What determines streamer speed and radius?

Nikolai G. Lehtinen

Birkeland Center for Space Science
University of Bergen, Norway

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Introduction

Streamer mechanism

Model

Goal and approach
Reduced system of equations
Analogy with flat-front theory

Results

Positive streamers
Negative streamers
Threshold fields

Conclusions
What is a streamer?

**Electric streamer discharges** are ionized columns in gas (or liquid) which advance by ionizing the material in front of them with the enhanced field at the streamer tip.

Shown here is a laboratory ~MV, 1 m gap discharge, with a complicated branched streamer tree.

Applications:

1. Lightning, sprites
2. Industry (suprathermal electrons)

[Kochkin et al., 2014, Fig. 8]
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Streamer mechanism
[Loeb and Meek, 1941]

Photons produced in the head of the streamer travel ahead, produce ion-electron pairs, and the electrons serve as avalanche seed in high electric field at streamer head.

Figure: Positive streamer [figure from Raizer, 1991, p. 335]
Streamer mechanism (negative streamer)

The avalanches started by photoelectrons are directed outward, but the streamer moves so fast that it catches up with them.

Figure: Negative streamer [figure from Raizer, 1991, p. 338]
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**Goal**
Understand the streamer basics and answer the question in the title of this talk

**Approach**
- look for a solution in a shape of a streamer;
- simplify microscopic physics PDEs which describe evolution of fields and particles and obtain a finite system of algebraic equations for a finite number of streamer parameters, such as radius, speed etc.;
- solve this system.
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Streamer is a cylinder (channel) with a hemispherical cap (head).

- External electric field $E_e$ and length $L$ are given.
- Want to find parameters: radius $a$, velocity $V$, etc.
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System of algebraic equations

1. Relation between $E$ fields, from electrostatic distribution of surface charge.
2. Continuity of total (conductivity + displacement) current through the streamer front.
3. Propagation stability criterion $\tau_M \sim \tau_{\text{ion}}$, connecting ionization with the maximum field.
4. Velocity-radius relation, from the photoionization mechanism [Pancheshnyi et al., 2001].

Problem: these equations do not give a unique solution! There is still one free parameter. 
I.e., we get something like $\mathcal{F}(V, a) = 0$, while all other parameters may be expressed in terms of $V$ and $a$.

Before giving up, let us look at another approach of reducing a system of PDEs to simpler equations: the flat front perturbation theory.
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Small flat-front perturbations: linear instability

- Start with a flat ionization front propagating as a whole to the right in the Figure.
- Small harmonic $\sim \cos ky$ perturbations grow as $e^{st}$ with growth rate $s$.
- Nonlinear stage.
Small flat-front perturbations: **linear instability**

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The growth rate as a function of transverse wavenumber $s(k)$ is called dispersion function.

- $k$ is a free parameter, evolution depends on initial conditions;
- Perturbation at maximum $s(k)$ grows fastest, so $1/k$ is the preferred transverse size $a$. 
Solution by Derks et al. [2008]

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Analogy of our system with flat-front theory

Flat-front theory [Derks et al., 2008]

- Shape: harmonic
- $k$ is a free parameter
- Velocity of protrusion $V = V_0 + s(k)L$

Our system

- Shape: streamer
- Not enough equations to fix $a \sim 1/k$
- No $s(k)$, but velocity $V(a, L, E_e)$

"Real" solution: $\max_k s(k) \Leftrightarrow \max_k V$

Is physical solution also at $\max_a V$?

**max-V criterion**

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![Graph showing $s(k)$ vs $k$ for different $E_e$ values and a graph showing $V$ vs $a$ for positive and negative $E_e$.]

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Dispersion functions $V(a)$ for positive streamers with $L = 120$ mm and several values of $E_e$.

Dots denote the max-$V$. 
Positive streamers

The following results are after application of $\max-V$. We compare to experimental results of Allen and Mikropoulos [1999].

Figure: Velocity and radius as a function of external field $E_e$, for three different values of $L$. 
Negative streamers

Below certain field $E_e$, there is no solution. Physically, the reason may be that the negative streamer must travel faster than electron drift speed.

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Results  Negative streamers

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Below certain field $E_e$, there is no solution. Physically, the reason may be that the negative streamer must travel faster than electron drift speed.
**Threshold field** $E_{\pm t}$ is the minimum $E_e$ at which propagation is still possible. It depends on $L$ and the physical reason is different for different polarities:

- **Positive streamers**: Three-body attachment inside the streamer quenches it.
- **Negative streamers**: Velocity drops below electron drift speed.
Streamers threshold fields

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![Graph showing different threshold fields $E_{\pm t}$ for various attenuation values and experimental data.](image-url)
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![Graph showing the variation of $E_{\pm t}$ with $L$ for different attenuation levels and experimental data.](image)
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![Graph showing the relationship between $L$ and $E_{\pm t}$ for different attenuation values.](image)
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We describe a streamer electric discharge in air by a system of algebraic equations, which have a solution as function of external field $E_e$, streamer length $L$ and streamer radius $a$.

By employing max-$V$ criterion, we obtain a unique solution which depends only on external conditions $E_e$ and $L$.

Calculations produce results for $V$ and $a$ compatible with observations.

Propagation thresholds are functions of $L$ and are determined by different reasons for positive and negative streamers and are compatible with experimental values $E_{+t} \approx 0.45$ MV/m, $E_{-t} \approx 0.75–1.25$ MV/m [Raizer, 1991, p. 362].
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Slides for extended presentation
  Reduced system of equations: details
System of equations

System of algebraic equations

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2. Continuity of total (conductivity + displacement) current flowing through the streamer front.
3. Propagation stability criterion $\tau_M \sim \tau_{\text{ion}}$, connecting ionization with the maximum field.
4. Velocity-radius relation, determined by the photoionization mechanism [Pancheshnyi et al., 2001].
Equation 1: Fields (relation between $E_s$, $E_f$)

- **External** $E_e$ (*given!*)
  - Inside $E_s < E_e$ due to high conductivity, all charges are at surface
  - Still $E_s > 0$ because there is a current in the channel $\propto n_s$
  - Just outside $E_f > E_e$

Use electrostatic model (method of moments).
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Equation 2: Currents ($E_s, n_s, V$)

- Charge on the surface per unit length $\lambda$ is from MoM and $E_s$
- The total current is $I = \lambda V$
- It is also calculated from $n_s$ and $E_s$ as $I = \int J_c \, dA_{\perp}$

Equivalent approach: total current continuity [Babaeva and Naidis, 1997]:

$$J_c(\text{inside}) = J_d(\text{outside}) = \varepsilon_0 \partial_t E$$
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Equation 3: The front \((n_s, E_f)\)

The flat front theory is used to relate \(n_s\) to \(E_f\). We also have corrections to this theory:

- to include the current \(J_0\) (on the axis)
- to include curvature
- maximum field is not \(E_f\) but corrected value \(E_m\) (which depends on \(d\))
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Equation 4: Photoionization \((V, E_f, a)\)

- Ionizing photons are produced in the front \(\propto\) ionization rate
- Photon production volume (and the number) \(\propto \pi a_{ph}^2, a_{ph} \sim a\)
- Ionization occurs remotely [Zheleznyak et al., 1982] \(\Rightarrow n_p/n_s\)
- Electron avalanche has length \(d\) in streamer frame, which depends on \(V\) and \(E_f\)
- The electron density in the end of avalanche must match \(n_s\)

Loeb [1965]:
\[
d \approx \frac{V}{\nu_i(E_f)} \Rightarrow V \approx \frac{a \nu_i(E_f)}{\log(n_s/n_p)}
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Equation 4: Photoionization ($V, E_f, a$)

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- Photon production volume (and the number) $\propto \pi a_{ph}^2$, $a_{ph} \sim a$
  - Ionization occurs remotely [Zheleznyak et al., 1982] $\Rightarrow n_p/n_s$
  - Electron avalanche has length $d$ in streamer frame, which depends on $V$ and $E_f$
  - The electron density in the end of avalanche must match $n_s$

Loeb [1965]: $d \approx V/\nu_i(E_f) \Rightarrow V \approx a\nu_i(E_f)/\log(n_s/n_p)$
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The model summary figure

Figure: The streamer model