Electric streamers as a nonlinear instability

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Goal and approach

Electric streamer discharges are ionized columns in gas or liquid, which advance by ionizing the material in front of them with the enhanced field at the streamer tip [1]. Streamers in air are an important stage of lightning discharge, and have been extensively studied by methods which include such computationally-intensive numerical approaches as adaptive mesh refinement 3D hydrodynamic models and PIC (Particle-In-Cell) simulations. The numerical approaches reproduce approximately the experimentally measured streamer parameters, but still leave open the question of what physical principles determine such streamer characteristics as their speed and transverse size (radius). Our goal is to isolate these physical principles and find a way to quickly determine streamer parameters without performing complicated numerical simulations.

We apply the following approach to achieve this goal: (1) look for a solution of physics equations in the shape of a streamer (i.e., a column attached to a conducting electrode, see Fig. 1); (2) simplify the PDEs describing the discharge to obtain a finite system of algebraic equations for a finite number of streamer parameters; and (3) solve it, with laboratory-condition fixed external electric field E_e and streamer length L.

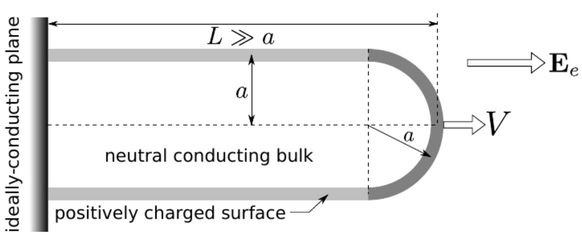

Fig. 1. Streamer shape

System of algebraic equations and its solution

We have identified the following algebraic equations: (1) relation between electric fields, determined by electrostatic redistribution of charges on the streamer surface; (2) continuity of total (conductivity + displacement) current flowing through the streamer front; (3) relation between the ionization and front electric field which follows from the flat front theory [2]; and (4) velocity-radius relation, determined by the photoionization mechanism [3].

This system, however, does not have a unique solution, as one of parameters remains free. It is convenient to choose streamer radius a as such parameter. In particular, we now can obtain functional dependences of the streamer velocity $V(a)$, such as those shown in Fig. 2. Some of the equations which comprise the above system were identified, e.g., by [4], who approached solving them by arbitrarily fixing some of the streamer parameters. We will show that a unique solution still may be found without such artificial fixing, but by applying same considerations as used in the flat-front perturbation theory.

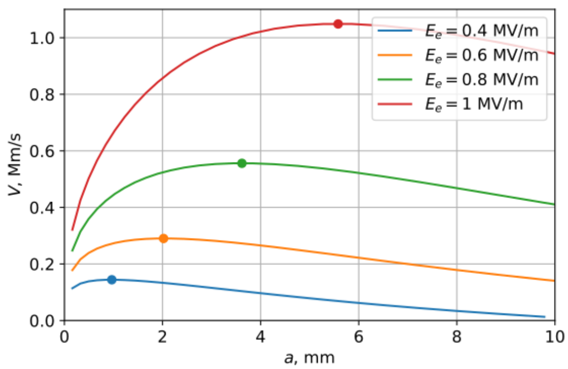

Fig. 2. Solutions of the simplified system of algebraic equations describing a positive streamer, in the form of velocity as a function of radius, $V(a)$. Here we present solutions for given streamer length L=120 mm and selected values of external electric field E_e (see legend). By analogy with flat-front perturbation theory, these curves may be called "dispersion curves," and each point of a curve is a streamer "mode." A dot on each curve denotes the maximum velocity, which corresponds to the "preferred" solution.

Analogy with the flat-front perturbation theory

We argue that non-uniqueness of the solution is inherent in the physical system. Let us consider the *flat-front perturbation theory* [5]. It solves the same physical system, but instead of a streamer shape (Fig. 1), the solution is sought in the shape of small perturbations of a flat ionization front, which are harmonic in the transverse direction with an arbitrary wavenumber k. Linearization reduces the system of PDEs describing the discharge to a linear system of ODEs. Some of the perturbations are unstable and grow in time exponentially with growth rate $s(k)$. This function is called "dispersion relation," and solutions at fixed k are called "modes." As we see, similarly to the streamer problem, there is also one free parameter, namely k, which characterizes the transverse size and is therefore analogous to streamer radius $a \sim 1/k$. The perturbation corresponding to the maximum of $s(k)$ grows the fastest, and is therefore the "preferred" solution, which on long time scales corresponds to the real physical outcome of the system if the initial perturbations were random. In the streamer problem, there is no direct analog of s, but we note that the velocity of the flat-front perturbation $V = s(k)L$ in respect to the front also has a maximum at the same value of k, if the perturbation "length" (i.e. spatial scale along propagation) L is fixed. Thus, we propose that the streamer problem also has the "preferred" solution at the maximum of $V(a)$, which now may also be called the "dispersion equation." The selection of the preferred solution will be referred to as "max-V criterion." Solutions with smaller V corresponding to other values of a may be called streamer "modes." They are valid physical solutions, but are suppressed by the faster-propagating max-V solution.

Results for laboratory conditions (STP)

The calculation results after application of the max-V criterion for positive streamers are presented in Fig. 3. The black curve in Fig. 3(a) is the experimental measurement of [6], for zero air humidity. We see that the presented theory gives reasonable values for streamer velocities and radii. Analogous curves for negative streamers will be presented in the full paper [7].

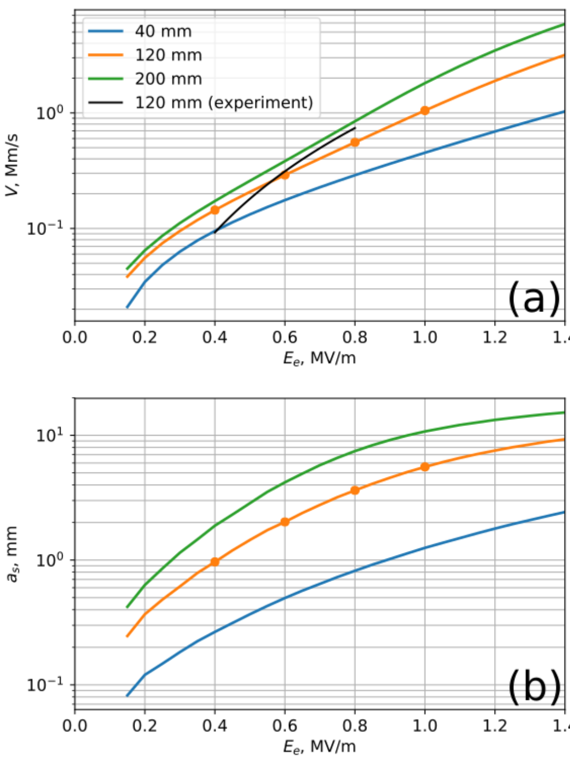

Fig. 3. Results for positive streamers: velocity and radius as a function of external electric field E_e, for three different values of streamer length L. The dots correspond to maxima marked in Fig. 2. Black curve is the experimental result of [6].

Streamer threshold fields

The threshold field $E_{\pm t}$ is the minimum E_e at which propagation is still possible. It depends on L and has a different physical reason for different streamer polarities. For *positive* streamers, the quenching of propagation is due to three-body attachment inside the streamer channel, so that conductivity is not constant but attenuates to a small value as electrons travel from the head of the streamer towards the electrode. For *negative* streamers, the threshold arises as the solution for *V(a)* disappears below certain external field. This may be related to the fact that the negative streamer velocity must always exceed the electron drift speed. The results of threshold field calculations using the presented theory are presented in Fig. 4. The calculated positive threshold is close to the experimental measurements of [6] which are also shown. The calculated negative threshold values also agree with the commonly accepted experimental value range [1, p. 362].

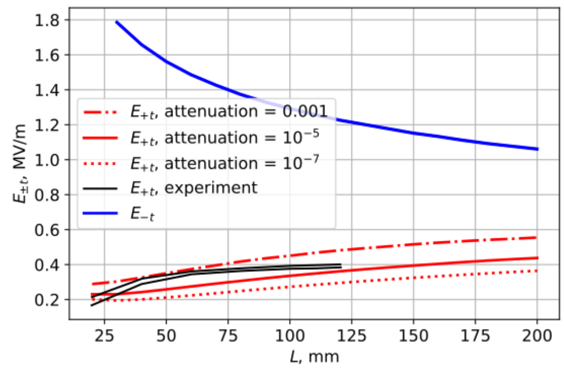

Fig. 4. Positive and negative streamer threshold fields as a function of streamer length L. For the positive threshold, several curves are given, corresponding to various values of conductivity attenuation in the streamer channel due to three-body attachment of electrons.

Summary

We present a new method of calculating streamer parameters, which introduces a streamer "dispersion equation," with the "preferred" solution selected by the max-V criterion. The terminology draws on the analogy with linear perturbation theory, so we may say that we describe the streamer as a nonlinear instability. The obtained streamer velocities, radii and threshold fields are in reasonable agreement with experimental values. The details of the algebraic equations, as well as the Python code for their solution, will be published in [7].

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References

1. Y. P. Raizer, *Gas discharge physics* (Berlin: Springer, 1991).
2. A. N. Lagarkov and I. M. Rutkevich, *Ionization Waves in Electrical Breakdown of Gases* (Springer-Verlag, New York, 1994).
3. S. V. Pancheshnyi, S. M. Starikovskaia, and A. Y. Starikovskii, *J. Phys. D: Appl. Phys.* **34**, 105 (2001).
4. M. Dyakonov and V. Kachorovskii, *Zh. Eksp. Teor. Fiz* **95**, 1850 (1989).
5. G. Derks, U. Ebert, and B. Meulenbroek, *Journal of Nonlinear Science* **18**, 551 (2008).
6. N. L. Allen and P. N. Mikropoulos, *J. Phys. D: Appl. Phys.* **32**, 913 (1999).
7. N. G. Lehtinen, submitted to PRL and PRE (2019).