BIRKELAND CENTRE
FOR SPACE SCIENCE

What determines streamer speed and radius?

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University of Bergen, Norway

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Outline

What is a streamer?

Electric streamer discharges are ionized columns in gas (or liquid) which advance by ionizing the material in front of them with the enhanced field at the streamer tip

Shown here is a laboratory ~MV, 1 m gap discharge, with a complicated branched streamer tree.

Applications:

1. Lightning, sprites
2. Industry (suprathermal electrons)

[Kochkin et al., 2014, Fig. 8]

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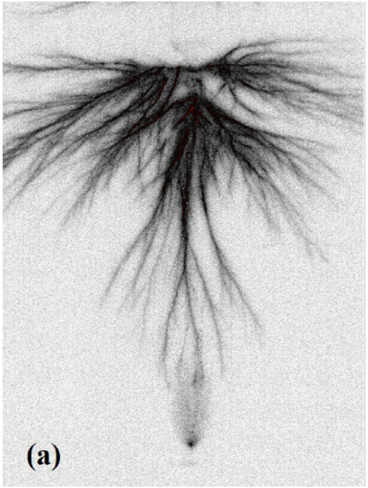

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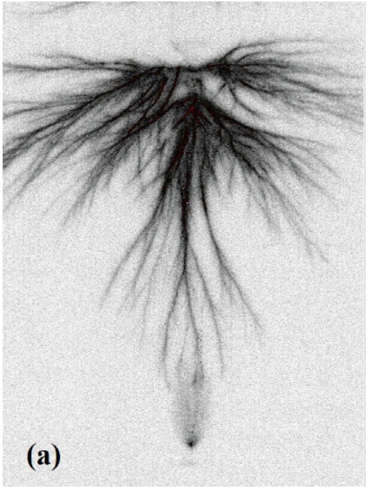

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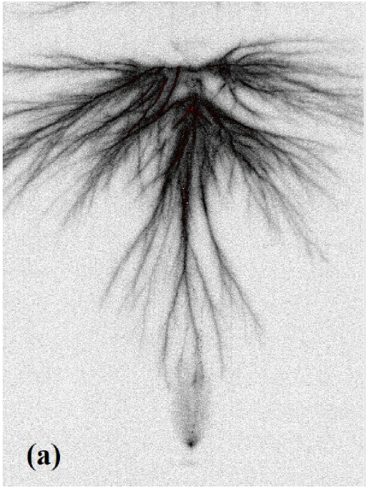

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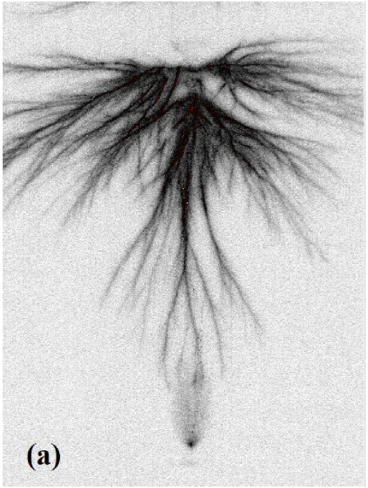

Streamer mechanism

[Loeb and Meek, 1941]

Photons produced in the head of the streamer travel ahead, produce ion-electron pairs, and the electrons serve as avalanche seed in high electric field at streamer head.

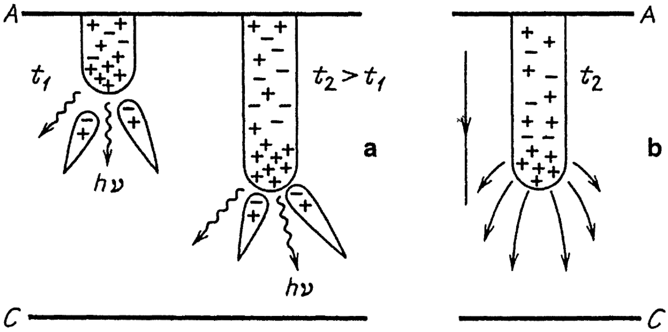

Figure: Positive streamer [figure from Raizer, 1991, p. 335]

Streamer mechanism (negative streamer)

The avalanches started by photoelectrons are directed outward, but the streamer moves so fast that it catches up with them.

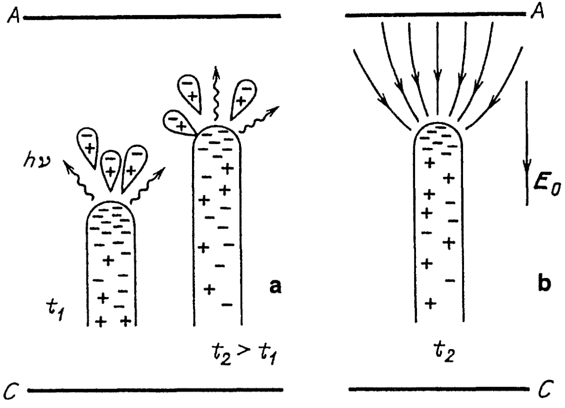

Figure: Negative streamer [figure from Raizer, 1991, p. 338]

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Goal and approach

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Goal

Understand the streamer basics and answer the question in the title of this talk

Approach

- look for a solution in a shape of a **streamer**;
- simplify microscopic physics PDEs which describe evolution of fields and particles and obtain a **finite system of algebraic equations** for a finite number of streamer parameters, such as **radius, speed** etc.;
- solve this system.

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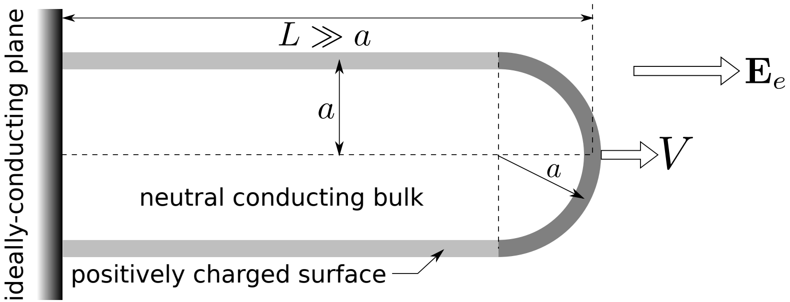
Streamer shape



- Streamer is a cylinder (**channel**) with a hemispherical cap (**head**).
- External electric field E_e and length L are given.
- Want to find parameters: radius a, velocity V, etc.

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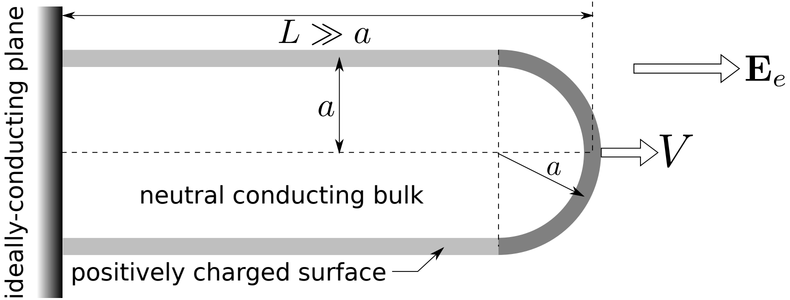

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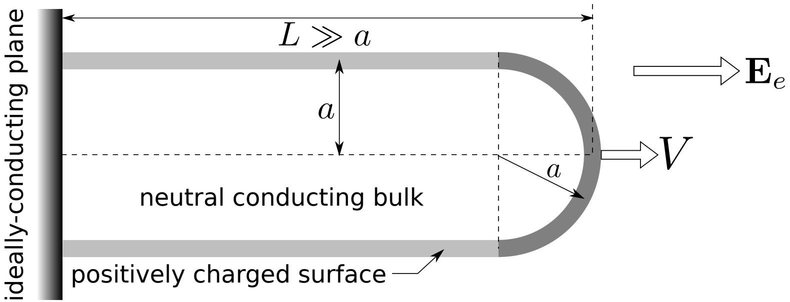

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System of equations

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System of algebraic equations

1. Relation between E fields, from electrostatic distribution of surface charge.
2. Continuity of total (conductivity + displacement) current through the streamer front.
3. Propagation stability criterion $\tau_M \sim \tau_{\text{ion}}$, connecting ionization with the maximum field.
4. Velocity-radius relation, from the photoionization mechanism [Pancheshnyi et al., 2001].

Problem: these equations do not give a unique solution! There is still **one free parameter**. I.e., we get something like $\mathcal{F}(V, a) = 0$, while all other parameters may be expressed in terms of V and a.

Before giving up, let us look at another approach of reducing a system of PDEs to simpler equations: the **flat front perturbation theory**

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Small flat-front perturbations: **linear instability**

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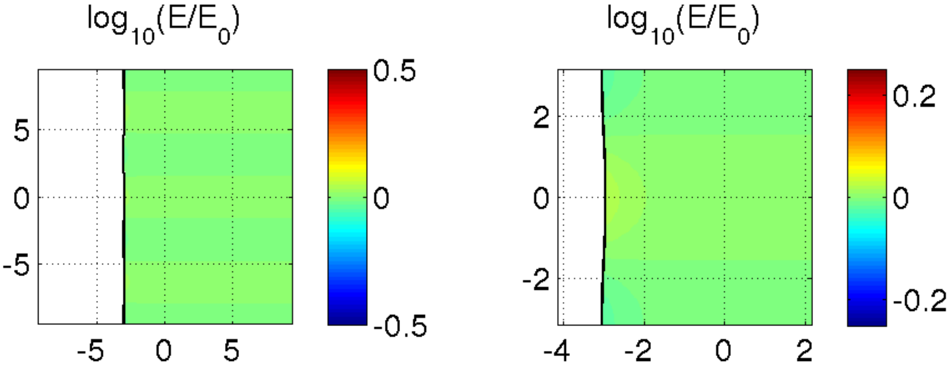
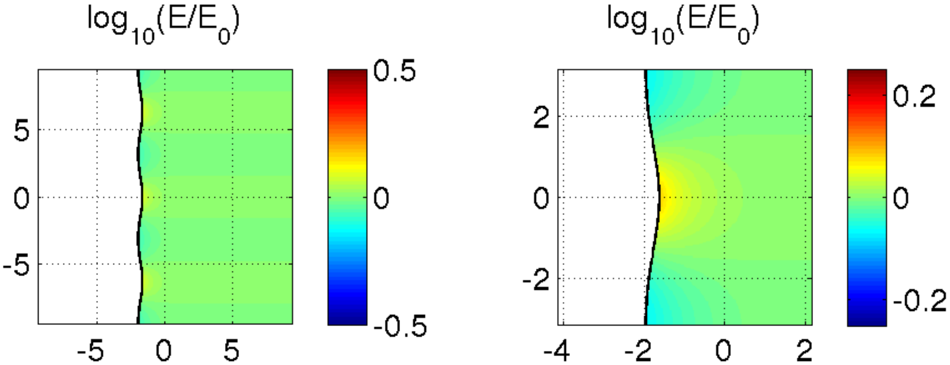


- Start with a flat ionization front propagating as a whole to the right in the Figure.
- Small harmonic $\sim \cos ky$ perturbations grow as e^{st} with **growth rate** s.
- Nonlinear stage.

Small flat-front perturbations: **linear instability**



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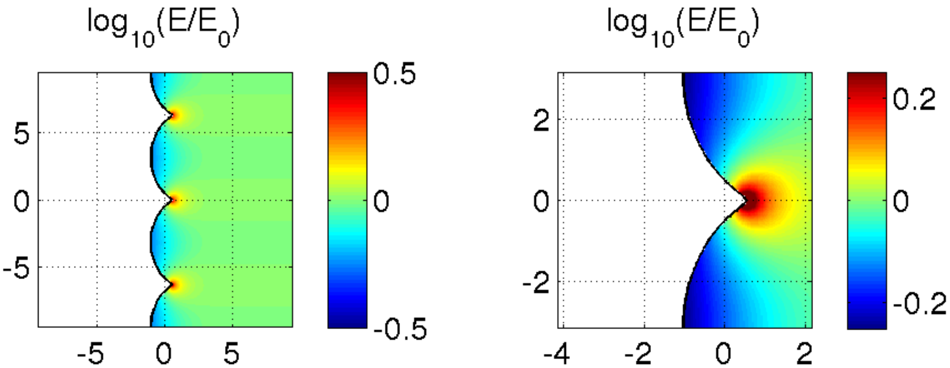

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Solution by Derks et al. [2008]

The growth rate as a function of transverse wavenumber $s(k)$ is called **dispersion function**.

- k is a **free parameter**, evolution depends on initial conditions;
- Perturbation at maximum $s(k)$ grows fastest, so $1/k$ is the **preferred transverse size** a.

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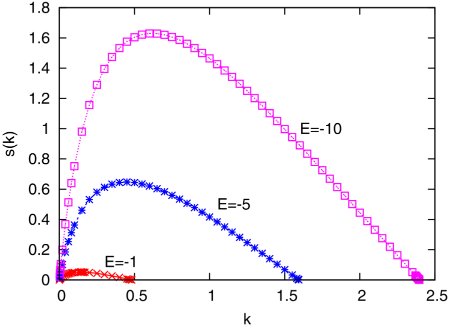

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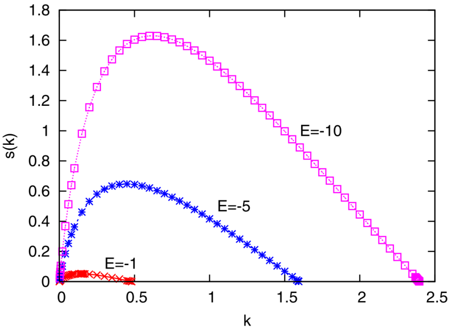

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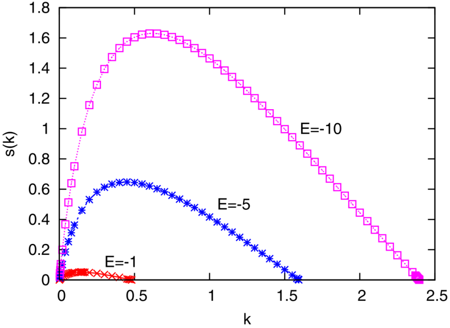

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Analogy of our system with flat-front theory

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Flat-front theory [Derks et al., 2008]

- Shape: harmonic
- k is a free parameter
- Velocity of protrusion $V = V_0 + s(k)L$

- "Real" solution: $\max_k s(k) \Leftrightarrow \max_k V$

Our system

- Shape: streamer
- Not enough equations to fix $a \sim 1/k$
- No $s(k)$, but velocity $V(a, L, E_e)$

- Is physical solution also at $\max_a V$?

max-V criterion

Radius a cannot be determined from equations, but may be fixed by maximizing velocity V.

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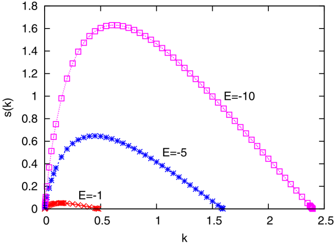

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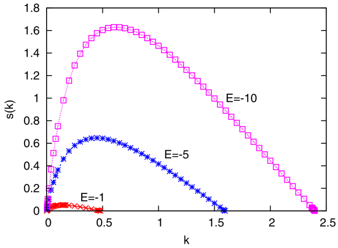

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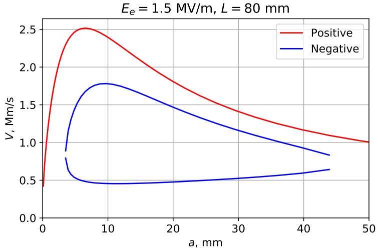

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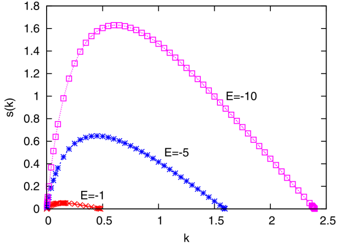

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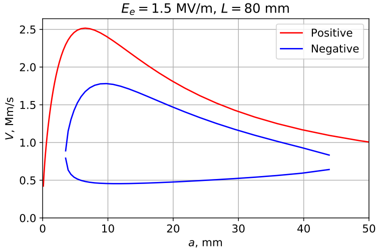

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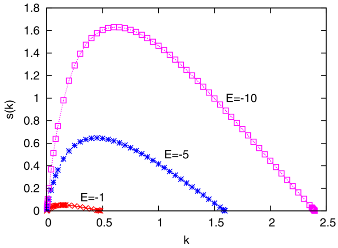

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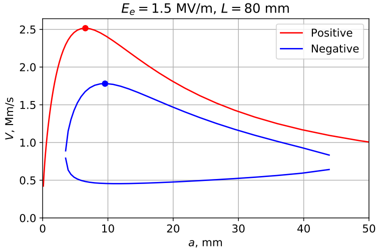

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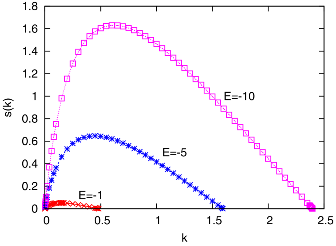

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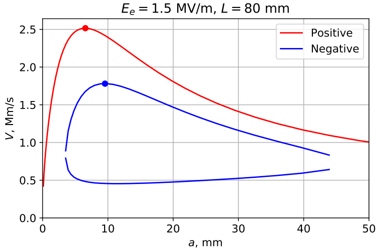

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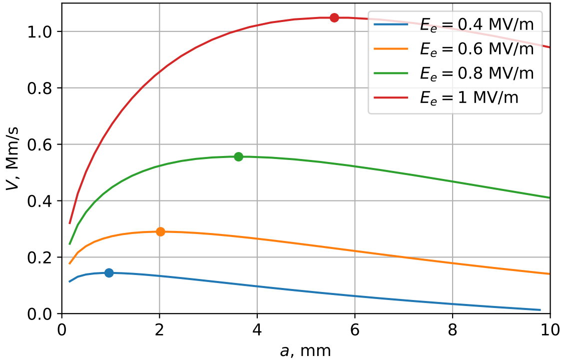
Results for lab conditions

Dispersion functions $V(a)$ for positive streamers with $L = 120$ mm and several values of E_e



Dots denote the max-V.

Positive streamers

The following results are after application of max-V.
We compare to experimental results of Allen and Mikropoulos [1999].

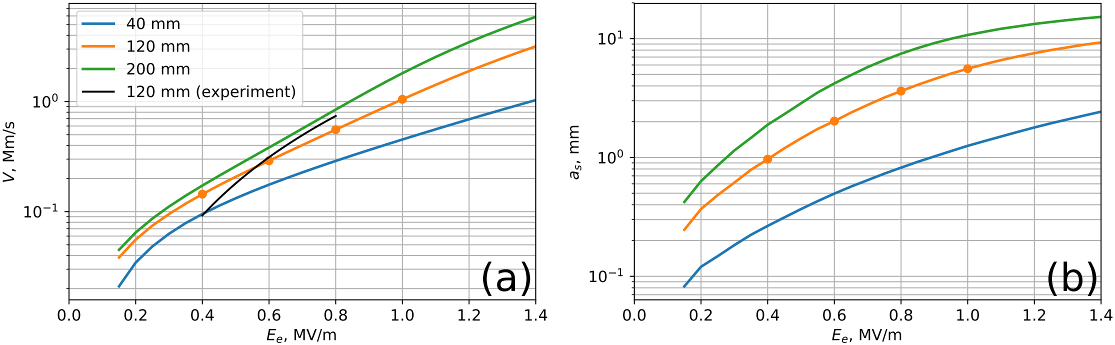


Figure: Velocity and radius as a function of external field E_e, for three different values of L.

Negative streamers

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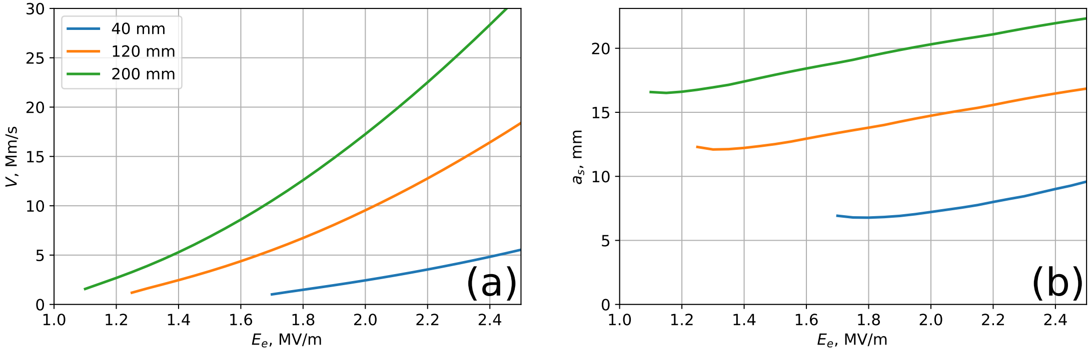

Figure: Velocity and radius as a function of external field E_e, for three different values of L.

Below certain field E_e, there is no solution. Physically, the reason may be that the **negative** streamer must travel faster than electron drift speed.

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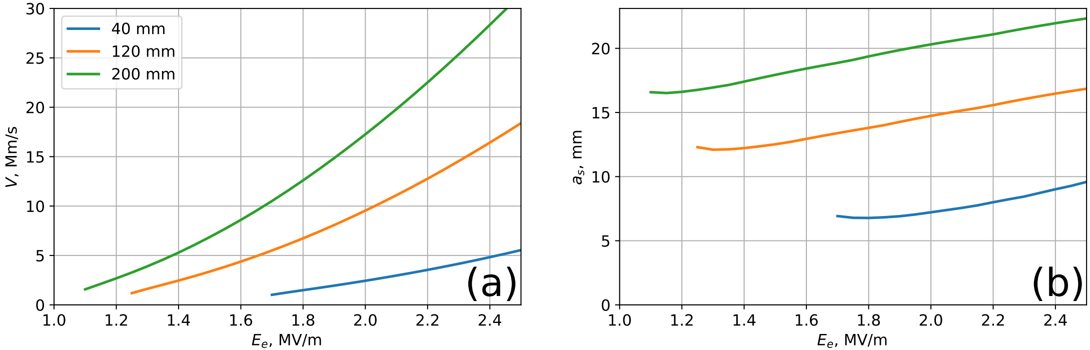

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Streamer threshold fields

Threshold field $E_{\pm t}$ is the minimum E_e at which propagation is still possible.

It depends on L and the physical reason is different for different polarities:

- **Positive streamers**: Three-body attachment inside the streamer quenches it.
- **Negative streamers**: Velocity drops below electron drift speed.

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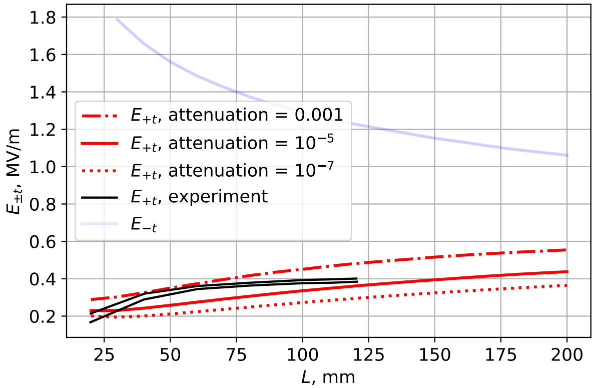

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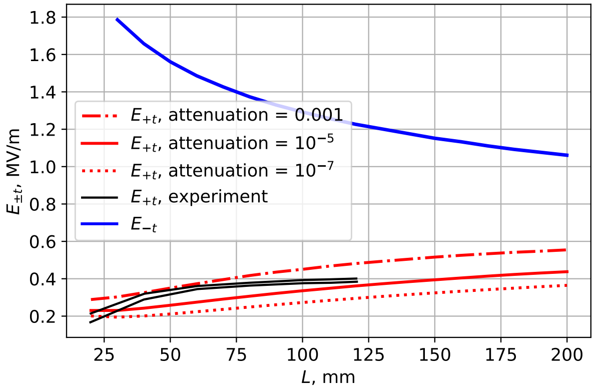

Outline

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Summary

- We describe a streamer electric discharge in air by a system of algebraic equations, which have a solution as function of external field E_e, streamer length L and streamer radius a.
- By employing max-V criterion, we obtain a unique solution which depends only on external conditions E_e and L.
- Calculations produce results for V and a compatible with observations.
- Propagation thresholds are functions of L and are determined by different reasons for positive and negative streamers and are compatible with experimental values $E_{+t} \approx 0.45$ MV/m, $E_{-t} \approx 0.75$–1.25 MV/m [Raizer, 1991, p. 362].

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References

N. L. Allen and P. N. Mikropoulos. Dynamics of streamer propagation in air. *J. Phys. D: Appl. Phys.*, 32(8):913, 1999. doi:10.1088/0022-3727/32/8/012.

N. Y. Babaeva and G. V. Naidis. Dynamics of positive and negative streamers in air in weak uniform electric fields. *IEEE Trans. Plasma Sci.*, 25(2):375–379, Apr 1997. doi:10.1109/27.602514.

G. Derks, U. Ebert, and B. Meulenbroek. Laplacian instability of planar streamer ionization fronts—an example of pulled front analysis. *Journal of Nonlinear Science*, 18(5):551, Jun 2008. doi:10.1007/s00332-008-9023-0.

P. O. Kochkin, A. P. J. van Deursen, and U. Ebert. Experimental study of the spatio-temporal development of metre-scale negative discharge in air. *J. Phys. D: Appl. Phys.*, 47(14):145203, 2014. doi:10.1088/0022-3727/47/14/145203.

L. B. Loeb. Ionizing waves of potential gradient. *Science*, 148(3676):1417–1426, 1965. doi:10.1126/science.148.3676.1417.

L. B. Loeb and J. M. Meek. *The Mechanism of the Electric Spark*. Stanford University Press, Stanford University, California, 1941.

S. V. Pancheshnyi, S. M. Starikovskaia, and A. Y. Starikovskii. Role of photoionization processes in propagation of cathode-directed streamer. *J. Phys. D: Appl. Phys.*, 34(1):105, 2001. doi:10.1088/0022-3727/34/1/317.

Y. P. Raizer. *Gas discharge physics*. Berlin: Springer, 1991.

M. B. Zheleznyak, A. K. Mnatsakanyan, and S. V. Sizykh. Photo-ionization of nitrogen and oxygen mixtures by radiation from a gas-discharge. *High Temp. (USSR)*, 20(3):357–362, 1982.

System of equations

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1. Relation between E fields, determined by electrostatic redistribution of charges on the surface.
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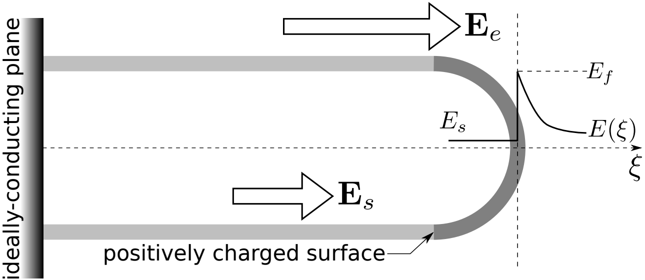
Equation 1: Fields (relation between E_s, E_f)



- External E_e (given!)
- Inside $E_s < E_e$ due to high conductivity, all charges are at surface
- Still $E_s > 0$ because there is a current in the channel $\propto n_s$.
- Just outside $E_f > E_e$

Use electrostatic model (method of moments).

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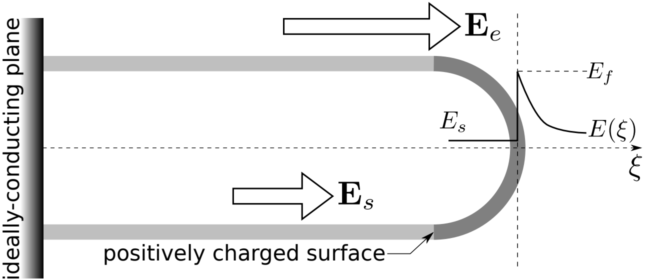

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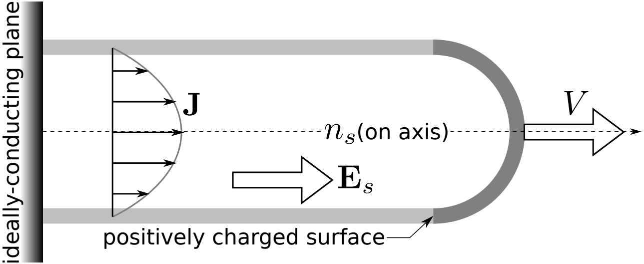

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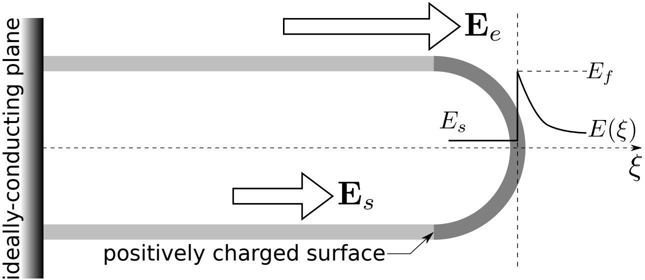

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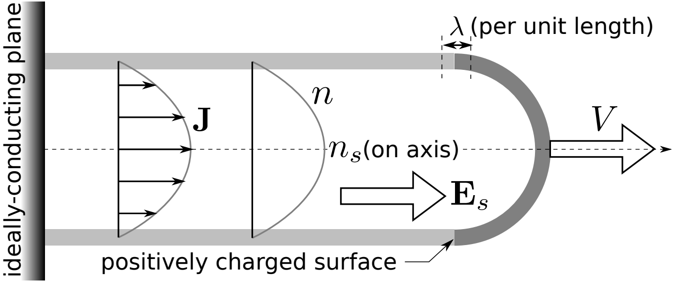
Equation 2: Currents (E_s, n_s, V)



- Charge on the surface per unit length λ is from MoM and E_s
- The total current is $I = \lambda V$
- It is also calculated from n_s and E_s as $I = \int \mathbf{J}_c \, dA_\perp$

Equivalent approach: total current continuity [Babaeva and Naidis, 1997]:

$$\mathbf{J}_c(\text{inside}) = \mathbf{J}_d(\text{outside}) = \varepsilon_0 \partial_t \mathbf{E}$$

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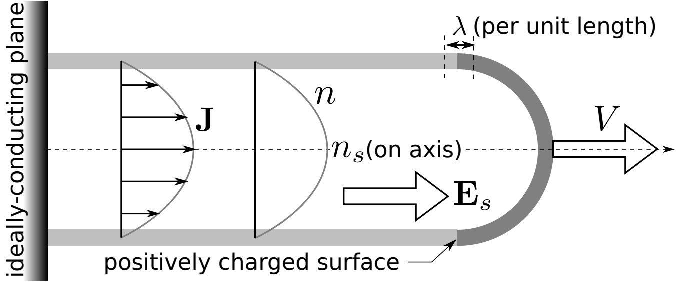

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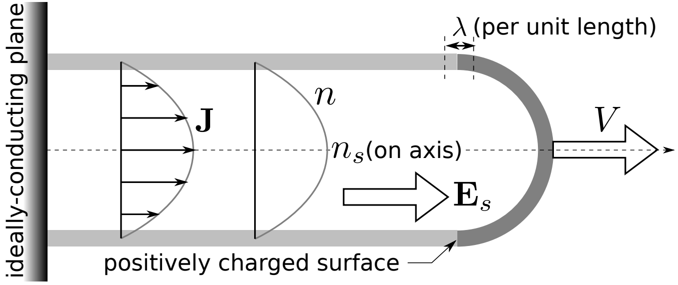

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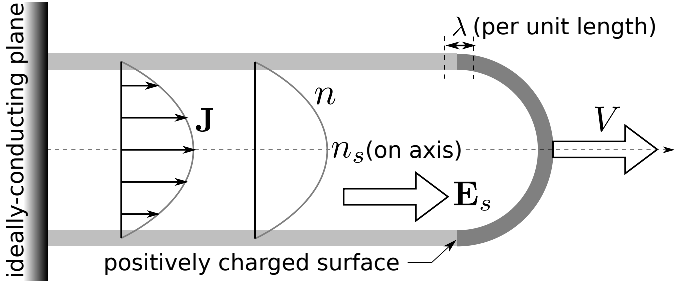

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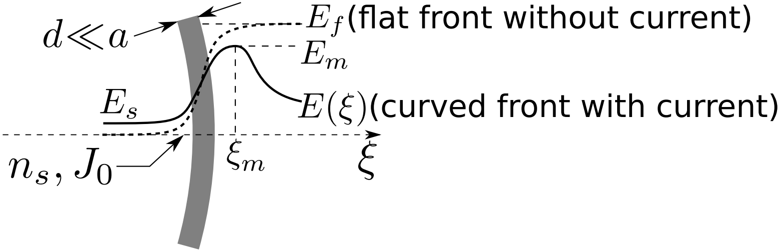
Equation 3: The front (n_s, E_f)



The flat front theory is used to relate n_s to E_f. We also have corrections to this theory:

- to include the current J_0 (on the axis)
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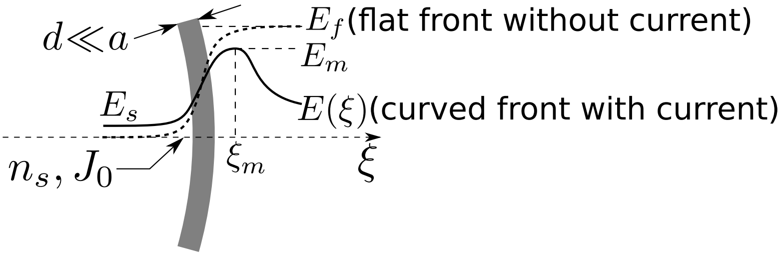

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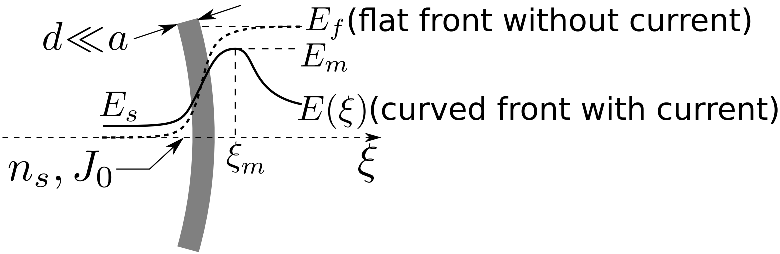

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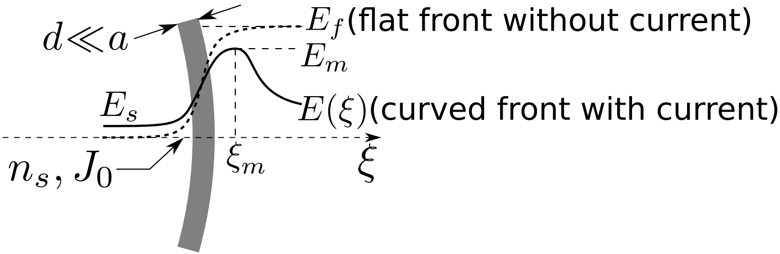

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Equation 4: Photoionization (V, E_f, a)

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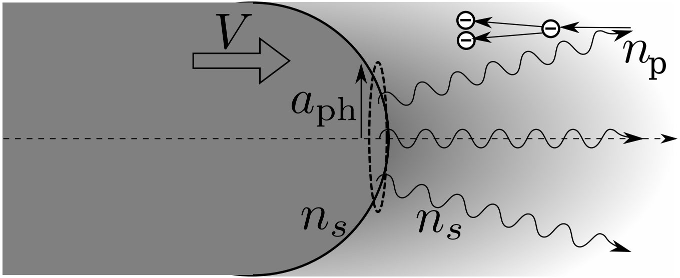

- Ionizing photons are produced in the front $\propto$ ionization rate
- Photon production volume (and the number) $\propto \pi a_{\mathrm{ph}}^2$, $a_{\mathrm{ph}} \sim a$
- Ionization occurs remotely [Zheleznyak et al., 1982] $\Rightarrow n_p/n_s$
- Electron avalanche has length d in streamer frame, which depends on V and E_f
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Loeb [1965]: $d \approx V/\nu_i(E_f) \Rightarrow V \approx a\nu_i(E_f)/\log(n_s/n_p)$

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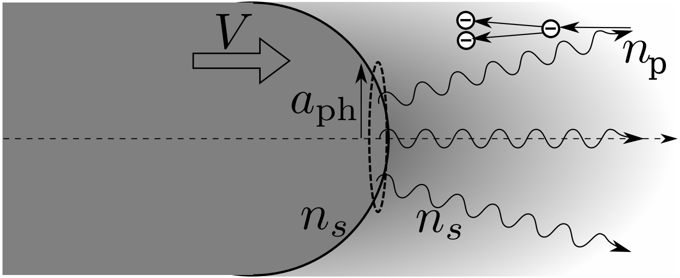

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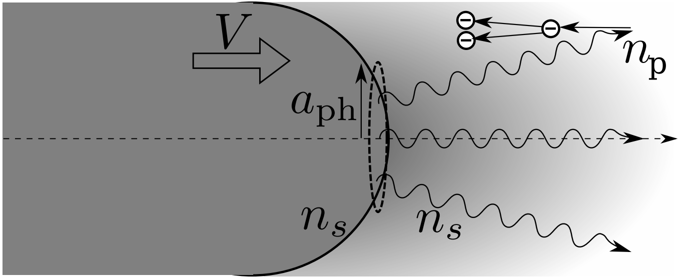

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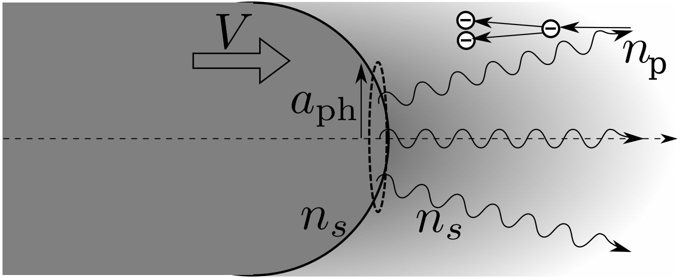

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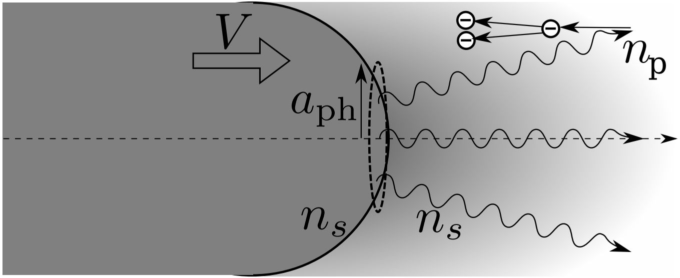

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The model summary figure

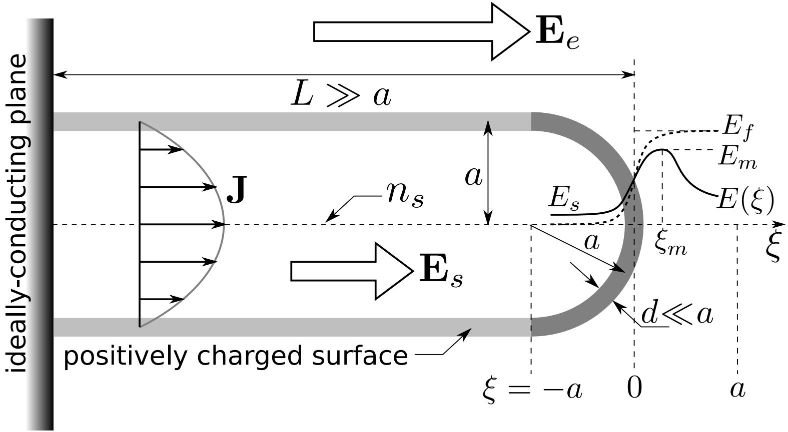

Figure: The streamer model