GEOPHYSICAL RESEARCH LETTERS, VOL. 27, NO. 8, PAGES 1095-1098, APRIL 15, 2000

Trapped energetic electron curtains produced by thunderstorm driven relativistic runaway electrons

N. G. Lehtinen, U. S. Inan, and T. F. Bell

STAR Laboratory, Stanford University, Stanford, CA 94305

Abstract. Relativistic runaway electron beams driven upward by intense lightning-generated quasielectrostatic (QE) fields undergo intense interactions with the background magnetospheric plasma, leading to rapid nonlinear growth of Langmuir waves. The beam electrons are strongly scattered by the waves in both pitch angle and energy, within one interhemispheric traverse along the Earth's magnetic field lines. While those electrons within the loss cone precipitate out, most of the electrons execute bounce and drift motions, forming detectable trapped curtains of energetic electrons.

1. Introduction

Energetic runaway electrons above thunderstorms, driven upward by intense QE field following positive cloud-to-ground (+CG) discharges, has been put forth [*Roussel-Dupré et al.*, 1998 and references therein] as a fundamentally new plasma acceleration process [*Gurevich et al.*, 1992], leading to the generation of terrestrial γ-ray flashes (TGF) [*Fishman et al.*, 1994; *Lehtinen et al.*, 1996; 1997]. In this paper, we consider the pitch angle and energy scattering of the electrons which escape from the ionosphere due to beam-plasma interactions in the radiation belts, leading to the formation of trapped electron "curtains".

2. Formation of the runaway beam

The upward driven relativistic runaway beam is produced as a result of the QE field which temporarily exists at high altitudes following +CG lightning discharges. The pre-discharge thundercloud can be modelled as a system of point or horizontal disk charges $+Q$ and $-Q$ at altitudes h_+ and h_-, respectively. The lightning discharge lowers $+Q$ to the ground in time $\tau = 1$ ms, thereby creating the QE field due to uncompensated space charge [*Pasko et al.*, 1997; *Lehtinen et al.*, 1997].

The intense downward QE field exists until the conducting upper atmosphere relaxes in several to tens of ms, during which time seed relativistic electrons (e.g., produced by cosmic ray showers) are accelerated upward, colliding with air molecules in an avalanche process, resulting in the formation of an intense relativistic runaway electron beam [*Roussel-Dupreé et al.*, 1998 and references therein; *Lehtinen et al.*, 1997; 1999].

Since the duration of the QE field is much longer than the time of travel for relativistic electrons from cloud tops to the ionosphere, a stationary continuity equation [*Bell et al.*, 1995] can be used to calculate the number density N_R of the runaway beam:

$$v_R \frac{dN_R}{dz} = \gamma_R(N_m, E)N_R + S_0(z) \tag{1}$$

where v_R is the runaway velocity, γ_R is the avalanche growth rate, $S_0(z)$ is the source of energetic electrons from cosmic rays. The growth rate $\gamma_R > 0$ only when the electric field exceeds the runaway threshold field E_t proportional to the neutral air density N_m [e.g., *Gurevich et al.*, 1992]. Using a Monte Carlo model [*Lehtinen et al.*, 1999], we calculate γ_R as a function of N_m and electric field E and the velocity v_R, which is found to be $v_R \simeq 0.9c$. The source $S_0(z)$ is due to cosmic rays, with S_0 proportional to N_m and $S_0 = 10\ \mathrm{m}^{-3}\mathrm{s}^{-1}$ at 10 km [*Bell et al.*, 1995].

To calculate the QE field, we assume an air conductivity profile $\sigma = \sigma_0 e^{z/H}$, with $H = 10$ km, consistent with measurements [*Holzworth et al.*, 1985]. Rapid removal of $+Q$ by lightning is equivalent to instantaneous placement of $Q_{\mathrm{eq}} = -Q$ at the same location. Since the E field relaxation time (ϵ_0/σ) at the altitudes of avalanche is relatively long, the field of Q_{eq} can be assumed to be the same as in vacuum. The driving QE field is then given as the sum of the stationary pre-discharge thundercloud field in the stratified conducting atmosphere [*Volland*, 1984, p. 34] and the vacuum field of Q_{eq}, as well as the fields of the corresponding image charges due to conducting Earth's surface, all of which can be expressed in terms of compact analytical expressions. The total E field is used to calculate γ_R (using results of [*Lehtinen et al.*, 1999]), in solving equation (1), for which we assume $N_R = 0$ at an initial altitude 18 km for $h_+ = 10$ km and 20 km for $h_+ = 20$ km. We take the upper limit of our solution domain to be the ionospheric boundary at 80 km.

Based on past work [*Lehtinen et al.*, 1999], the magnitude of the runaway electron flux escaping upward from the ionosphere is insignificantly affected by the geomagnetic field for latitudes >45°. The density N_R of the runaway beam escaping from the lower ionosphere determined by numercal solution of equation (1) is shown in Figure 1 for $h_+ = 10$ km and $h_+ = 20$ km. We see that N_R depends nonlinearly on the discharge value Q, and is also greater for a higher h_+.

3. Interaction of the runaway beam with the magnetospheric plasma

The relativistic runaway electron beam entering the magnetosphere can interact with the background plasma during its field-aligned transport between hemispheres. The problem of a cold relativistic beam travelling between geomagnetically conjugate points was considered by *Khazanov et al.* [1999]. In our case, the beam exhibits a wide range of electron energies [*Lehtinen et al.*, 1999], so that we must consider the growth rate Γ of a hot beam-plasma instability.

Paper number 1999GL0140765.
0094-8276/00/1999GL0140765$05.00

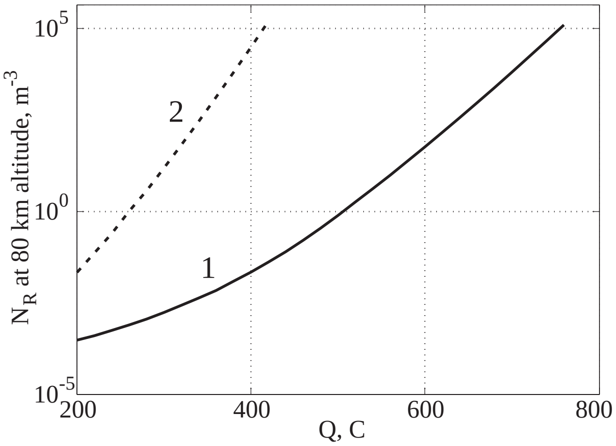

Figure 1. The number density N_R of the runaway electron beam escaping upward from the lower ionosphere: (1) $h_+ = 10$ km; (2) $h_+ = 20$ km. The negative charge altitude is $h_- = 5$ km in both cases.

We set out to determine whether Γ is high enough to lead to significant growth of Langmuir waves during a single interhemispheric traverse. If such growth does occur, the beam loses energy to waves and is also scattered in pitch angle. If, on the other hand, Γ is small, then we can conclude that the beam remains largely intact during its traverse, with most of the particles arriving at the conjugate hemisphere with pitch angles well below the loss cone and thus precipitating into the lower ionosphere.

To determine Γ, we adopt the usual procedure of using the dispersion relation describing the complex permittivity $\epsilon(\omega, k)$ of the system to evaluate the imaginary part of frequency ω.

3.1. Growth rate Γ of the beam-plasma instability

The beam-plasma permittivity for a set of beams α (all having velocities parallel to the same axis) is given by [e.g., *Stix*, 1962, p. 111]:

$$\epsilon = 1 - \sum_\alpha \frac{q_\alpha^2 N_\alpha}{\epsilon_0 m_{\alpha\|}} \frac{1}{(\omega - kV_\alpha)^2}$$

where N_α are the densities of the beams, m_α and q_α are the masses and the charges of the particles constituting the beams. For relativistic beams, we use the mass $m_\| = m/(1-\beta^2)^{3/2}$, where $\beta = v/c$. An individual relativistic "hot" beam having a range of parallel momenta p can be represented as a superposition of beams each with density $N_\alpha = N_R f(p)\Delta p$, with $f(p)$ being the momentum distribution function normalized to 1, so that the permittivity of a "hot beam-cold background plasma" system is given by

$$\epsilon(\omega, k) = 1 - \frac{\omega_0^2}{\omega^2} - \omega_0^2 \frac{N_R}{N_0} \int \frac{(1-\beta^2)^{3/2} f(p) dp}{(\omega - kc\beta)^2}, \qquad (2)$$

where N_0 is the magnetospheric ambient plasma density and $\omega_0 = \sqrt{e^2 N_0/(\epsilon_0 m_e)}$ is the corresponding plasma frequency.

The momentum distribution of electrons in the relativistic runaway beam escaping upward from the lower ionosphere has been evaluated using a Monte Carlo method [*Lehtinen et al.*, 1999] and can be approximated with a log-normal analytical fit:

$$f(p) = \frac{1}{p} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{1}{2\sigma^2} \left[\ln\left(\frac{p}{p_0}\right) \right]^2 \right\}, \qquad (3)$$

where $p_0 = 8.9 m_e c$ and $\sigma = 0.94 m_e c$ give the best fit to electron distribution for a discharge with $Q = 500$ C at altitude of 400 km, above which Coulomb collisions can be neglected for our calculations. This distribution is shown in Figure 2. Monte Carlo calculations further indicate that we can neglect $p_\perp$ compared to p.

We now calculate Γ under the assumption that $N_R \ll N_0$. To utilize published beam-plasma system growth rate formulas for non-relativistic beams, we denote

$$F(\beta) \equiv (1-\beta^2)^{3/2} f(p) \frac{dp}{d\beta} = m_e c f(p(\beta)).$$

We find Γ ($\equiv \mathrm{Im}\ \omega$ for time dependence $e^{-i\omega t}$) from the formula [e.g., *Krall and Trivelpiece*, 1986, p. 389]:

$$\Gamma = -\frac{\mathrm{Im}\ \epsilon}{\partial \mathrm{Re}\ \epsilon / \partial\omega}$$

where the values are taken at the point (ω, k) where $\mathrm{Re}\ \epsilon = 0$. Substituting for ϵ from equation (2) we find $\partial \mathrm{Re}\ \epsilon/\partial\omega \simeq 2/\omega_0$ for small N_R/N_0 and

$$\Gamma = \frac{N_R}{N_0} \frac{\pi\omega_0}{2} \left(\frac{\omega_0}{ck}\right)^2 \left.\frac{dF}{d\beta}\right|_{\beta = \omega_0/(ck)} \qquad (4)$$

with a maximal value of

$$\Gamma \simeq 25 \frac{N_R}{N_0} \omega_0 \simeq 0.05 N_R \ \mathrm{s}^{-1}, \quad N_R \text{ in m}^{-3} \qquad (5)$$

for an assumed value $N_0 \simeq 10^9$ m^{-3} and the beam distribution (3).

Interaction of the beam with the background plasma becomes significant if the bounce time $t_B \simeq 0.2$ s for the electron beam is comparable to the characteristic growth time

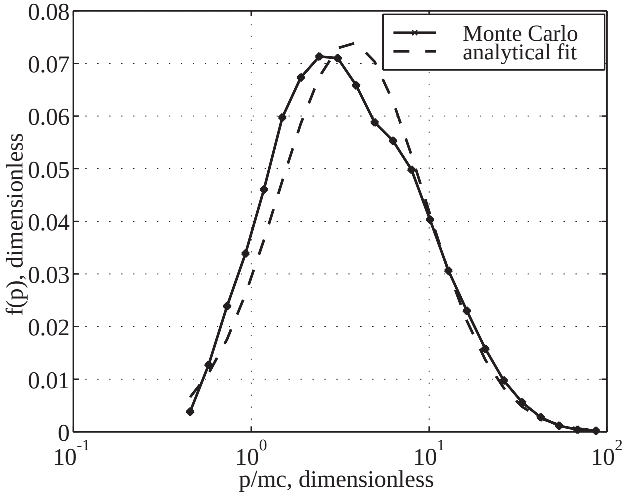

Figure 2. The momentum distribution of the runaway electrons calculated with a Monte Carlo model and an analytical (log-normal) fit.

Γ^{-1}. From (5), $t_B\Gamma \simeq 1$ for $N_R \simeq 100$ m^{-3}, so that significant pitch angle scattering and energy degradation of the beam occurs for $N_R \gtrsim 100$ m^{-3}. Note that Γ is proportional to $N_0^{-1/2}$, so that the maximum interaction occurs near the equatorial plane, where N_0 is a minimum.

It is instructive to compare the Γ calculated above to the instability Γ_{cold} of a cold relativistic beam. If we use assume that all electrons in the beam have the momentum equal to the mean momentum from Monte Carlo calculations, we find Γ_{cold} from a nonrelativistic expression [e.g., *Tsytovich*, 1995, p. 224] by substitution $m_e \to m_\parallel \equiv m_e\gamma^3$ for the electrons in the beam which have a typical relativistic factor $\gamma = (1 - v^2/c^2)^{-1/2} \simeq 10$:

$$\Gamma_{\mathrm{cold}} = \frac{\sqrt{3}}{2^{4/3}}\frac{\omega_0}{\gamma}\left(\frac{N_R}{N_0}\right)^{1/3} \simeq 150 N_R^{1/3}\ \mathrm{s}^{-1}, \qquad (6)$$

where N_R is in m^{-3}, $N_R \ll N_0$, $N_0 \simeq 10^9$ m^{-3}. Comparison of Γ_{cold} with (5) for typical parameters indicate that the assumption of a cold monoenergetic beam results in a much higher growth rate.

3.2. Nonlinear growth and resultant electron distribution

The number density of the energetic electrons in the beam is estimated from comparison with experimental data on the terrestrial γ-ray flashes (TGF) [*Fishman et al.*, 1994]. The γ-photon bremsstrahlung production rate is proportional to $N_m N_R$. According to numerical solutions of equation (1), most γ-ray emissions are emitted at altitudes 60–75 km. The correct observed γ-ray flux is then obtained for energetic electron densities of $N_R \simeq 10^4$ to 10^5 m^{-3}. Similar results were also obtained by *Lehtinen et al.* [1997]. Other models predict maximum γ-photon emissivity at heights $\sim$40 km [*Milikh and Valdivia*, 1999] and a different maximum value of N_R.

For such high values of N_R, it is clear from (5) that the linear growth rate is very high, so that the instability rapidly grows into the nonlinear regime during the traverse of the runaway beam between hemispheres. In such a case, the evolution of the distribution of the beam electrons can only be determined via detailed computer simulations, which are beyond the scope of this work. Nevertheless, we can estimate the electron distribution subject to certain assumptions and with reference to published simulation results.

Based on qualitative estimates confirmed by computer simulation work [*Birdsall and Langdon*, 1991, p. 117], in the advanced nonlinear stage of interaction of a one-dimensional cold and non-relativistic electron beam with longitudinal Langmuir waves, the maximum energy density of the wave field has the value:

$$W_E = \frac{1}{4}\epsilon_0 E^2 = \left(\frac{N_R}{2N_0}\right)^{1/3} W_R, \qquad (7)$$

where W_R is the energy density in the beam. Assuming that the results do not change qualitatively for the case of a relativistic beam, and that the initial beam density is $N_R = 10^5$ m^{-3}, we obtain the relative energy loss of $\sim (N_R/2N_0)^{1/3} \lesssim 10\%$. Note that based on the comparison of (5) and (6), the growth rate and thus the energy degradation of the beam should be even smaller for a hot initial distribution, so that the relative energy loss of $\sim$10% is an upper bound for our case.

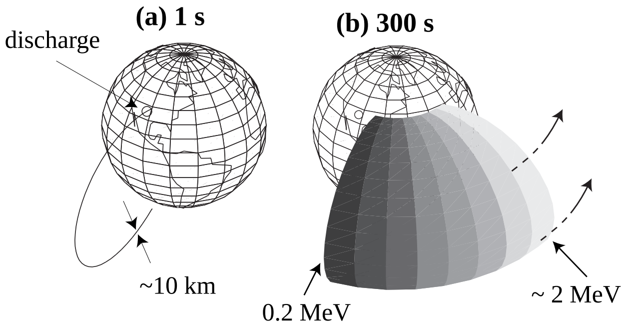

Figure 3. Schematic description of the formation of trapped electron curtains.

Interaction of a monodirectional beam with oblique Langmuir waves leads to isotropization of the distribution [*Tsytovich*, 1995, p. 70]. With the beam density being so high as to lead to very large growth rate and indeed nonlinear growth, we can expect very significant pitch angle and energy scattering, so that the electrons may acquire an isotropic thermal distribution (over pitch angles and energies) during one traverse along the field line.

On the other hand, the resonance interaction of beam electrons with plasma waves with an increment given by (4) occurs only for electrons with energies $\mathcal{E} < 2.3$ MeV, with the maximum growth at $\mathcal{E} = 1.45$ MeV. This, and the mechanism of nonlinear stabilization [*Tsytovich*, 1995, p. 184] would tend to prevent the thermalization of the distribution at higher energies. Thus, it is difficult to predict the resultant electron distribution function without a nonlinear computer simulation analysis, well beyond the scope of this paper.

3.3. Formation of trapped electron curtains

For the sake of discussion, and with the above caveats in mind, we proceed by assuming that the intense beam-plasma interaction transforms the relativistic runaway beam consisting of electrons with pitch angles near zero and with an initial momentum distribution as given in (3) to an isotropic thermal distribution with typical energy $\gtrsim$1 MeV. Assuming an initial beam density of up to 10^5 m^{-3}, a beam radius of $\sim$10 km, and a process duration of $\sim$1 ms [*Lehtinen et al.*, 1997], a total of $N_e^{\mathrm{tot}} \simeq 3 \times 10^{18}$ electrons are initially injected into the radiation belts. After isotropization, only a small fraction ($\sim$2–10%) of these electrons are in the loss cone and precipitate in the conjugate hemisphere. The remaining electrons are trapped, and bounce back and forth between hemispheres, while at the same time drifting eastward in longitude. After a few bounces, the electrons that are not precipitated fill up the geomagnetic field tube. For $\sim$45° geomagnetic latitude, the length of the geomagnetic field line of $\sim 2 \times 10^7$ m, with the geomagnetic field at the equator being smaller by a factor of $\sim$10, resulting in electron density $N_e \simeq 150$ m^{-3} at the equator, corresponding to a differential energy flux of electrons at $\sim$1 MeV of $\Phi_{\mathcal{E}} \simeq 3 \times 10^2$ el-cm^{-2}-s^{-1}-keV^{-1}.

The trapped electrons drift eastward due to curvature and gradient of the geomagnetic field, with a period τ_d given in equation (4.47) by *Walt* [1994] on p. 49. For 1 MeV electrons at 45° invariant magnetic latitude (corresponding to $L = 2$), and for equatorial electron pitch angle $\alpha_{\mathrm{eq}} \simeq 90°$, we have $\tau_d \simeq 10^3$ s. Due to the fact that electrons with higher energies drift in longitude at a greater rate, the trapped electrons eventually form electron curtains as shown in Figure 3. After several drift periods, when the curtain wraps around the Earth and electrons of different energies mix together, the omnidirectional flux of electrons can be estimated by comparing the 10 km longitudinal beam radius with the distance around the globe at $L = 2$. Based on these considerations, we find the flux of electrons at energy ~1 MeV at the geomagnetic equator to be $\Phi_{\mathcal{E}} \simeq 7 \times 10^{-2}$ el-cm^{-2}-s^{-1}-keV^{-1}.

Preliminary calculations indicate that such fluxes may be detectable on satellites with high time resolution and sensitive detectors. Noting that the latitudinal extent of the original beam is ~10–20 km, the curtains would be traversed by a polar orbiting satellite within a few seconds. A detector with a geometric factor of ~1 cm^2-sr would measure a total number of ~100 electrons of >1 MeV energy.

4. Summary and conclusions

We considered the fate of energetic runaway beams driven upward by intense thundercloud fields produced by large positive cloud-to-ground discharges. Based on the velocity space distribution function of such beams as determined by Monte Carlo methods [*Lehtinen et al.*, 1999], we have determined that the runaway electron beam intensely interacts with the background magnetospheric plasma, leading to rapid nonlinear growth of Langmuir waves and pitch angle and energy scattering of the beam electrons. The end result of this interaction is the isotropization and thermalization of the electron distribution function, leading to the trapping of most of the beam electrons in the radiation belts, and the formation of detectable trapped electron curtains (Figure 3).

Acknowledgments. This work was supported by NASA under grant NAGW4738 to Stanford University.

References

Bell, T. F., V. P. Pasko, and U. S. Inan, Runaway electrons as a source of red sprites in the mesosphere, *Geophys. Res. Lett., 22,* 2127, 1995.

Birdsall, C. K. and A. B. Langdon, *Plasma Physics via Computer Simulation*, Adam Hilger, New York, 1991.

Fishman, G. J., P. N. Bhat, R. Malozzi, J. M. Horack, T. Koshut, C. Kouveliotou, G. N. Pendleton, C. A. Meegan, R. B. Wilson, W. S. Paciesas, S. J. Goodman, and H. J. Christian, Discovery of intense gamma-ray flashes of atmospheric origin, *Science, 264*, 1313, 1994.

Gurevich, A. V., G. M. Milikh, and R. A. Roussel-Dupré, Runaway mechanism of air breakdown and preconditioning during a thunderstorm, *Phys. Lett. A, 165*, 463, 1992.

Holzworth, R. H., M. C. Kelley, C. L. Siefring, L. C. Hale and J. T. Mitchell, Electrical measurements in the atmosphere and the ionosphere over an active thunderstorm. 2. Direct current electric fields and conductivity, *J. Geophys. Res., 90,* 9824, 1985.

Khazanov, G. V., M. W. Liemohn, E. N. Krivorutsky, J. U. Kozyra, and B. E. Gilchrist, Interhemispheric transport of relativistic electron beams, *Geophys. Res. Lett., 26,* 581, 1999.

Krall, N. A., and A. W. Trivelpiece, *Principles of Plasma Physics*, San Francisco Press, Inc., San Francisco, 1986.

Lehtinen, N. G., M. Walt, U. S. Inan, T. F. Bell and V. P. Pasko, γ-ray emission produced by a relativistic beam of runaway electrons accelerated by quasi-electrostatic thundercloud fields, *Geophys. Res. Lett., 23,* 2645, 1996.

Lehtinen, N. G., T. F. Bell, V. P. Pasko, and U. S. Inan, A two-dimensional model of runaway electron beams driven by quasi-electrostatic thundercloud fields, *Geophys. Res. Lett., 24,* 2639, 1997.

Lehtinen, N. G., T. F. Bell, and U. S. Inan, Monte Carlo simulation of runaway MeV electron breakdown with application to red sprites and terrestrial gamma ray flashes, *J. Geophys. Res., 104,* 24699, 1999.

Milikh, G. and J. A. Valdivia, Model of Gamma Ray Flashes due to Fractal Lightning, *Geophys. Res. Lett., 26*, 525, 1999.

Pasko, V. P., U. S. Inan, T. F. Bell and Y. N. Taranenko, Sprites produced by quasi-electrostatic heating and ionization in the lower atmosphere, *J. Geophys. Res., 102,* 4529, 1997.

Roussel-Dupré, R., E. Symbalisty, Y. Taranenko, and V. Yukhimuk, Simulations of high-altitude discharges initiated by runaway breakdown, *J. Atmos. Sol. Terr. Phys., 60*, 917, 1998.

Stix, T. H., *The Theory of Plasma Waves*, McGraw-Hill, New York, 1962.

Tsytovich, V. N., *Lectures on Non-linear Plasma Kinetics*, Springer, New York, 1995.

Volland, H., *Atmospheric Electrodynamics*, Springer-Verlag, New York, 1984.

Walt, M., *Introduction to Geomagnetically Trapped Radiation*, Cambridge University Press, New York, 1994.

N. G. Lehtinen, U. S. Inan, and T. F. Bell, STAR Laboratory, Stanford University, Stanford, CA 94305

(Received November 17, 1999, accepted January 4, 2000.)