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EXCITATION OF AN IONOSPHERIC ALFVÉN RESONATOR BY A PLASMA WAVE DISCHARGE

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We present a theoretical interpretation of the excitation of an ionospheric Alfvén resonator (IAR) by an electron beam propagated along the external magnetic field. The growth rate of generated Alfvén waves is found and the IAR excitation efficiency is estimated. The theoretical results are in qualitative agreement with the experimental data.

The investigation of active [1, 2] methods of generating extremely low-frequency (ELF) electromagnetic fields in the ionospheric plasma is of interest both for solving applied problems of low-frequency radio communication [3] and for a correct understanding of the physical processes in terrestrial space when the ionosphere is acted upon by ground-based and space sources [2, 4]. In the present paper, we give experimental data on the excitation of ELF oscillations in the ionosphere by a plasma-wave discharge, which were received from a meteorological rocket, and we propose a simplified theoretical model for a qualitative explanation of the observed effect.

1. A meteorological rocket MR-12 was launched in Kapustin Yar at 8.30 p.m. Moscow Time on the sixth of February, 1991. The rocket was oriented vertically with a deviation $\sim 5°$ to the east. A dipole antenna in the form of two semicylinders made of a metal screen with diameter 2 m and with amplitude ~ 1.5 kV at frequency 480 kHz was applied to the semicylinders. The high-frequency signal was modulated by a telegraphic signal at $f_1 = 240$ Hz and $f_2 = 120$ Hz in accordance with s special cyclogram. The parameters of the antenna-excited discharge were determined by a Langmuir probe. They amounted to $N_e \sim 10^6$ cm^{-3} and $T_e \sim 20$ eV at altitudes $h \geq 140$ km. The ground-based receiver recorded a noise signal at the discharge modulation frequencies f_1 and f_2. Spectral processing of those signals showed the Doppler-shifted narrow-band radiation of a twinkling discharge $f_{1,2}(1 + 3 \cdot 10^{-3})$ and a few modulated ELF peaks (see Fig. 1, where $S(f)$ is the spectral density of the signal) corresponding to the frequency range of an ionospheric Alfvén resonator (IAR) [5]. The mechanisms of radiation from an active plasma antenna (APA) produced by a discharge from the rocket were discussed in detail in [6]. It was shown in that paper that the APA-emitted power depends on the geomagnetic field intensity in the antenna area. Consequently, each modulation component in the geomagnetic field, which occurs, for example, when an IAR is excited, is manifested as a peak in the ELF spectrum of the APA.

The plasma-wave discharge in the Earth's ionosphere [7] represents a very long and narrow plasma inhomogeneity. The longitudinal scale of this inhomogeneity is determined by the hot plasma scattering from the active region of the discharge along the geomagnetic field, and it can reach $L_{\|} \geq 10^5$ m [8]. The transverse scale of the inhomogeneity depends on the speed of the rocket, and it reached $L_{\perp} \sim 10^2$ m in the experiment. In the night winter ionosphere at middle latitudes, this inhomogeneity is possibly due to the scattering of hot electrons from the discharge area along the magnetic field and the transverse diffusion of ions [9]. The longitudinal velocity of injected electrons $v_{\|} \sim v_e \geq 2 \cdot 10^6$ m/s is greater than the Alfvén velocity C_A in the F_2 layer in which the electron density $N_0 > 10^4$ cm^{-3}. Under these conditions, the

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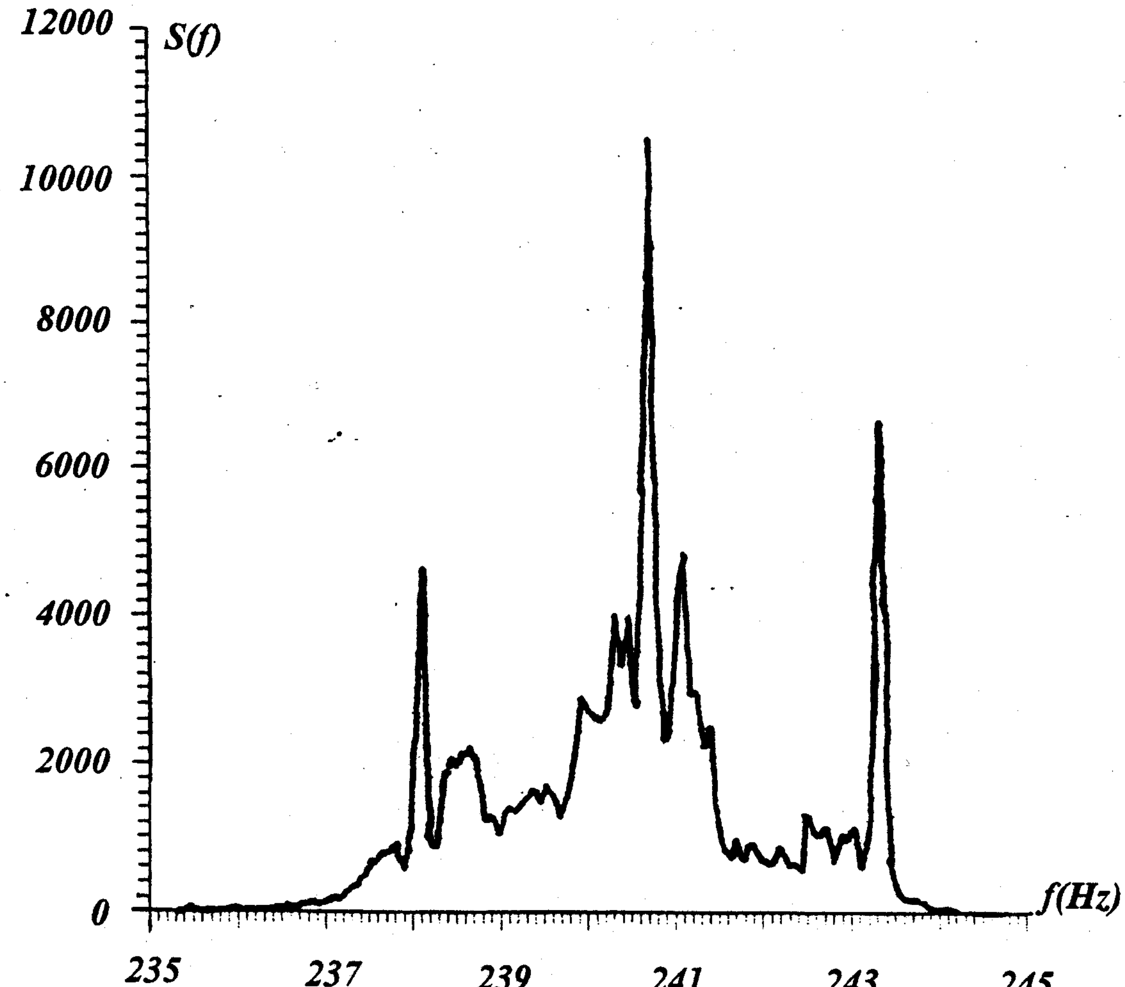

Fig. 1.

excitation of Alfvén waves (AW) by a flow of fast ($v_{\|} > C_A$) electrons is possible by virtue of the Cerenkov radiation mechanism (see, e.g., [10–12])

2. Consider a cylindrical axisymmetic electron beam of radius a with electron density βN_0 and speed $v_{\|}$, which genetrates a background plasma of density N_0 along the external magnetic field $\mathbf{H}_0$. Collisions between particles and the dependence on the longitudinal coordinate z ($\mathbf{H}_0 \parallel \mathbf{z}_0 \parallel \mathbf{v}_{\|}$) are neglected for simplicity. In the Alfvén range of frequencies ($\omega \ll \Omega_i, \Omega_i$ is the gyrofrequency of the background ions), the dielectric permittivity tensor of an electron beam—plasma system in cylindrical coordinates has the form [10]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & i\dfrac{\varepsilon_5}{k_0}\dfrac{\partial}{\partial r} \\ i\varepsilon_2 & \varepsilon_1 & -\dfrac{\varepsilon_4}{k_0}\dfrac{\partial}{\partial r} \\ i\dfrac{\varepsilon_5}{k_0}\hat{L} & \dfrac{\varepsilon_4}{k_0}\hat{L} & -\varepsilon_3 - \dfrac{\varepsilon_3'}{k_0^2}\hat{L} \end{pmatrix}. \tag{1}$$

Here,

$$\hat{L} = \frac{1}{r}\Big(r\frac{\partial}{\partial r}\ldots\Big), \quad k_0 = \frac{\omega}{c}, \quad \varepsilon_1 = \Big(\frac{C}{C_A}\Big)^2, \quad C_A^2 = \frac{H_0^2}{4\pi M N_0},$$

$$\varepsilon_2 = \varepsilon_1\Big(\frac{\omega}{\Omega_i} - \beta\frac{\Omega\omega'}{\omega^2}\Big), \quad \varepsilon_3 = \Big(\frac{\omega_0}{\omega}\Big)^2 + \beta\Big(\frac{\omega_0}{\omega'}\Big)^2, \quad \omega_0^2 = \frac{4\pi N_0 e^2}{m},$$

e/m, e/M is the specific charge of electron (ion), $\omega' = \omega - kv_{\|}$, k is the longitudinal wavenumber, $\varepsilon_3' = \beta(m/M)\varepsilon_1(v_{\|}/c)^2 \ll 1$, $\varepsilon_4 = -\beta\varepsilon_1(v_{\|}/c)(\Omega_i/\omega)$, $\varepsilon_5 = \beta(m/M)\varepsilon_1(v_{\|}/c)(\omega'/\omega) \ll 1$. In the background plasma outside the electron beam, it is required to put $\beta = 0$ in (1).

From the Maxwell equations, using (1), we can write the system of differential equations for azimuthal components of the electric (E_φ) and magnetic (H_φ) fields through the remaining components of the wave fields:

$$\left\{\varepsilon_3\alpha_2 - (\alpha_2\alpha_5 - \alpha_1\alpha_4)k_0^{-2}\hat{L_0}\right\}E_\varphi + \left\{\varepsilon_3\alpha_3 - (\alpha_5\alpha_3 + \alpha_1^2)k_0^2\hat{L_0}\right\}iH_\varphi = 0,$$
$$\left\{\alpha_2\alpha_4 - \varepsilon_1\alpha_1(\alpha_3 - \varepsilon_2^2\varepsilon_1^{-2}) - \alpha_1 k_0^{-2}\hat{L_0}\right\}E_\varphi + \left\{\alpha_4\alpha_3 - \alpha_1\alpha_2\right\}iH_\varphi = 0. \tag{2}$$

Here, $\alpha_1 = 1 + p\varepsilon_5\varepsilon_1^{-1}$, $\alpha_2 = p\varepsilon_2\varepsilon_1$, $\alpha_3 = 1 - p^2\varepsilon_1^{-1}$, $p = kk_0^{-1}$, $\alpha_4 = \varepsilon_4 - \varepsilon_2\varepsilon_5\varepsilon_1^{-1}$, $\alpha_5 = \varepsilon_5^2\varepsilon_1^{-1} - \varepsilon_3'$. The eigenfunctions of the operator $\hat{L_0}$ are the solutions of the Bessel equation

$$\hat{L_0}F(k_0qr) = \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\Big(rF(k_0qr)\Big)\right] = -k_0^2q^2F(k_0qr). \tag{3}$$

Substituting (3) into (2), we find the relation between the dimensionless longitudinal (p) and transverse (q) wavenumbers

$$q_1^2 \approx \varepsilon_1 - p^2 - \varepsilon_2^2(\varepsilon_1 - p^2)^{-1}, \quad q_2^2 \approx -\varepsilon_3\alpha_3(\alpha_5\alpha_3 + \alpha_1^2)^{-1}, \tag{4}$$

where it is taken into account that $|\varepsilon_3|$ is the largest component of the tensor (1).

Deriving a dispersion equation requires that the eigenfunctions (3) satisfy the boundary conditions for $r = a$

$$E_\varphi = \tilde{E}_\varphi, \quad E_z = \tilde{E}_z,$$
$$H_z = \tilde{H}_z, \quad H_\varphi + \varepsilon_5 E_r - i\varepsilon_4 E_\varphi + i\varepsilon_3' k_0^{-1}\frac{\partial E_z}{\partial r} = \tilde{H}_\varphi, \tag{5}$$

(here, $\mathbf{E}$ and $\mathbf{H}$ denote the field for $r < a$ and $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ denote the field for $r > a$). Moreover, we require that these fields be finite when $r = 0$ and satisfy the radiation conditions at infinity. In deriving dispersion relations from (5), taking (3) and (4) into account and assuming $|\varepsilon_3| \gg 1$ and $|\varepsilon_3||\tilde{\varepsilon}_3|^{-1} \sim 1$, we find

$$q_1 J_0(k_0q_1a)\left[J_1(k_0q_1a)\right]^{-1} = q_1 H_0^{(2)}(k_0q_1a)\left[H_1^{(2)}(k_0q_1a)\right]^{-1}, \tag{6}$$

$$\varepsilon_3^{-1}q_2 J_0(k_0q_2a)\left[J_1(k_0q_2a)\right]^{-1} = \varepsilon_3^{-1}q_2 H_0^{(2)}(k_0q_2a)\left[H_1^{(2)}(k_0q_3a)\right]^{-1}, \tag{7}$$

where $J_{0,1}$ and $H_{0,1}^{(2)}$ are the Bessel and Hankel functions. Equation (6) corresponds to fast magnetosound and Alfvén waves, and Eq. (7) corresponds to quasitransverse waves associated with the "plasma resonance." Assuming for the background plasma ($r > a$) $p \approx CC_A^{-1}$, $\varepsilon_1 \approx p^2$, $\varepsilon_3 \sim \omega_0^2\omega^{-2}$ and $qp^{-1} \sim |(\varepsilon_3\varepsilon_1^{-1})|^{1/2}$, from (6) we obtain

$$p \approx Cv_{\|}^{-1}(1 \pm i\sqrt{\beta}). \tag{8}$$

Relation (8) is valid under the condition $x_{1n}a_0a^{-1} \ll 1$, where x_{1n} is the n-root of the Bessel function of the first order and the coefficient $a_0 \approx C\omega_0^{-1} \sim 15$ m.

3. For a layer-inhomogeneous ionospheric plasma with the geomagnetic field $\mathbf{H}_0 \parallel \nabla N_0$ and the refractive index for AW described by the model function [5]

$$n^2(z) = C^2C_A^{-2}(z) = \begin{cases} n_0^2, & h_1 < z < h_2, \\ n_0^2\left(\varepsilon^2 + \exp\left[-2(z - h_2)L^{-1}\right]\right), & \end{cases} \tag{9}$$

where $n_0^2 = 4\pi c^2 M N_{\max} H_0^{-1}$, $\varepsilon^2 \sim 10^{-3} - 10^{-4}$, $L \approx 400$ km, and $h = h_2 - h_1 \approx 150$ km, there is a solution to the Maxwell equations by which we can find the eigenfrequencies (f_n) and the Q-factor (Q_n) of the IAR eigenmodes [5]

$$f_n = C_A\left[n + 1/4 + \varphi_i(2\pi)^{-1}\right]\left[2(h+L)\right]^{-1}, \tag{10}$$

$$Q_n \approx \frac{h+L}{\pi\varepsilon L} \sim 10. \tag{11}$$

Here, φ_i is the phase shift when the wave is reflected at the lower edge of the resonator ($z = h_1$), and the energy losses are due to the partial penetration of the wave through the upper wall of the resonator where the geometrical optics condition is broken. The energy losses can be compensated by resonance amplification of the wave by a fast electron beam. From the energy balance condition for the IAR, we can find the limits on electron density in the beam

$$\frac{c}{v_{\|}} k_0(L_{\|}\sqrt{\beta} - \pi\varepsilon L) = 0. \tag{12}$$

For a characteristic thickness of the F_2 layer $L_{\|} \approx 150$ km, from (12) we have

$$\beta_{kp} = (\pi\varepsilon L L_{\|}^{-1})^2 \approx 10^{-2} - 10^{-3}.$$

Thus, if the length of the electron beam $L_\beta \sim L_{\|}$ and the characteristic transverse scale $a > a_0 = 15$ m, then swinging of IAR oscillations is possible for electron flow densities exceeding 1% of the background plasma density. At altitudes $h \geq 200$ km, the free path of electron $l_e > 10$ km and the concentration of fast ($T_e \geq 20$ eV) electrons in the discharge $> 10^6$ cm^{-3} [8]; therefore, satisfying the inequality $\beta > \beta_{kp}$ under conditions of the above experiment is quite realistic.

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