].

More relevant to this work is the manifestation of the magnetic field variation in the form of both longitudinal and hemispheric differences in the loss cone angles, as seen in Figure 4a. The importance of these longitudinal and hemispheric differences in loss cone angle were first noted by Inan et al. [1988b] and predicted to influence the occurrence rate and precipitated flux levels of LEP events. Figure 4a shows the variation in the equatorial loss cone angle for both hemispheres and as a function of longitude at $L = 2.5$ and is based upon the IGRF geomagnetic field model [Tsyganenko and Sitnov, 2005]. The loss cone — shown by the shaded region—is defined as the larger of these two angles on any specific field line (conjugate longitudes are connected by the thin diagonal lines) and it is clear that equatorial electrons at varying longitudes behave very differently. The classic example is illustrated by comparison of two electrons with the same pitch angle (e.g., $\alpha_{eq} = 16^\circ$) at longitudes of $\varphi_1 = 0^\circ$E and $\varphi_2 = 100^\circ$E. As shown by Figure 4a it is clear that the electron at $\varphi_1$ is inside the bounce loss cone while the electron at $\varphi_2$ is well outside the bounce loss cone, but inside the drift loss cone (the global maximum loss cone angle [e.g., Blake et al., 2001]).

Taking this scenario a step beyond the simple bounce loss cone example, consider an electron scattered to a pitch angle of $\alpha_{eq} = 16^\circ$ due to a south-going whistler wave launched by a lightning flash in the Northern Hemisphere. In this case the specific bounce motion of the first electron (at $\varphi_1 = 0^\circ$) can again be differentiated. It is evident that a southward traveling (co-streaming) electron scattered to a pitch angle of $\alpha_{eq} = 16^\circ$ penetrates below 100 km and interacts with the atmosphere (possibly backscattering). On the other hand a northward traveling (counter-streaming) electron scattered to a pitch angle of $\alpha_{eq} = 16^\circ$ is far above the northern loss cone. This northward traveling electron does not interact with the northern atmosphere, but instead mirrors in the north (well above 100 km) and returns to the Southern Hemisphere where it only then interacts with the atmosphere.

In order to discuss these differences in the rest of this paper it is necessary to introduce some new terminology. An electron which is below the southern loss cone but above the northern loss cone (e.g., at $\varphi = 0^\circ$ and $\alpha_{eq} = 16^\circ$) is ‘half-trapped’ above the northern atmosphere. Likewise, an electron which is below the northern loss cone, but above the southern loss cone (e.g., at $\varphi = 200^\circ$ and $\alpha_{eq} = 11^\circ$) is half-trapped above the southern atmosphere.

Finally, as shown in subsequent sections, the difference between the two loss cones at conjugate points of the same field line plays a large role in determining the location (i.e., which hemisphere) of LEP precipitation as well as the temporal evolution (i.e., which atmospheric interaction) of the precipitation signature. Figure 4b shows this explicitly where $\Delta \alpha_{eq}^{L} = \alpha_{eq,S}^{L} - \alpha_{eq,N}^{L}$. For example, an electron which is pitch angle scattered by 0.2° (a fairly large equatorial pitch angle-scattering [Bortnik et al., 2006a, 2006b]) inside the loss cone at a longitude of $\varphi = 0^\circ$ is still half-trapped more than 4° above the northern loss cone, while an electron which is scattered 0.2° at a longitude of $\varphi = 200^\circ$ is inside both loss cones. Combining the effects of the loss cone variation with longitude and hemisphere as well as the properties of backscatter discussed in section 2.2 allows the prediction of LEP event characteristics as a function of longitude and hemisphere.

### 3.2. Hemispheric Dependence of Electron Deposition and Backscatter

As discussed briefly in section 2.2, the effect of backscatter is dominant in the hemisphere where the loss cone angle is smaller because absent the case of very large pitch angle scattering (or closely spaced loss cones) the hemisphere with a smaller loss cone cannot have precipitation until backscattered electrons arrive from the conjugate hemisphere. With the inclusion of atmospheric backscatter it is evident that observed LEP event characteristics vary with different longitudes/hemispheres. The best way to illustrate these differences is to show the evolution of an incident distribution of electrons as it interacts with the atmosphere in the two conjugate hemispheres, being incident first on one hemisphere then the conjugate hemisphere, and repeating this process therein. With each atmospheric interaction some (dynamically changing) percentage of the total energy fluence is backscattered or deposited until all the energy is gone or the amount remaining is small enough to be below the background diffusion precipitation rate.
This concept is illustrated in Figure 5, showing the evolution of electrons incident first on the Southern Hemisphere at a longitude of 235°E, with incident energy 0.3 MeV, and pitch angle (at 200 km) of 78°. Note that it is now necessary to specify a longitude (and hemisphere) to completely define the input. In this case the full input parameters are abbreviated as (\(E_0 = 0.3\) MeV, \(\alpha_0 = 78°\), \(\phi_0 = 235°E/S\)).

The evolution of the electron distribution is shown over the first six interactions with the atmosphere (three in each hemisphere). The abscissae show energy (in keV) and the ordinates show the pitch angle of both precipitating and backscattered electrons together on the same axis (at an altitude of 150 km, \(\alpha_{150km}\)). Precipitating (backscattered) electrons are shown between 0°–90° (90°–180°). The color bar shows the fluence of backscattered electrons (on a log scale). The dotted line in each plot is at 90° for reference. The solid line in the top row shows the region of electrons half-trapped above the Northern Hemisphere (pitch angles above 90° and below the solid line). These electrons mirror before reaching the northern hemisphere atmosphere, at or above an altitude of 200 km.

In Figure 5 (top left), the large dot shows the energy and pitch angle of the example monoenergetic precipitation input as it is incident on the atmosphere. The backscattered distribution is shown in the same panel above the dashed line. As indicated by the thick solid horizontal line, only shows the electrons at the conjugate point of 260°E/N.

Finally, each column shows the interaction number (\(\kappa\)) which designates the total number of times the electrons have interacted with the atmosphere (1, 3, 5, ... in the Southern Hemisphere and 2, 4, 6, ... in the Northern Hemisphere). In addition, in the hemisphere with the larger loss cone (the Southern Hemisphere in this case) the solid line indicates the portion of particles which are half-trapped above the conjugate hemisphere. In this case the definition is expanded slightly so that any electron which mirrors at an altitude of 200 km or lower is input into the conjugate hemisphere to ensure that all atmospheric effects on the precipitating electrons are accounted for. Additionally, in order to limit computational requirements, energy which is deposited at altitudes above \(\sim 130\) km, and electrons backscattered with energy \(\leq 50\) keV are not included in subsequent atmospheric interactions (though their contribution is accounted for in terms of conservation of total energy).

In Figure 5 (top left), the large dot shows the energy and pitch angle of the example monoenergetic precipitation input as it is incident on the atmosphere. The backscattered distribution is shown in the same panel above the dashed line. As indicated by the thick solid horizontal line, only
those electrons with pitch angles above $\sim 102^\circ$ are below the loss cone in both hemispheres, so only these electrons are incident in the northern hemisphere as shown in Figure 5 (bottom left). Note that because the magnetic field strength in the conjugate (Northern) hemisphere is larger than that in the Southern Hemisphere, the backscattered electrons which leave the Southern Hemisphere with pitch angles of $102^\circ$–$180^\circ$ (corresponding to $0^\circ$–$78^\circ$) broaden to a range of $0^\circ$–$90^\circ$ in the conjugate hemisphere.

[42] It can be shown as a product of Liouville’s theorem [e.g., Walt, 2005, p. 64] that the flux—$j(\mathbf{E}, \theta)$—along a field line between these two points is conserved. This is due to the fact that the spreading in solid angle due to increasing $B$ (which decreases the flux), is exactly counteracted by the converging magnetic field (concentrating the flux) and therefore the flux of electrons is constant along a field line. Since the fluence is just the time-integrated flux, it is also conserved along the field line, but both are only constant in the direction perpendicular to the velocity of the electron. Since the input for the Monte Carlo simulations needs to be in terms of flux/fluence per unit horizontal area (not area perpendicular to the velocity of the electron), it is necessary to compensate for the changing angle of the magnetic field as well as for the contraction of the flux tube.

[43] The downward directed flux can be calculated through scaling the flux perpendicular to the electron velocity by a factor of $\cos \alpha \cos \theta_B$, where $\theta_B$ is the angle of the magnetic field with the vertical. Likewise, it is possible to relate the backscattered (upward traveling) fluence (per unit horizontal area) leaving one hemisphere to the input (downward traveling) flux/fluence incident upon the conjugate hemisphere through the ratio:

$$\frac{J_d(\mathbf{E}, \alpha_u)}{J_u(\mathbf{E}, \alpha_d)} = \frac{\cos \alpha_d \cos \theta_B}{\cos \alpha_u \cos \theta_B},$$

where the $u$ indicates upward traveling (i.e., backscattered electrons) and the $d$ indicates downward traveling electrons incident in the conjugate hemisphere. Using the above fluence conservation calculation, the portion of the distribution above the solid horizontal line in Figure 5 (top left) is mapped to the Northern Hemisphere and is incident upon the atmosphere as shown in Figure 5 (bottom left) ($0^\circ$–$90^\circ$).

[44] Recall from sections 2.1 and 2.2 that the backscattered distribution can vary significantly with input energy and pitch angle. It is therefore necessary to calculate the response of every input ($\mathbf{E}_0, \alpha_0$) individually and to combine their response into a single distribution. The ABS model calculates this backscattered distribution by interpolating between specifically calculated ($\mathbf{E}_0, \alpha_0$) pairs and combining the results of the backscatter distribution for each ($\mathbf{E}_0, \alpha_0$) in the input distribution in a process very similar to a 2-D convolution. As expected, only those electrons which are near the local loss cone angle experience significant backscatter, the rest are deposited and lost from the magnetospheric system, a fact which can easily be seen by comparing the incident and backscattered distributions in the Figure 5 (bottom left).

[45] The electrons which are half-trapped ($90^\circ$–$102^\circ$ in Figure 5) still mirror at an altitude above 200 km and are therefore not considered in the northern hemisphere backscatter calculation, instead they return to the Southern Hemisphere where they combine with the (now) twice backscattered electrons. The combined distribution is then incident once again upon the Southern Hemisphere (Figure 5, top middle). This process then repeats until all electrons are lost from the magnetosphere as seen in the subsequent panels. Based upon this evolution it is evident that at this longitude there is significantly more precipitation in the Southern Hemisphere than in the Northern Hemisphere. In addition note that the ABS model provides a specific and calculable relationship between the fluence incident in the two hemispheres, linking them, so that knowledge (or measurements) of an electron distribution (or deposition profile) in one hemisphere can also give valuable information about the electron distribution in the conjugate hemisphere even in the absence of observations.

[46] In many cases, these measurements are from VLF remote sensing so while the atmospheric backscatter determines the evolution of electrons in the magnetosphere, it is the secondary ionization created in the ionosphere which can be reliably observed and can therefore be used to predict systematic differences between the characteristics of LEP events in different hemispheres or at varying longitudes, as shown in the next section.

3.3. Longitudinal Variation of Electron Deposition and Backscatter

[47] The previous section showed the hemispheric differences in the evolution of backscattered electrons due to the Earth’s asymmetric magnetic field. This section shows how this asymmetry can also affect precipitation at different longitudes. For example, at $\varphi_1 = 235^\circ$E/S, the loss cones are relatively close together ($\Delta_{\alpha} \sim 0.4^\circ$), however at longitudes where the loss cones are further apart such as at $\varphi_2 = 295^\circ$E/S ($\Delta_{\alpha} \sim 1.5^\circ$), the results are significantly different despite an identical input. This result is shown in Figure 6 which shows the change in electron density as a function of altitude with one profile shown for each atmospheric interaction ($\kappa$). Figures 6a and 6c show the deposition profiles for electrons initially incident at $\varphi_1^N = 235^\circ$E/S (corresponding to Figure 5), and Figures 6b and 6d show the deposition profiles for the same initial distribution of electrons incident at $\varphi_2^S = 295^\circ$E/S with each panel showing the first three depositions at each location.

[48] Notice that the first deposition in the Southern Hemisphere is nearly identical at both longitudes. As previously discussed this result comes about because the backscatter from the atmosphere is primarily determined by the neutral density. The differences in subsequent depositions in the Southern Hemisphere are a result of the differing range of half-trapped electrons at the two longitudes. In the Northern Hemisphere at $\varphi_2$, the portion of electrons which are half-trapped is significantly higher than at $\varphi_1$, which translates to roughly twice as much deposition in the Northern Hemisphere (for $\kappa = 2, 4, 6$) at $\varphi_1^N$ than at $\varphi_2^S$.

[49] The ABS model also indicates a limited backscatter dependence on the dip angle of the magnetic field. However, extensive simulations (not shown here) indicate that there is little difference in backscatter distributions for dip
angles ranging from 77° ($L = 3.0$, $\varphi = 100°$E/N) to 54° ($L = 2.0$, $\varphi = 200°$E/N) and for the energy range in question (i.e., 0.05 MeV–30 MeV). At first sight, this result is in apparent contrast to past work [Wulff and Gledhill, 1974] in which a magnetic dip angle dependence on deposition was calculated for electrons precipitating in relation to the aurora. However, this contrast is easily understood in terms of the limited range of magnetic dip angles which are necessary here, and also because the electrons of interest here are of much higher energy ($\geq 50$ keV versus $\sim 1$ keV).

4. Modeling of LEP Event Characteristics

The previous sections have shown how the backscatter and deposition of electrons can vary as a function of energy and pitch angle as well as how the asymmetry of the Earth’s magnetic field can affect the evolution of backscattered electrons. With this background it is now possible to use the ABS model to investigate the evolution of a realistic distribution of precipitating electrons calculated using a complete wave-particle interaction model as detailed by Bortnik et al. [2006a, 2006b]. [51] The choice of geographic location at which to apply the model is primarily determined by the location of VLF transmitters and receivers available for comparison. The three primary areas of interest for the modeling calculations are $\varphi_1 = 260°$E/N, $\varphi_2 = 290°$E/N, and $\varphi_3 = 295°$E/S. These longitudes correspond to the geographic location of transmitter and receiver pairs used for data comparison (shown in Figure 7 and discussed further in section 5). These three locations are (i) the Central United States, using GCPs between the NML transmitter (in ND) and the HAIL array [Johnson et al., 1999; Peter and Inan, 2004] (in WY, CO and NM) shown in Figure 7a, (ii) the region off the East Coast of the United States using GCPs between the NAA and NAU transmitters (in ME, and PR respectively) and VLF receivers in Boston, MA, Arecibo, PR, and Bermuda shown in Figure 7b, and (iii) the GCP between the NPM transmitter (in HI) and Palmer Station, Antarctica shown in Figure 7c. Because $\varphi_3$ is quite close to the conjugate point of $\varphi_2$, (and because the differences in Southern Hemisphere deposition are negligible at these longitudes as shown in Figure 6) it is sufficient to model $\varphi_2$ and $\varphi_3$ together, referring to each case by the longitude and hemisphere. To clarify the differences between the longitudes each different
Figure 8. Realistic distribution of precipitating electrons between 0.1 MeV and 0.3 MeV incident upon the atmosphere as the result of a whistler interaction [Bortnik et al., 2006a, 2006b; Peter and Inan, 2007]. (a) Change in equatorial pitch angle ($\Delta \alpha_{\text{RMS}}$) as a function of energy and time. (b) Example probability density function based upon a square loss cone at the equator. (c) Same example PDF transformed to the altitude of input into the ABS model (not showing fluence change from equator to 200 km). (d) Scaled (AE8) distribution of precipitating electron fluence input into the ABS model.

case is referred to by the longitude and a N or S superscript to indicate hemisphere: $\varphi_1 = 260^\circ \text{E/N}$, $\varphi_2 = 290^\circ \text{E/N}$, and $\varphi_2 = 295^\circ \text{E/S}$.

4.1. Wave Particle Interaction and Initial Precipitation

[55] The complete modeling of electron precipitation due to lightning requires sophisticated models of lightning discharge waveforms [e.g., Commer, 1997, and references therein], transionospheric propagation (coupling) of VLF waves [Helliwell, 1965; Lehtinen and Inan, 2009], magnetospheric wave propagation [e.g., Inan and Bell, 1977], magnetospheric particle motion [e.g., Walt, 2005] and wave–particle interactions [e.g., Inan et al., 1978, 1989; Bell, 1984] among many others.

[57] Previous work by Peter and Inan [2007] has shown that a forward modeling approach can be successful in determining the amplitude characteristics of individual LEP events. However, the present paper focuses on the systematic differences which can be statistically observed in the characteristics of LEP events at different longitudes. To accomplish this task it is necessary to calculate a realistic input distribution of precipitating electrons which can be input into the ABS model at various longitudes. The input distribution is calculated by the whistler-induced particle precipitation (WIPP) model of Bortnik et al. [2006a, 2006b] which calculates the pitch angle scattering due to a representative lightning flash.

[54] To make comparison with previous work as easy as possible, the same input parameters are used as in the work by Peter and Inan [2007, Figure 7, Case 1]. In this case the +133 kilo-ampere cloud-to-ground causative lightning flash was located at 33.34°N, 260°E. At latitudes of ±10° surrounding the causative flash the lightning energy is coupled through the ionosphere to an altitude of 1000 km using attenuation coefficients taken from Helliwell [1965, Figure 3–35]. Once the wave energy is in the magnetosphere the WIPP model calculates 41 raypaths for each of 130 frequency components ranging from 200 Hz to 60 kHz (spaced roughly logarithmically) resulting in 5330 total rays. The raypaths are calculated using a cold plasma density model based upon the work of Tarcsai et al. [1988], and then interpolated every 0.01° in latitude and every 1 Hz in frequency for a total of ~120 million rays. These rays are assumed to enter the magnetosphere with vertical wave–normal angles due to the horizontally stratified sharp edge of the lower ionosphere. The pitch angle scattering of electrons is calculated using the previously discussed gyroresonance condition [e.g., Chang and Inan, 1983], for each harmonic resonance mode ranging from −5 to 5. This calculation is carried out for each frequency component, accounting for scattering contributions at geomagnetic latitudes of −40° to 40° and for energy components within a factor of two around the resonant energy.

[55] The result is a calculation of pitch angle change as a function of energy and time with each pitch angle change referred back to the equator using the first adiabatic invariant. The WIPP model then accounts for the travel time to the ionosphere for each electron bunch and the root-mean-square (RMS) pitch angle change is placed in the appropriate $\varepsilon$–$t$ bin. The pitch angle scattering for Case 1 of Peter and Inan [2007] is shown in Figure 8a where the abscissa shows time (in seconds), the ordinate shows the energy of resonant electrons and the color bar shows the equatorial RMS pitch angle change. The portion of Figure 8a highlighted by the black box (i.e., 0.1 MeV–0.3 MeV, and 0–5 s) indicates the portion of the overall calculation which is used as an input into the ABS model. The energy range was chosen as a representative sample of those electrons which are typically involved in the production of LEP events [Voss et al., 1998] and the time range was chosen to match previous work [Peter and Inan, 2007] though it is evident from further analysis [Peter, 2007, Figure 4.1b] that the bulk of 0.1–0.3 MeV precipitation is incident upon the atmosphere within the first second.

[56] The next step is to convert the $\varepsilon$–$t$ calculation of RMS pitch angle change to a distribution of precipitating electrons. In order to do this it is necessary to know the pitch angle distribution of the trapped magnetospheric electrons. Since this distribution is not typically known a priori it is assumed to be either a square–shaped distribution (a step function, with a sharp cutoff at the loss cone) or a more realistic sine–shaped distribution (where the electron density is zero below the loss cone and increases as $\sin \alpha$ above the loss cone). In either case the loss cone angle is defined as the larger of the northern and southern hemispheric loss cones. The probability density function (PDF) of the new (scattered) pitch angle distribution can then be calculated as shown by Bortnik et al. [2006a, 2006b, Figure 10]. This PDF represents the scattering distribution and is calculated for each time and at each energy. The contributions are superimposed to form a single scattered pitch angle PDF at each energy. An example of a single (normalized) PDF based upon an initially square pitch angle distribution is
Figure 9. Longitudinal dependence of electron backscatter between hemispheres. Similar to Figure 5, the abscissae show energy in keV, the ordinates show pitch angle in degrees at an altitude of 150 km and the color bar shows fluence in units of cm$^{-2}$ str$^{-1}$ keV$^{-1}$. The dashed line is at 90° for reference. The solid line in the bottom row shows electrons half-trapped above the Northern Hemisphere (between 90° and the solid line) (a) Evolution of electron backscatter for electrons initially incident at 290°E/N. (b) Evolution of electron backscatter for electrons initially incident at 260°E/N.

shown in Figure 8b showing both the initial and scattered equatorial distributions. Figure 8c shows the distribution mapped to an altitude of 200 km using the first adiabatic invariant (not accounting for the changing angle of the magnetic field or the contraction of the flux tube from the equator to 200 km). Note that because the goal is to model an impulsive injection of electrons, only those electrons below the local loss cone (vertical dashed line) are considered to be incident upon the atmosphere. The final step is to scale the distribution to a realistic population. While the actual population of radiation-belt electrons can vary significantly based upon geomagnetic conditions [e.g., Nagai, 1988], the goal of this paper is to compare the LEP signatures at different longitudes under similar conditions and therefore the exact available flux of electrons is not of great importance. However, to simplify the comparison to past work the distribution is scaled using the AE8 MAX radius-invariant (not accounting for the changing angle of the magnetic field as they are represented in the loss cone angle (as shown in Figure 4). These differences can then be incorporated into the input distribution as shown in Figures 8c and 8d.

4.2. LEP Event Backscatter and Deposition Calculations

The process described above in conjunction with Figure 8 is based entirely on previous work, but is just the first step in the calculation of LEP event characteristics. In this section the calculated equatorial pitch angle distribution is used as an input into the ABS model for causative lightning flashes located at an $L$-shell of 2.5 and longitudes of $\varphi^N_1 = 260°E/N$ and $\varphi^S_1 = 290°E/N$ (chosen for comparison with LEP event data recorded at those longitudes as discussed in section 5). The expected LEP event characteristics are then calculated from the results of the evolution of the backscatter and deposition as shown in Figures 9 and 10, respectively. The first four backscatter calculations (two in each hemisphere) for both longitudes are shown in Figure 9 with the incident and backscattered distributions as a function of energy and pitch angle.

[57] The evolution of the backscattered distribution of electrons determines the timing, energy and pitch angle of electrons incident upon the conjugate atmosphere; however, it is the deposition profile (i.e., change in electron density due to deposited electrons) which can be observed using VLF remote sensing. These changes in electron density in turn result in differences in the characteristics of observed LEP events. Figure 10 shows the electron density profile calculations which are carried out in conjunction with the backscatter calculation for each of the three locations, with a separate curve corresponding to each atmospheric interaction $\kappa = 1−7$ (the first four of which are shown in Figure 9).

[58] In VLF data, the LEP event amplitude ($\Delta A$) is defined as the total change (positive or negative, in dB) in the amplitude of the recorded VLF signal. The onset duration ($t_d$) is defined as the time over which the amplitude change occurs (from the 10% to 90% change). The recovery time is defined as the time between the end of the onset of the event (time of maximal deviation from ambient conditions) to when the signal amplitude returns to a value within 10% of the initial (unperturbed) value that it would have had.
in the absence of an event. Finally, the onset delay ($\Delta t$) is defined as the time between the causative lightning flash to when the onset of the event begins (the 10% change point in amplitude). The time of the lightning flash was determined by identifying the causative sferic (signature of a lightning flash in the frequency band of the VLF transmitter signal and observed in the same VLF data [Inan et al., 1996, and references therein]). Understanding precisely what is being measured in data gives a greater understanding of the relationship between observed LEP event characteristics and those determined through modeling as discussed below.

4.2.1. LEP Event Onset Delay ($\Delta t$) Characteristics

The first feature to notice from these figures is that there is no initial backscatter or deposition from the first atmospheric interaction with the Northern Hemisphere ($\kappa = 1$). This result can be seen by noting that Figures 9a (top left) and 9b (top left) are empty and also by noting that there is no solid curve in either Figure 10a or Figure 10b. This lack of initial precipitation is due to the fact that at both longitudes, the Southern Hemisphere loss cone is larger and the population of (counter-streaming) electrons which are initially scattered into the loss cone are still half-trapped above the Northern Hemisphere. The electrons mirror at an altitude above 200 km and traverse the magnetic field line to the Southern Hemisphere where they are below the local loss cone and are incident upon the southern hemispheric atmosphere as shown in Figure 9a (bottom left) and 9b (bottom left). Some electrons are then deposited—as shown by the solid curve corresponding to $\kappa = 2$ in Figure 10c—and others are backscattered, returning to the Northern Hemisphere where those which are now inside the northern hemispheric loss cone (above the solid line in Figure 9 (bottom)) are incident upon the northern hemispheric atmosphere. The resulting deposition is shown by the long-dashed curve ($\kappa = 3$) in Figures 10a and 10b.

The fact that there is no deposition in the Northern Hemisphere until the $\kappa = 3$ interaction means that an LEP event observed at $\phi_N^1$ or $\phi_N^2$ has an onset delay ($\Delta t$) which is increased by one extra bounce period ($\tau_b$) between the causative lightning flash and the onset of the LEP event as previously predicted [Voss et al., 1998; Peter and Inan, 2007]. Likewise, the fact that there is energy deposited into the Southern Hemisphere on the $\kappa = 2$ interaction as shown in Figure 10c means that the onset delay time for LEP events observed at $\phi_S^2$ (caused by northern hemispheric lightning) would have an onset delay time which is $\frac{1}{2}\tau_b$ shorter than corresponding LEP events in the Northern Hemisphere.

4.2.2. LEP Event Magnitude ($\Delta A$) Characteristics

Another important feature to note when comparing the backscatter and deposition results for the two cases is that there is a significantly lower total energy fluence incident at $\phi_S^2$ than at $\phi_N^2$ which can be seen by comparing Figures 9a (bottom) and 9b (bottom) (between 0° and 90°). This lower total incident energy fluence also results in significantly different deposition profiles at the two longitudes and can also be seen by comparing the peak in the deposition profiles at the two longitudes, (long-dashed curves) in Figures 10a and 10b where the total electron density enhancement at $\phi_N^1$ is more than twice as large as that at $\phi_S^2$. The physical reason for these differences is the asymmetry of the Earth’s magnetic field. As shown by the solid lines in Figure 9 (bottom), a significantly larger portion of electrons are half-trapped above the Northern Hemisphere at $\phi_N^2$ than at $\phi_N^1$. At $\phi_N^1$ the difference between the northern and southern loss cones (at the altitude of 150 km shown) is $\sim 15^\circ$, whereas at $\phi_S^1$ the difference is $\sim 30^\circ$.

Similar longitudinal differences in LEP event precipitation magnitude and occurrence rate were previously predicted by Inan et al. [1988b]. Rather than assuming an empty loss cone, their approach presumed (based upon satellite measurements) that the population of trapped electrons available for scattering would vary (with longitude and hemisphere) according to $\Delta A_\text{ecr}$. While the initial approach was different, the net result is similar to that shown in the present work. Their predictions of higher LEP event occurrence rate and event magnitude over the Central/Western United States compared to the East Coast are similar to that presented here and assessed quantitatively below.

Without significant further modeling (beyond the scope of this paper) it is not possible to calculate the expected amplitude perturbation for the above deposition profiles; however, using the fact that there is a significant difference between the deposition profiles means that there should (on average) be a longitudinal variation in the observed amplitude change of LEP events which can be qualitatively calculated using past work [Peter and Inan, 2007].

It is possible to qualitatively estimate the magnitude of the signal perturbations by noting that the paths in the experimental setup (at $\phi_N^1$ and $\phi_S^2$) are both nearly directly north-south, and are both relatively short paths, (i.e., $\lesssim 2$ Mm) and are therefore similar enough for a direct comparison. The path from NPM to Palmer Station, Antarctica on the other hand is $\sim 10$ Mm, and therefore direct comparison between the deposition profiles at $\phi_S^2$ with the two Northern
Hemisphere sites is not possible. However, it is clear from Figure 10c that the deposition in the Southern Hemisphere (i.e., $\eta = 2, 4, 6$) is significantly larger than the deposition in the Northern Hemisphere (at either longitude) and while a direct comparison is not likely to be accurate, it seems logical that the amplitude changes observed at $\varphi_S$ would be larger than either of the Northern Hemisphere locations.

Bearing in mind the similarities (and dissimilarities) of the investigated paths it is possible to quantitatively estimate the amplitude change by applying metrics from previous work [Peter and Inan, 2007, Figure 12b] relating the change in electron density to the amplitude of an observed LEP event. While the integrated line density enhancement ($N_{ILDE}$) metric derived in this past work is technically only valid for the exact path for which it was calculated, the linear relationship between ionization and amplitude implies that a similar (linear) relationship should also exist between LEP events observed on similar GCPs (such as $\varphi_S$).

The $N_{ILDE}$ metric is a single number which characterizes, for the purpose of estimating $\Delta A$, the total integrated ionization created between the transmitter and the receiver (shown in Figure 11a) and between the altitudes of 80 km and 85 km (as shown in Figure 11b). Since the location of each precipitation region is unknown, it is necessary to estimate both the size of the disturbance region and the distance from the observed GCP on which it occurred. Estimating the size of the precipitation region from Peter and Inan [2007, Figure 8d], the deposition region was assumed to have a Gaussian shape with a standard deviation of $s = 90$ km. It was then assumed that an LEP event would only occur when the center of the deposition region was within one standard deviation of the GCP (i.e., $d_c \leq s$). This simple model estimates a mean amplitude change of 0.57 dB for $\varphi_S$, 0.22 dB for $\varphi_N$, and 1.5 dB for $\varphi_S$.

**4.2.3. LEP Event Onset Duration ($t_d$) Characteristics**

The next important feature shown in Figure 10 is that it suggest the possibility for determining a longitudinal dependence of the onset duration time for observed LEP events. The onset of an LEP event persists as long as sufficient energy continues to be deposited into the atmosphere and thus depends on the amount of new ionization created during each atmospheric interaction. Below some minimum detectability threshold the newly introduced ionization is sufficiently small so as to constitute a negligible change in the overall electron density. The onset duration is therefore determined by the number of atmospheric interactions of the inter-hemispherically bouncing electron bunch which con-
tribute new ionization at a level large enough to be detected using VLF remote sensing.

[70] In the specific cases of the calculated deposition profiles at \( \varphi_1^N \) and \( \varphi_2^N \) (shown in Figure 10) the onset duration at each longitude can be inferred by comparing the peak deposition at each of the atmospheric interactions. The secondary ionization created by the first two Northern Hemisphere depositions (i.e., \( \kappa = 3, 5 \)) for both longitudes is on the same order of magnitude as the ambient electron density near the VLF reflection height and it is therefore likely for these ionospheric disturbances to create perturbations on the VLF signal. Conversely, the peak of the \( \kappa = 7 \) profiles are smaller than the ambient electron density, begging the question as to whether or not they create sufficient ionization to be observed. The deposition peak at \( \varphi_1^N \) is more than 40\% larger than that at \( \varphi_2^N \) so if the minimum detectability threshold lies between these values then the onset duration at \( \varphi_1^N \) will be \( 3\tau_b \) while the onset duration at \( \varphi_2^N \) will only be \( 2\tau_b \). The \( \kappa = 6 \) peak deposition at \( \varphi_2^N \) is larger than at either location in the Northern Hemisphere, and while not directly comparable to those two locations as discussed in section 4.2.2, it is likely that the corresponding onset duration observed is \( \geq 3\tau_b \).

4.2.4. LEP Event Recovery Time (\( t_r \)) Characteristics

[71] The last measurable characteristic of LEP events is their recovery time. Like the amplitude change of an LEP event, the recovery time depends significantly on the ambient ionospheric density profile because the recovery of transient ionization is, to first order, determined by the percent change in electron density relative to the ambient electron density profile (i.e. \( \Delta N_e/N_{eo} \)) [Pasko and Inan, 1994]. In addition, previous work [Inan et al., 1988c; Glukhov et al., 1992; Pasko and Inan, 1994] has shown that \( D \)-region ionization at lower altitudes recover at a faster rate than ionization at higher \( D \)-region altitudes. Based upon this evidence alone, the ABS model can also be used to quantitatively predict the recovery time of LEP events at different longitudes. To do this, it is necessary to determine the altitude of peak deposition for each of the three geographic locations.

[72] The differences seen in the peak altitude of deposition are again due to the asymmetric magnetic field. Recalling from Figure 2 that the altitude of deposition is strongly correlated with the energy of precipitating electrons, it is evident that the differences in the altitude profiles for each of the cases are due to the incidence of electrons with a different energy range at each longitude/hemisphere. As previously discussed (at both longitudes in question) the southern hemispheric loss cone is larger and hence a significant range of electrons are half-trapped above the Northern Hemisphere. This fact coupled with the previous demonstration that electrons incident at near loss cone angles experience relatively little energy loss and small pitch angle scatterings means that only the electrons which have lost significant energy have diffused far enough inside the southern loss cone to also be inside the northern loss cone (note again the half-trapped range of electrons shown by the solid horizontal line in Figure 9 (bottom)). This feature can be seen graphically by comparing the incident distributions for each of the three cases in Figure 9. The distribution of input electrons for \( \varphi_1^N \) can be seen in Figure 9a (top right), the range for \( \varphi_2^N \) can be seen in Figure 9b (top right), and the range for \( \varphi_3^N \) can be seen in Figure 9b (bottom). Note that Figure 9a (bottom) represents the conjugate point of \( \varphi_1^N \) but as there are no transmitter to receiver GCPs in this region it is not possible to compare the predicted results to measurements. In each case, the incident electrons are shown between the pitch angles of 0\° and 90\°. It is evident by comparing these three plots that not only is the total incident energy different, but the fluence of high energy electrons (i.e., those near 0.3 MeV) is significantly different at the different geographic locations.

[73] The longitudinal differences in deposition altitude are shown in Figure 10 where the peak deposition at \( \varphi_1^N \) occurs at 83 km, at \( \varphi_2^N \) the peak deposition occurs at 84 km, and at \( \varphi_3^N \) the peak deposition occurs at 81 km. Though the differences in altitude may seem relatively small it is important to note that the attachment and detachment coefficients of electrons (critical factors for determining the chemical relaxation of the atmosphere) vary exponentially with altitude [e.g., Pasko and Inan, 1994] so that even small differences in the altitude profile can have a relatively large influence on the recovery of the LEP event. In addition, because the chemical relaxation of the atmosphere is not dependent on GCP configuration (as is the amplitude change), the recovery time for each of the three locations can all be reasonably compared. For the cases presented, the ABS model predicts the fastest recovery for LEP events observed at \( \varphi_3^N \), the slowest recovery at \( \varphi_2^N \), and an intermediate value at \( \varphi_1^N \).

[74] Before modeling the recovery time it is important to note that because the recovery time is so dependent on the percent change relative to the background levels, that the differences (and unknowns) in the ambient electron density profile introduces significant uncertainty into the recovery time calculations. In an effort to address the variability of ambient electron density, a number of different ambient profiles are used in recovery calculations. In addition to the three profiles described by Inan et al. [1992], two ionospheric profiles from the International Reference Ionosphere (IRI) are used. These five profiles are shown in Figure 11d and display the ambient electron density (in \( \text{cm}^{-3} \)) as a function of altitude.

[75] The recovery of the transient ionization incident upon the atmosphere is calculated using a 5-species model (separated into electrons, positive ions and ion clusters, negative light and heavy ions) of ionospheric chemistry [Lehtinen and Inan, 2007], which calculates the recovery of the ionization as a function of altitude. The model includes the effects of electron attachment and detachment as well as recombination of electrons with positive ions, ion-ion recombination and various paths for conversion between ion species. For the current simulations most of the important rate coefficients were taken from Pasko and Inan [1994], with the electron detachment rate taken from Alexandrov et al. [1997]. An example of an ionospheric recovery is shown in Figure 11c. In this figure the change in ionization relative to the ambient ionosphere (\( \Delta N_e/N_{eo} \)) is shown as a function of altitude and the corresponding time is indicated by the color of the line (shown by the color bar on a log scale with \( t = 0 \) s corresponding to dark blue and \( t = 1000 \) s corresponding to dark red). Note that as expected [e.g., Pasko and Inan, 1994; Lehtinen and Inan, 2007], the recovery of ionization is a strong function of altitude with
higher $D$-region altitudes ($\sim 90$ km) generally recovering more slowly than lower $D$-region altitudes ($\sim 80$ km).

[76] Using the first-order approximation of the recovery time, $t_r \simeq \Delta N_e/N_{e0}$ (between the altitudes where $\Delta N_e/N_{e0}$ is maximum to $5$ km above this altitude), to estimate the recovery time for each of the three cases yields results consistent with the qualitative predictions above. As shown in Figure 11e (top), for every ambient profile the predicted recovery times for $\varphi_1^N$ (ranging from $14$ s to $72$ s) are consistently shorter than the predicted recovery times for $\varphi_2^N$ (ranging from $14$ s to $108$ s), and consistently longer than the predicted recovery times for $\varphi_2^S$ (ranging from $13$ s to $50$ s).

[77] It is clear that due to differences in initial values of $\Delta N_e/N_{e0}$, the variation of recovery time between different ambient profiles ($N_{e0}$) is significantly larger than the recovery time variation of the different deposition profiles ($\Delta N_e$). It is therefore important to compare recovery times for the same ambient profile, and not to compare recovery times between profiles. It is also useful to compare the relative differences between the recovery times (for each separate ambient electron density profile). For the five ambient electron density profiles investigated, the recovery at $\varphi_2^S$ is an average of $28\%$ longer than at $\varphi_2^N$, and $14\%$ longer than at $\varphi_1^N$, while the recovery at $\varphi_1^N$ is an average of $13\%$ longer than at $\varphi_2^S$. This data is shown graphically for the five profiles in Figure 11e (bottom), where despite the variation, the relative length of recovery time is consistently shorter for deposition which occurs at lower altitudes.

[78] Finally as a single number with which to compare to data, it is interesting to note that for the Amb 1 profile shown in Figure 11d (and used extensively in past work as Profile 1 [e.g., Inan et al., 1992; Pasko and Inan, 1994; Peter and Inan, 2007]), the calculated recovery times for the three longitudes are $52$ s, $43$ s, and $37$ s for $\varphi_2^N$, $\varphi_1^N$, and $\varphi_2^S$, respectively.

### 4.2.5. Bounce Period Calculation

[79] As indicated above, to quantitatively determine the onset delay and onset duration times, it is necessary to calculate the bounce period for the distribution of backscattered electrons. Complicating this calculation is the fact that the bounce period of trapped (or half-trapped) magnetospheric electrons is a strong function of energy and pitch angle. Therefore the packet of incident electrons continues to disperse in time as it mirrors between hemispheres. To assess the range of bounce periods which are present in a distribution of backscattering $0.1$ MeV-$0.3$ MeV electrons it is useful to compare the variability of $\tau_b$ with the energy and pitch angle which are present in a typical distribution of backscattered electrons. This comparison is shown in Figure 12 where contours of constant bounce period (calculated using a dipole magnetic field model, but accounting for relativistic factors [Walt, 2005, p. 44]) are overlaid on backscatter distributions for electrons incident at $(E_0 = 0.1$ MeV, $\alpha_0 = 78^\circ)$ and $(E_0 = 0.3$ MeV, $\alpha_0 = 78^\circ)$. Notice that in both cases the vast majority of the backscattered electrons lie along one of the contours of constant bounce period. This fact indicates that for electrons incident upon the atmosphere near the loss cone the effect of energy loss on $\tau_b$ (increasing $\tau_b$) is compensated by the decrease in pitch angle (decreasing $\tau_b$) and electrons of the same initial energy incident upon the atmosphere continue to stay bunched together in agreement with previous work [Voss et al., 1998].

[80] In this context it is important to note (not shown here) that the time over which the atmospheric backscatter takes place is a negligible (<5%) contribution to the bounce period. For the specific distributions shown, the bounce period of $0.1$ MeV electrons are bunched near $0.45$ s, and $0.3$ MeV electrons are bunched near $0.3$ s which define the limits of the bounce period for the modeled distribution. These values indicate that for the results shown in section 4.2.1 the onset delay in the Northern Hemisphere ($\Delta t_b$) is increased by $\tau_{b,\text{min}} = 0.3$ s while the onset delay in the Southern Hemisphere ($\Delta t_b$) is decreased by $\frac{1}{2}\tau_{b,\text{min}} = 0.15$ s. Based upon the modeling work of Peter and Inan [2007], this result indicates onset delay times of $\Delta t_b = 0.7$ s, and $\Delta t_b = 0.55$ s. Likewise for $\varphi_1^N$ and $\varphi_2^S$, the predicted onset duration is $c_\Delta = 3\tau_b$. Based upon the range indicated in Figure 12 these onset durations lie between $0.9$ s and $1.3$ s depending on the relative contributions of each energy band to the complete distribution. For the $\varphi_2^N$ longitude, the predicted onset duration is $c_\Delta = 2\tau_b$ which corresponds to a range of $0.6$–$0.9$ s.

### 5. Comparison With Observations

#### 5.1. Ground-Based Observations of VLF Transmitter Signals

[81] The previous section laid out the methodology for calculating LEP event characteristics and predicted results at three separate longitudes/hemispheres. It is now possible to compare the predicted characteristics of LEP events to those observed in data. The challenge in this comparison is that characteristics of LEP events vary widely on an individual...
basis due to the number of variables which contribute to their creation. For example, the peak current of the lightning flash relates to the amplitude and frequency spectrum of the whistler wave in the magnetosphere [e.g., Bortnik et al., 2006a, 2006b]. The whistler wave amplitude and frequency spectrum in turn affects the number and energy of electrons scattered into the loss cone [Chang and Inan, 1983], as does the flux of trapped particles available for scattering [Peter and Inan, 2004]. In addition, the magnetospheric cold plasma density can affect the individual raypaths thus changing the location of the precipitation footprint on the ionosphere [Peter and Inan, 2007]. Finally, the electron density of the ambient ionosphere affects the modal composition of the subionospheric VLF waves [e.g., Poulsen et al., 1993b; Lehtinen and Inan, 2007] which can affect the amount of scattering observed from any disturbance region on a subionospheric VLF signal for a given GCP.

[s2] With all of these variables affecting the characteristics of LEP events the most tractable way to separate the longitudinal dependence from the variation among individual events is with a statistical analysis of event characteristics. To create this statistical database it is necessary to examine LEP events from transmitter to receiver GCPs which are as close to north-south as possible to limit the measured longitudinal extent. As discussed in section 4 and shown in Figure 7, the paths chosen for this study are at (geographic) longitudes ($\phi$) of 260°E/N, 290°E/N and at 295°E/S. The selected data sets came from a cursory search of several years of data to identify time periods (at each location) in which a large number of LEP events occurred. The spring (April and May) and autumn months (September and October) from the years of 2005, 2006 and 2008 provided the best data for this study. Each data set was then analyzed in detail with every LEP event large enough to be observed above the noise floor counted for the statistics with a standard set of criteria used to characterize the events.

[s3] Due to varying signal-to-noise ratios (SNR) throughout the observation periods and between different observation sites, some parameters were not always discernible. While the amplitude change and onset duration time are (nearly) always well defined and easily identifiable, the recovery time and specifically the onset delay are sometimes obscured. Identification of the onset delay can often be very difficult primarily due to the large occurrence rate of lightning relative to the number of LEP events. On some occasions there are several possible causative lightning flashes identified in the data and in still others the SNR is low enough that it is impossible to identify any causative lightning flash. In the cases where no lightning flash could be identified the event was removed from the statistics entirely due to the fact that the causative mechanism could not conclusively be attributed to lightning. In the cases where it is clear that the perturbation of the VLF signal is caused by lightning, but there are multiple possible causative flashes, then the onset delay is considered unknown but the other characteristics of the event are recorded. The measurements of recovery time are also treated in this manner. Typically lasting many tens of seconds, the recovery time can be interrupted on occasion by the natural variation of the ionosphere or by a subsequent LEP event (or other sudden ionospheric disturbance). In these cases (since the classification of the event is not based upon the recovery time), the recovery time is listed as unknown, but the other characteristics are recorded.

[s4] These complications are reflected in the number of events ($n$) included in the statistical analysis as shown in Figures 13a–13c. As can be surmised by the number of events with some unidentified characteristics the NPM signal received at Palmer ($\phi_2^E$) had an excellent SNR and all characteristics could be identified for each event. The paths from the NML transmitter to the HAIL Array ($\phi_1^E$) and those comprising the paths off the East Coast of the U. S. ($\phi_2^W$) had a lower SNR resulting in greater difficulty identifying all event characteristics. In addition, the large number of lightning flashes in the vicinity of these two regions further complicated the identification of the causative sferic.

[s5] The results of the statistical data analysis are summarized in Figure 13 and though there is an obvious spread in each of the observed characteristics (due to factors described above) it is useful to look at the mean value of each characteristic at each location. Each panel displays the mean and standard deviation for the plotted histogram. Taking this mean value to represent a ‘typical’ LEP event at the specified location it is clear that the prediction of a longitudinal variation is borne out in the data and that it is now possible to evaluate the ABS model results discussed in section 4.2. Table 1 shows the comparison between the modeled LEP characteristics and those observed in the data. Table 1 is divided up into the predictions which come directly from the ABS model (onset delay and onset duration) and the predictions which require further modeling and unknowns (amplitude change and recovery time).

[s6] The amplitude predictions match the measurements very well with the exception of the predicted amplitude change for the southern hemispheric path (i.e., $\phi_2^W$). Even expecting the linearity of the integrated line density enhancement approximation [Peter and Inan, 2007] to fail when extended to such a long path, it is still surprising that the mean amplitude change of events at $\phi_2^W$ is smaller than at either $\phi_1^E$ or $\phi_2^E$. One possible explanation for this discrepancy is the fact that on such a long over-sea path there is likely only a single dominant subionospherically propagating mode [Inan and Carpenter, 1987]. The presence of only a single mode can contribute to generally smaller signal perturbations in a couple of ways. First, since there is only a single mode it may be that the disturbance region does not have significant ionization in the altitude range where the electric field of the mode is large, resulting in a small scattering [e.g., Lehtinen and Inan, 2007]. In addition, as shown by Marshall [2009, Figure 4.5], the presence of interference nulls in the ground amplitude of subionospheric wave propagation can contribute to the observation of larger signal perturbations. Since there are no modal interference nulls from only a single subionospherically propagating mode it may be that the signal perturbations due to ionospheric disturbances are generally smaller than on shorter paths. It is useful to note here that VLF phase perturbations associated with LEP events may, in contrast, be much more clearly observable on all-sea-based paths such as that from NPM to Palmer, as was previously noted [Wolf and Inan, 1990; Inan and Carpenter, 1987]. Phase perturbation events are indeed much more numerous in data sets on this path; however, phase perturbations are not used in the cur-
rent work since data from other longitudes have many more amplitude events. The observed mean of all remaining LEP characteristics match the predicted values quite closely. The prediction of recovery time based upon the atmospheric chemistry model match well with the observed results where in general the mean recovery time observed at $\varphi^N_1$ is the shortest followed by $\varphi^S_2$, and the longest mean recovery time is observed at $\varphi^N_2$. Additionally, for one ambient electron density profile (Amb 1) used extensively in previous work [e.g., Pasko and Inan, 1994; Peter and Inan, 2007], the match between observed and predicted recovery is rather exceptional. For $\varphi^N_1$ the mean observed and predicted recovery times are 42 s and 43 s, respectively. For $\varphi^S_2$ the mean observed and predicted recovery times are 52 s and 50 s, respectively and for $\varphi^N_2$ the mean observed and predicted recovery times are 35 s and 37 s, respectively. The recovery calculation is strengthened by noting that even within the wide range of recovery times predicted using different ambient profiles, the relative differences between the recovery times are consistent. These results are summarized in the bottom section of Table 1 in which the average percent difference calculated by the recovery model is compared with the average differences observed in data.

For all three cases the results are quite similar, giving further confidence in the model results.

The two timing characteristics (onset delay and onset duration) which are most closely tied to the results of the

Figure 13. Statistically identified characteristics of LEP events at different longitudes. Each panel above is a histogram (with peak normalized to unity) for each of the four LEP characteristics: Onset delay ($\Delta t$), Onset duration ($t_d$), Amplitude ($|\Delta A|$), and Recovery time ($t_r$). (a) Characteristics of LEP events observed at $\varphi^N_1 = 260^\circ$E/N. (b) Characteristics of LEP events observed at $\varphi^N_2 = 290^\circ$E/N. (c) Characteristics of LEP events observed at $\varphi^S_2 = 295^\circ$E/S.

Table 1. Comparison of Model Predictions (Section 4) and Data (Section 5) for the Three Longitudes Considered

<table>
<thead>
<tr>
<th>Location</th>
<th>Characteristic</th>
<th>Model</th>
<th>Data ($\mu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi^N_1$</td>
<td>$\Delta t$</td>
<td>0.7 s</td>
<td>0.77 s</td>
</tr>
<tr>
<td>$\varphi^N_1$</td>
<td>$t_d$</td>
<td>0.9–1.3 s</td>
<td>0.92 s</td>
</tr>
<tr>
<td>$\varphi^N_1$</td>
<td>$</td>
<td>\Delta A</td>
<td>$</td>
</tr>
<tr>
<td>$\varphi^N_1$</td>
<td>$t_r$</td>
<td>43 s$^b$</td>
<td>42 s</td>
</tr>
<tr>
<td>$\varphi^N_2$</td>
<td>$\Delta t$</td>
<td>0.7 s</td>
<td>0.73 s</td>
</tr>
<tr>
<td>$\varphi^N_2$</td>
<td>$t_d$</td>
<td>0.6–0.9 s</td>
<td>0.63 s</td>
</tr>
<tr>
<td>$\varphi^N_2$</td>
<td>$</td>
<td>\Delta A</td>
<td>$</td>
</tr>
<tr>
<td>$\varphi^N_2$</td>
<td>$t_r$</td>
<td>52 s$^b$</td>
<td>50 s</td>
</tr>
<tr>
<td>$\varphi^S_2$</td>
<td>$\Delta t$</td>
<td>0.55 s</td>
<td>0.5 s</td>
</tr>
<tr>
<td>$\varphi^S_2$</td>
<td>$t_d$</td>
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<td>0.95 s</td>
</tr>
<tr>
<td>$\varphi^S_2$</td>
<td>$</td>
<td>\Delta A</td>
<td>$</td>
</tr>
<tr>
<td>$\varphi^S_2$</td>
<td>$t_r$</td>
<td>37 s$^b$</td>
<td>35 s</td>
</tr>
<tr>
<td>$\varphi^N_1$ versus $\varphi^S_2$</td>
<td>$\Delta t_{13}/t_1$</td>
<td>28%</td>
<td>30%</td>
</tr>
<tr>
<td>$\varphi^N_2$ versus $\varphi^S_2$</td>
<td>$\Delta t_{12}/t_1$</td>
<td>14%</td>
<td>18%</td>
</tr>
<tr>
<td>$\varphi^N_1$ versus $\varphi^S_2$</td>
<td>$\Delta t_{23}/t_1$</td>
<td>13%</td>
<td>15%</td>
</tr>
</tbody>
</table>

$^a$The first three sections compare each longitude separately. The last three rows compare the relative recovery times between the longitudes.

$^b$For Amb 1 profile.
ABS model (they need no further modeling as do the amplitude change and recovery time) are in good agreement with the mean observed LEP characteristics. The predicted onset delay for all three cases is within 10% of the mean observed onset delay and the predicted onset duration matches well with the lower bound of the predicted range for the onset duration. The fact that the mean observed onset duration is consistently at the lower bound of the predicted results indicates that the higher energy electrons (0.3 MeV) likely play a more important role in determining the onset duration of the LEP event. This can be explained in terms of Figure 2 by noting that (for grazing incidence i.e., $\alpha > 45^\circ$) and the altitude of peak deposition for precipitating electrons of initial pitch angle $\alpha_i$. The result of this is an onset duration more than twice the amount of total ionization created by the 0.1 MeV electrons. The result of this is an onset duration still larger than the peak deposition due to a 0.1 MeV electrons and additionally the 0.3 MeV electrons create more than twice the amount of total ionization created by the 0.1 MeV electrons. The result of this is an onset duration which is likely dominated by the 0.3 MeV electrons with a shorter bounce-period.

While the mean observed characteristics match the typical events modeled in section 4, Figure 13 shows that for each characteristic there is considerable spread around the mean. This spread is expected due to the large number of variables which contribute to the production of an LEP event. Most of the variation can be easily explained by the variation in the causative lightning flash peak current and frequency content. In the case of a strong lightning flash the generated whistler also has a large magnetic field component and likely scatters electrons more effectively into the loss cone. In such a case it is plausible that the scattering is of sufficient magnitude to scatter electrons deep enough into the loss cone that observable levels of precipitation can occur without the necessity of atmospheric backscatter. Likewise, if the scattering is very large, then a larger number of electrons are scattered into the loss cone and the deposition profile for subsequent atmospheric interactions (beyond the $\kappa = 6$ or 7 scattering for the Southern or Northern Hemisphere respectively) may be above the minimum detectability threshold. In the case of very weak scattering it is possible that the deposition profile only remains above the minimum detectability threshold for one atmospheric interaction. Additionally, differences in the frequency content of the whistler wave can result in variations in the energy of precipitated electrons [e.g., Lauben et al., 2001] which can in turn affect the characteristic magnitude, onset duration and recovery time of observed events. Finally, a long onset delay can be explained by the possibility of lightning at lower latitudes which can cause precipitation at higher latitudes due to scattering by magnetospherically reflecting whistlers [Bortnik et al., 2006a, 2006b]. In such a case the whistler wave energy reflects several times in the magnetosphere (moving in a poleward direction) before it reaches the latitude of observation, each whistler bounce taking ~0.8 seconds between equatorial crossings.

### 5.2. Satellite Observations of Backscattering Electrons

[90] The accuracy and validity of the ABS model can be further evaluated by comparison with in-situ observations of electrons scattered into the bounce loss cone as reported by Voss et al. [1998]. This rare and impressive data set shows the detailed evolution of electrons scattered into the bounce loss cone by lightning, and tracks the backscattered electrons as they bounce between hemispheres. These events were recorded by a number of particle detectors on-board the low-altitude (200 km) S81-1/SEE satellite. The two detectors most important for comparison with this work are the TE2 and ME1 detectors, oriented at 90° and 0° from zenith as shown graphically in Figure 14a. The best-defined of the recorded events [Voss et al., 1998, Figure 5b] was recorded at an altitude of 200 km at $L = 2.23$ and a geographic longitude of $\varphi = 277^\circ E/N$. For this case the magnetic field had a magnetic dip angle of $\sim 65^\circ$, meaning that the TE2 detector was oriented to detect precipitating electrons at grazing incidence (high pitch angles of $45^\circ \lesssim \alpha_{200km} \lesssim 85^\circ$), and the ME1 detector was oriented to detect backscattered electrons with pitch angles deep inside the loss cone ($\alpha_{200km} \lesssim 55^\circ$).

[91] To compare to this data set, it is necessary to model the physical conditions under which the observations by Voss et al. [1998] were made as closely as possible. This matching is accomplished using the same pitch angle scattering calculation shown in Figure 8a but limiting the input distribution of electrons to between 0.1 MeV and 0.2 MeV as shown by Voss et al. [1998, Plate 1]. This distribution was then transformed to a 200 km altitude at $\varphi = 277^\circ E/N$ for input into the ABS model.

Figure 14. (a) Illustration of the detector setup on-board the SEE satellite showing the TE2 and ME1 detectors, as well as their approximate orientation relative to the magnetic field ($\theta_B$) and precipitating electrons of initial pitch angle $\alpha_i$. (b) Comparison of the ABS model and data from the SEE satellite [Voss et al., 1998, Figure 5b] showing the evolution of backscattered electrons in data.
[92] Because the ABS model calculates only the total fluence incident and backscattered from each hemisphere it is necessary to make assumptions as to the duration of the incident pulse. Consistent with previous work [Lauben et al., 1999], the chosen pulse shape is Gaussian with a standard deviation of 0.5 s. To calculate the time of arrival for each pulse on the atmosphere, a mean bounce period \( \tau_b = 330 \text{ ms} \) (corresponding roughly to the bunching time of a backscattered distribution incident at the intermediate energy of 150 keV) is used. The results of this analysis are shown in Figure 14b overlaid upon the original data points from [Foss et al., 1998, Figure 5] for the TE2 and ME1 detectors. Note that the range of pitch angles included in the two curves (grazing incidence and deep incidence) match closely to those detected by the respective detectors. Because the number of precipitating electrons is primarily determined by the lightning peak current and the background density of electrons in the magnetosphere available for scattering, the ABS model results are normalized such that the first backscattered peak (green curve at \(-0.4 \text{ s}\)) is equal in magnitude to the first backscattered peak observed in data. Figure 14b shows excellent agreement between the ABS model and the observed backscatter distribution. In particular the number of electrons observed on the TE2 detector relative to that on the ME1 detector is in excellent agreement (i.e., \(-10 \text{ times}\) larger) with the predicted ratio. Furthermore the rate of decay predicted for both the TE2 and ME1 detector match well through \( \kappa = 7 \) backscatters before the remaining distribution is comparable to the noise floor of the measurements. It is also important to note that this same calculation carried out at other longitudes (e.g., \( \varphi = 260^\circ \text{E/N, not shown} \)) does not match the data presented by Voss et al. [1998] (at \( \varphi = 277^\circ \text{E/N} \)) in terms of either relative backscatter magnitude, or in decay rates of the two curves.

[93] The excellent agreement between the ABS model and the observed distribution provides further confidence in the accuracy of the ABS model and indicate its usefulness for in-situ as well as ground-based predictions.

6. Summary and Conclusions

[94] This paper details a novel method for predicting the longitudinal dependence of LEP event characteristics at any midlatitude L-shell by accounting for the effects of precipitating electrons which are backscattered from the atmosphere. The model of atmospheric backscatter is based upon the calculation of \(-16,000\) individual atmospheric backscatter responses for monoenergetic electron beams with a single incident pitch angle using the Monte Carlo model detailed by Lehtinen et al. [1999]. Treating the backscatter from the atmosphere as a linear system, each of these backscatter responses can be thought of as an atmospheric response to an input pair \( (E_0, \omega_0) \). In this way, not only can these backscatter responses be used to determine the backscatter of any arbitrary input electron distribution, but looking at the characteristics of the individual atmospheric responses reveals insight into the previously undervalued importance of incident pitch angle on deposited and backscattered electron characteristics.

[95] The next major component of the ABS model is the inclusion of the asymmetric magnetic field in calculations of the pitch angle of backscattered electrons entering the conjugate hemisphere. The varying magnetic field strength at conjugate points of the same field line can cause an expansion or contraction of the flux tube in the conjugate hemisphere. An electron just inside the local loss cone in one hemisphere may therefore be either deep inside the loss cone in the conjugate hemisphere or conversely may be so far above the loss cone that it cannot reach the conjugate atmosphere at all. The ABS model accounts for these differences through use of the first adiabatic invariant and by scaling the distribution in each hemisphere (using a factor of \( \cos \cos \theta_l \theta_g \)) to conserve the total horizontal input energy in transferring the electron bunch from one hemisphere to the other. These calculations are critical especially because the difference between hemispheric loss cones means that for many longitudes (especially near the South Atlantic Anomaly), backscatter from the Southern Hemisphere is the only explanation which can account for LEP events observed in the Northern Hemisphere without exceptionally large equatorial pitch angle scattering (of \(-4^\circ\)), much larger than predicted in the literature [Chang and Inan, 1983; Lauben et al., 1999; Bortnik et al., 2006a, 2006b].

[96] The final step for predicting the characteristics of LEP events using the ABS model is to create a realistic distribution of precipitating electrons as input. As detailed in section 4.1 this is done using the WIPP model of Bortnik et al. [2006a, 2006b] which calculates the equatorial pitch angle change for magnetospheric electrons. This scattering is then combined with a realistic sinusoidal pitch angle distribution and scaled by the AE8 MAX trapped flux model to produce the final number density (and pitch angle distribution) of precipitating electrons. This distribution is then converted to the input longitude and altitude for input into the ABS model.

[97] Using the above methodology to calculate deposition profiles and backscatter distributions, section 5 discusses the comparison between predicted LEP characteristics at three longitudes and those observed in data. The model predictions match data quite well in each case.

[98] Confirming the prediction of Peter and Inan [2007], there is a one bounce-period delay in the Northern Hemisphere, and a corresponding one-half bounce period advance in the Southern Hemisphere in the observed onset delay \( (\Delta t) \) at the longitudes investigated which was previously undocumented.

[99] The onset duration of LEP events also exhibit a longitudinal and hemispheric variation based upon the number of times an incident distribution of electrons can backscatter and still create atmospheric ionization above a minimum detectability threshold. At the longitudes investigated, the three bounce-period duration predicted by the ABS model matches well with observations over the Central United States and in the Southern Hemisphere near Antarctica. Likewise, the prediction of a two bounce-period duration for LEP events observed off the East Coast of the United States is also observed in the data.

[100] The longitudinal and hemispheric variation of LEP event amplitude change occurs due to the difference between the northern and southern hemispheric loss cones, \( \Delta \omega_{eq} \). However, the ABS model provides only atmospheric deposition profiles which must be further modeled using the methods of Peter and Inan [2007] to quantify the exact variation in amplitude. A preliminary investigation using the
integrated line density enhancement approximation [Peter and Inan, 2007] yields results which are consistent with observations on pathways of similar length and location (in the Northern Hemisphere), but as expected do not match well with the long distance path in the Southern Hemisphere.

[10] The recovery time also requires significant modeling beyond the scope of the ABS model to compare with observed results. However, a first-order approximation of the recovery time using a 5-species model of atmospheric chemistry to calculate the change in electron density relative to the ambient (ΔN_e/N_0) yields results which are qualitatively consistent over a range of different ambient ionospheric profiles. Furthermore, for the ambient ionosphere Amb 1, used extensively in past work, the quantitative predictions for all three longitudes match exceptionally well with the mean recovery time observed in the data.

[102] The preceding sections have shown the importance of atmospheric backscatter in determining the characteristics of LEP events. By accounting for atmospheric backscatter it is possible to accurately predict all the observable characteristics of LEP events. Furthermore, by combining the effects of atmospheric backscatter with previously calculated radiation belt electron loss rates due to lightning at a single longitude [Peter and Inan, 2007] it is possible to develop a global estimation of radiation belt electron loss due to lightning. Finally, by accounting for the asymmetry of the Earth’s magnetic field in backscatter calculations it is possible to link the total electron precipitation at conjugate points of the same field line and to estimate radiation belt electron loss rates due to lightning even in remote portions of the world where there are few VLF receivers and even fewer VLF transmitters (e.g., over much of Africa).


References


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