Full Wave Modeling of VLF Wave Scattering and Propagation in Curvilinear Stratified Ionosphere

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Abstract – **We describe two enhancements to Stanford Full Wave Method (StanfordFWM) for calculation of electromagnetic field radiated into a stratified medium by arbitrarily positioned monochromatic sources [1,2]. In the first part, we consider the problem of scattering on strong ionospheric disturbances. We introduce a novel computational technique which combines StanfordFWM with the method of moments (MoM), instead of the previously used Born approximation method [3]. In the second part, we demonstrate a method to include the curvature of stratification layers into the full-wave approach. This method is used to calculate the attenuation and height gains for ELF/VLF modes in the Earth-ionosphere waveguide. The results are significantly different from those obtained with the flat-Earth assumption, especially for the least attenuated modes.**

1 INTRODUCTION

Stanford Full Wave Method (StanfordFWM) [1-3] has the capability of calculating the electromagnetic field radiated by arbitrarily positioned monochromatic electric and magnetic current sources into an arbitrary plane-stratified anisotropic local medium. This method has been applied previously to waves in ELF/VLF frequency range in the Earth's ionosphere which may be represented as horizontally-stratified magnetized plasma with an arbitrary direction of the geomagnetic field. It provides a full-wave 3D solution for both whistler waves launched into the Earth-ionosphere waveguide. The method is stable against the numerical "swamping" instability by evanescent waves [4, p.574-576] and makes efficient use of computer resources by being easily parallelized.

The typical problems to which StanfordFWM has been applied are: (1) transport of plane waves upward through the ionosphere and their attenuation; (2) radiation from ionospheric regions, such as electrojet currents modulated by HF heating; (3) radiation from ground-based transmitters; (4) pulse radiation from lightning, i.e. the so-called sferics (time waveform being obtained by inverse-Fourier transforming a set of monochromatic solutions); (5) Earth-ionosphere waveguide modes; (6) scattering on ionospheric disturbances in Born approximation [5]. In this work, we enhance the results of the last two applications by calculating the scattering on strong disturbances using the method of moments (MoM) (Section 2) and modification of the attenuation coefficient and height gains of the Earth-ionosphere waveguide modes by including the effects of the Earth's finite radius (Section 3).

2 SCATTERING ON STRONG IONOSPHERIC DISTURBANCES

Various artificial and natural processes, such as heating by ground-based powerful VLF and HF transmitters or electromagnetic pulses from lightning, may modify the conductivity in the *D*-region ionosphere. The VLF waves from Navy transmitters scatter off such a disturbance, which results in a perturbation of the narrow-band VLF signal. For small disturbances of conductivity, the Born approximation was previously used together with StanfordFWM [3]. However, a strong disturbance may strongly affect the propagation of the VLF wave and render the use of the Born approximation invalid. In order to tackle this problem, we introduce a novel computational technique which combines the StanfordFWM with the method of moments (MoM). This novel technique may be used for general problems of calculation of scattering on spatially localized strong perturbations in stratified media. The Green's functions used in MoM are calculated for the stratified Earth-atmosphere-ionosphere system by application of the StanfordFWM. We show that the Born approximation method overestimates the field scattered by strong disturbances. In particular, we now may demonstrate the saturation effects in the VLF perturbations for strong HF.

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2.1 Summary of the method of moments (MoM)

We use StanfordFWM to calculate the 3×3 matrix Green's function $\hat{G}$ with components $G_{i,j}(\mathbf{r}_o,\mathbf{r}_s)=E_i(\mathbf{r}_o)$ which describes the electric field $\mathbf{E}$ at observation point $\mathbf{r}_o$ created by a point electric current source $\mathbf{J}(\mathbf{r})=\hat{x}_j\delta(\mathbf{r}-\mathbf{r}_s)$ located at position $\mathbf{r}_s$. In our case, the field is in the stratified medium, and the currents are due to the conductivity perturbation, $\mathbf{J}=\Delta\hat{\sigma}\mathbf{E}$. The integral equation for the scattered field $\mathbf{E}_s$ is

$$\mathbf{E}_s(\mathbf{r})=\int\hat{G}(\mathbf{r},\mathbf{r}')\Delta\hat{\sigma}(\mathbf{r}')[\mathbf{E}_0(\mathbf{r}')+\mathbf{E}_s(\mathbf{r}')]d^3\mathbf{r}' \quad (1)$$

where the integration is over the perturbed region ($\Delta\hat{\sigma}\neq 0$). The method of moments (MoM) makes use of spatial discretization of $\mathbf{J}$ and $\mathbf{E}$. Then, the discretized integral equation is solved numerically, which involves an inversion of a large matrix. Note that Born approximation corresponds to neglecting $\mathbf{E}_s$ on the right-hand side of (1).

2.2 VLF scattering by an HF heater

We consider the scattering of VLF signal produced by NLK Navy Transmitter on a *D*-region ionospheric disturbance produced by HAARP HF heater. The VLF transmitter operates at frequency f = 24.8 kHz with power P = 250 kW. The HF heater operates at frequency f_{HF} = 5 MHz and has the effective radiated power (ERP) of 1 GW. We assume a Gaussian shape HF beam of width ~23 km at *D*-region altitudes.

The perturbation in ionosphere is due to the change $\Delta\nu_e$ in the electron-neutral collision rate caused by the change in electron energy distribution which is found by solving the kinetic equation [6]. The change $\Delta\nu_e$ in turn produces the perturbation of the VLF conductivity tensor $\Delta\hat{\sigma}$. The motivation for using the MoM instead of Born approximation comes from comparing the calculated $\Delta\hat{\sigma}$ to the background conductivity: Figure 1 shows that the eigenvalues of the conductivity tensor are changed by a large fraction as a result of ionospheric heating.

2.3 Results

As shown in Figure 2, the currents induced in the ionosphere, when calculated in Born approximation, are significantly different from those using the more precise MoM approach. In particular, the currents at the top of the heated region had been overestimated. This leads to a decrease in scattered field by about a factor of ~2-3, as shown in Figure 3.

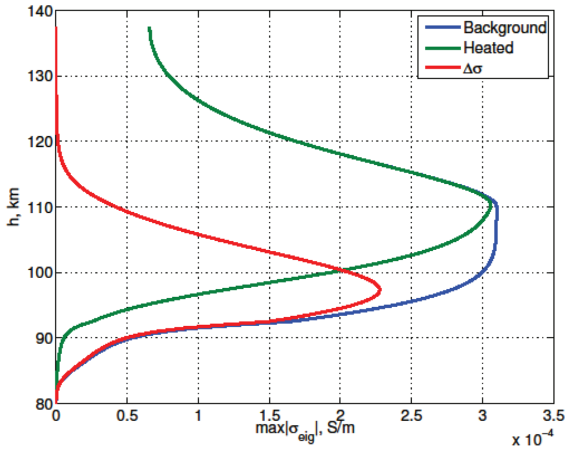

Figure 1. Change in conductivity by HF heating.

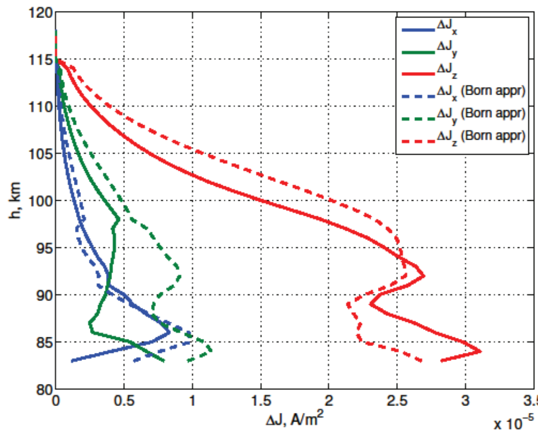

Figure 2. Comparison of currents induced in the ionosphere, calculated using MoM (solid) and Born approximation (dashed).

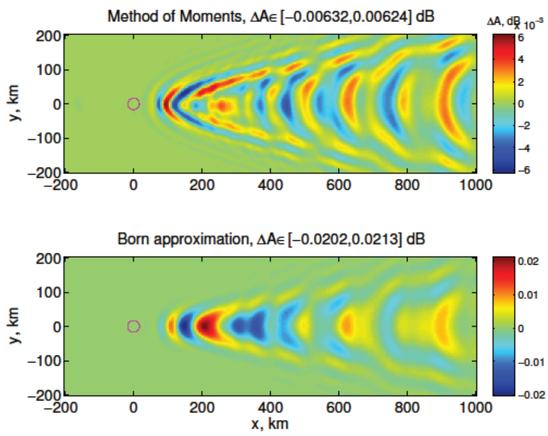

Figure 3. The VLF amplitude perturbation (in dB) of the total horizontal magnetic field on the ground, as a function of position, calculated using MoM and in Born approximation. The oncoming VLF wave propagates from left to right; the disturbance is shown with a circle.

3 EFFECTS OF CURVATURE IN STRATIFICATION

In this Section, we demonstrate, by considering a general curvilinear stratified system with anisotropic media, an approach to include the curvature into the full-wave method of calculation of electromagnetic fields. We use this approach to calculate the attenuation and height gains for VLF modes in the Earth-ionosphere waveguide while taking into account the finite Earth radius. The results show improved accuracy compared with the plane full-wave method which assumes an infinite Earth radius.

3.1 Modification of the Booker equation

Consider a curvilinear coordinate system $\{\xi_1, \xi_2, \xi_3\}$ with scale factors $\{h_1, h_2, h_3\}$, such that

$$h_{1,2} = 1 + \alpha_{1,2}\xi_{1,2}, \; h_3 = 1, \; \xi_3 \equiv Z \qquad (2)$$

with constant curvatures $\alpha_{1,2}$ of the horizontal coordinates $\xi_{1,2}$. Due to translational symmetry along $\xi_{1,2}$, the wave numbers along that direction are conserved, i.e., the fields are $\propto \exp(ik_0[\beta_1\xi_1 + \beta_2\xi_2])$. Here $k_0 = \omega/c$ is the vacuum wave number and we use normalized (dimensionless) wave numbers $\beta_{1,2} = const$.

Consider first a flat case ($\alpha_{1,2} = 0$). In a uniform medium, the vertical wave numbers $\beta_3 \equiv q$ for different waves are the solution of the Booker equation. The ratios between different components of the electromagnetic field $\{E_1, E_2, E_3, H_1, H_2, H_3\}$ may be found by substituting into Maxwell's equations. It has been also shown by Clemmow and Heading [7] that the horizontal fields (which may be written as a vector $\mathbf{e}=\{E_1, E_2, H_1, H_2\}$) and q are found as eigenvectors and eigenvalues of an operator $\hat{L}$ which describes propagation along the vertical coordinate Z:

$$\hat{L}\mathbf{e} = \frac{1}{ik_0}\frac{\partial \mathbf{e}}{\partial Z} \qquad (3)$$

It may be shown [8] that in the curvilinear case described by equation (2) the operator $\hat{L}$ at $Z = 0$is modified as

$$\hat{L} = \hat{L}_{\text{flat}} - \frac{\alpha_1}{ik_0}\hat{I}_1 - \frac{\alpha_2}{ik_0}\hat{I}_2 \qquad (4)$$

where $\hat{L}_{\text{flat}}$ is the Clemmow-Heading operator for Cartesian case and $\hat{I}_{1,2}$ are the projection operators on $\xi_{1,2}$. Thus, in the curvilinear case, we find the field components $\mathbf{e}$ and vertical refractive index q as eigenvectors and eigenvalues of the modified operator (4).

3.2 Validation

A validation of the modification described above may be done by comparing a known analytic solution (in a system for which such a solution can be found) to a numerical solution by a modified full-wave method using (4). For initial validation, let us consider a system consisting of an outside region of perfectly conducting cylinder of radius $a = 2$ filled with vacuum and a cylindrical source current sheet of radius $b = 2.5$ with uniform surface current parallel to the axis. The analytical solution may be obtained in cylindrical coordinates $\{r, \phi, z\}$, in terms of Bessel functions. For each fixed R, we can choose the curvilinear coordinates $\xi_1 = R\phi$, $\xi_2 = z$, $\xi_3 \equiv Z = r - R$, so that the curvatures at $Z = 0$ are $\alpha_1 = 1/R$, $\alpha_2 = 0$. Since the given surface current source is uniform, the field will be axially symmetric (i.e., in our notation β_1=0). We considered various k_0 and β_2. The full-wave method solution was obtained in the region $2 < r < 3$ with various stratum thicknesses Δr. Both analytic and StanfordFWM solutions are plotted in Figure 4 for $k_0 = 4$, $\beta_2 = 0$ and show excellent agreement with each other, even for a coarse discretization $\Delta r = 0.1$. The maximum error for smaller Δr was ~0.2% and was due to imperfect radiation conditions used in StanfordFWM at the outer boundary of $r = 3$. It is possible to demonstrate that the curvilinear solution is visibly different from the flat (Cartesian) geometry case.

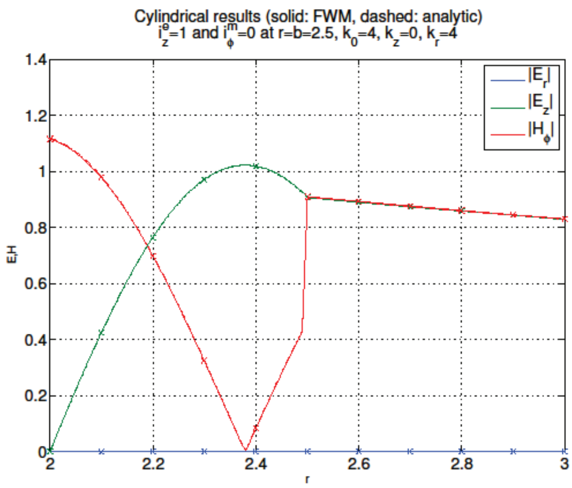

Figure 4. StanfordFWM (solid) and analytic (dashed) solutions for the field in the system used in the validation example.

3.3 Results for Earth-ionosphere waveguide

The Earth-ionosphere waveguide modes may be found by solving the modal equation which equates the field in a plane wave to itself after being reflected both from ionosphere and the ground [e.g., 9, eq. (9.28)]. We solve it using StanfordFWM like in [3], except for the modification (4) for the solution of the Booker equation. Figure 5 shows the results of including the finite Earth's radius (i.e., $\alpha_{1,2}=1/R_{\text{Earth}}$) for phase velocities (at ground level) and attenuation coefficients of several least-attenuated Earth-ionosphere waveguide VLF modes. In particular, we see that taking $R_{\text{Earth}}=\infty$ significantly underestimates the attenuation of the second least-attenuated mode (~1.5 dB/Mm instead of ~0.3 dB/Mm). Also, the phase velocity may be less than c by when the curvature of the Earth is taken into account.

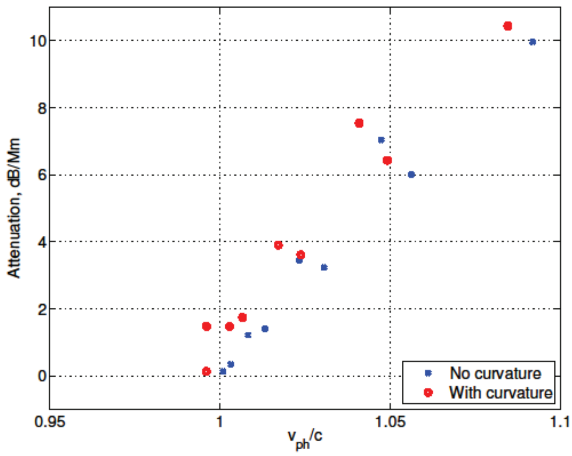

Figure 5. Phase speeds and attenuation coefficients for the least attenuated modes.

4 CONCLUSIONS

We have implemented two enhancements to the Stanford full-wave method (StanfordFWM) of calculation of fields in stratified media. The first enhancement allows calculation of scattering of a given incident wave on an arbitrary spatially localized perturbation of the stratified medium. The second allows for constant curvature in the stratification layers. These enhancements were applied to problems in VLF propagation in the Earth's ionosphere and have shown that the new more accurate results may be significantly different from those obtained using previously used approximations.

Acknowledgments

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References

[1] N. G. Lehtinen and U. S. Inan, "Radiation of ELF/VLF waves by harmonically varying currents into a stratified ionosphere with application to radiation by a modulated electrojet", *J. Geophys. Res.*, **113**, A06301, doi:10.1029/2007JA012911, 2008.

[2] N. G. Lehtinen and U. S. Inan, "Full-wave modeling of transionospheric propagation of VLF waves", *Geophys. Res. Lett.*, **36**, L03104, doi:10.1029/2008GL036535, 2009.

[3] N. G. Lehtinen, R. A. Marshall, and U. S. Inan, "Full-wave modeling of "early" VLF perturbations caused by lightning electromagnetic pulses", *J. Geophys. Res.*, **115**, A00E40, doi:10.1029/2009JA014776, 2010.

[4] K. G. Budden, "The propagation of radio waves: the theory of radio waves of low power in the ionosphere and magnetosphere", Cambridge University Press, Cambridge, 1985.

[5] N. G. Lehtinen, T. F. Bell and U. S. Inan, "Full-wave method of calculation of electromagnetic fields in stratified media", *Proceedings of ICEAA'11*, paper 231, p. 327, COREP, Torino, Italy, 2011.

[6] N. G. Lehtinen and U. S. Inan, "Non-Maxwellian model of ionospheric heating by HF radiation"; *Eos Trans. AGU,* **87**(52), Fall Meet. Suppl., Abstract SA53B-1379, 2006.

[7] P. C. Clemmow and J. Heading, "Coupled forms of the differential equations governing radio propagation in the ionosphere", *Math. Proc. of the Cambridge Phil. Soc.*, **50**(2), pp. 319-333, doi:10.1017/S030500410002939X, 1954.

[8] L. Qiu, L., N. G. Lehtinen and U. S. Inan, "Effects of Earth's curvature in full-wave modeling of VLF propagation", *2011 Fall Meeting, AGU*, Abstract AE33A-0282, 2011.

[9] K. G. Budden, "The wave-guide theory of wave propagation", Prentice-Hall Inc., Englewood Cliffs, 1961.