Monte Carlo Simulations of Relativistic Runaway Electrons and Terrestrial Gamma Ray Flashes (TGF)

Nikolai G. Lehtinen
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Terrestrial Gamma Flashes (TGF)

- ~1 ms duration
- Photon energies of up to 20 MeV
- Hard photon spectrum \( \sim E_{\text{ph}}^{-1} \)
- 10-100 J total energy (assuming isotropic emission)
- First observed by BATSE, ~1/month
- RHESSI detects ~10-20/month, \( \Rightarrow \)
  ~50/day globally
**BATSE**

- Burst and Transient Source Experiment
- Was located on CGRO (Compton Gamma Ray Observatory)
- In orbit Apr 5, 1991 – June 4, 2000
- 450 km altitude
- 28.5 deg inclination
- 20 keV - 1.9 MeV photons
- ~2000-10000 cm$^2$ effective area
BATSE TGFs
[Fishman et al 1994]
RHESSI

- **Reuven Ramaty High Energy Solar Spectroscopic Imager**
- Launched on Feb 5, 2002
- Altitude 600 km
- 38 deg inclination
- 3 keV – 17 MeV photons
- \(\sim 10-100 \text{ cm}^2\) effective area
RHESSI TGFs [Smith et al, 2005]

Lightcurves (4 events)

Scatter plot (1 event)
Bremsstrahlung mechanism

- Electrons are accelerated by E field following a +CG discharge
- Number of electrons is increased due to ionization
- Gamma rays are produced by bremsstrahlung from energetic electrons (~35 MeV energy)
- Emission is forward-directed
- Direction of electron motion is determined by elastic collision rate and B.
Spectrum [Smith et al., 2005]

- Black: average measured spectrum
- Red: isotropic bremsstrahlung spectrum (35 MeV electrons)
- Blue: bremsstrahlung with instrumental response
Post-discharge electric field

- Exceeds relativistic runaway threshold
- Accelerates electrons upward
- In atmosphere with exponential conductivity profile:

\[ \Phi_{\text{disch}} = \frac{Q}{4\pi\varepsilon_0} \exp \left[ -\frac{(r + z)}{2H} \right] - 1 \]
Quasi-Electrostatic Field

BEFORE DISCHARGE

E (small)

negative screening charge

10 km
5 km

+ Q
- Q
1 ms

AFTER DISCHARGE

Large E causes runaway breakdown

negative screening charge

10 km
5 km

- Q
Relativistic Runaway Mechanism and Dynamic Friction Force

43 inelastic processes:
- Rotational, vibrational, electronic level excitations
- Dissociative losses
- Ionization

In townsend (1 Td = \(10^{-21}\) V-m²):

\[
E/N = 8 \text{ Td, } 130 \text{ Td, } 1000 \text{ Td}
\]
Relativistic Runaway Electron Avalanche

- Cosmic ray primaries
- Ionization with production of relativistic electrons
- Acceleration

Electron thermalizes due to collisions

Cosmic ray shower
incident primary particle

Cosmic rays produce $\sim 10^{-5}/cm^3\cdot sec$
$>1$ MeV electrons at $\sim 10$ km altitude
Monte Carlo simulations

Method:

- Relativistic motion

\[ \frac{dp}{dt} = -eE - \frac{e}{m\gamma} p \times B + G(t) \]

- \(G(t)\) includes inelastic energy losses and elastic scattering (but excludes energy losses from ionization)

- New electrons from ionization

Results:

- Electron distribution
- Avalanche growth rate
- Drift velocity
Electrons in momentum space

\[ E = 5E_t, \ B = 0 \]
Growth rate

- \( N = N_0 e^{Rt} \)
- \( R = R_0 / \tau \)
- Proportional to atmosphere density \( N_m \)

\[
\tau = (2\pi N_m Z_m r_0^2 c)^{-1}
\]
- \( r_0 \) – classical electron radius
- \( Z_m \) – molecule charge

- \( R_0 \sim (\delta - 1) + 0.04(\delta - 1)^2, \quad \delta = E/E_t \)
Fluid modelling of runaway avalanche above thundercloud

\[ \frac{\partial N}{\partial t} + \nabla (vN) = RN + S \]

- **Notations:**
  - \( v \) – drift velocity
  - \( R(E/E_t) \sim N_m \) – avalanche rate
  - \( S \sim N_m \) – source from cosmic rays, \( =10 \text{ m}^{-3} \text{ s}^{-1} \) at 10 km

- **Cylindrical**
  - Vertical magnetic field

- **Cartesian**
  - Arbitrary direction of magnetic field
  - Horizontally extended thundercloud
Fluid modelling results: Cylindrical

Q=420 C, h+=20 km, t=3 ms

Runaway velocity

Electric field

Runaway density: 2D structure

\( \log(N_R), m^3; N_{R_{max}} = 118.64 \, m^3 \)

Runaway density: time evolution

\( t=0.5 \, ms, \quad t=1 \, ms, \quad t=2 \, ms, \quad t=3 \, ms \)
Fluid modelling results: Cartesian

Q=12 C/km, h=10 km, t=3 ms
Simulated BATSE photon counts

- \( Q = 1200 \text{ C}, \text{100 km long cloud (in EW direction)} \)
- \( 45^\circ \text{ N latitude} \)
- \( 500 \text{ km altitude} \)
- \( 100-300 \text{ keV} \)
- Beam width \( \sim mc^2/E_{el} \)
- Spectrum is \( E_{ph}^{-1} \) on average, harder in the center, softer on the edges
Highly nonlinear dependence on the charge removed and altitude

\[
v \frac{dN}{dz} = RN + S
\]

- \( v = 0.9c \) – runaway drift velocity
- \( R(E/E_t) \sim N_m \) – avalanche rate
- \( S \sim N_m \) – source from cosmic rays, \( =10 \text{ m}^{-3} \text{ s}^{-1} \) at 10 km
Gamma Ray Emissivity

\[ \varepsilon = N N_m v \frac{d\sigma}{dE_{ph}} \]

- \( \frac{d\sigma}{dE_{ph}} \) – Heitler’s differential bremsstrahlung cross-section
The power produced in gamma rays

Integrate over volume, assume ~10km transverse size of the electron beam
Escaping runaway electrons precipitate at the conjugate point.

- cloud-to-ground discharge
- conjugate point
- energetic electrons interact with plasma waves
- magnetic field line
Gamma rays from precipitating electrons

- Initial gamma ray direction is downward
- They are backscattered
- Soft spectrum
- Observed spectrum is $\sim E_{ph}^{-1}$

Gamma ray energy spectrum

$\sim E_{ph}^{-2.5}$
Runaway Electron Avalanche and TGFs at the Magnetic Equator
TGFs occur in regions with high lightning activity
RHESSI TGF locations

Color shows lightning activity
Numerous RHESSI observations

Contradiction:

- Suppressed by geomagnetic field
- ExB drift in the horizontal direction

$\Rightarrow$ gamma rays are not emitted vertically?
Electrons in momentum space

\[ E = 5 \ E_t, \ B = cE, \ B \perp E \]

NOTE: There is no avalanche for \( B \perp E, B=2cE \)
The avalanche growth rates with perpendicular magnetic field

- The rate decreases slowly with increasing B at small B
- The avalanche is quenched approximately when $B > \frac{2E}{c}$
Nonuniform Monte Carlo results at the equator
Number of particles (MC simulation at the equator)

Number of particles with energy > 2 keV

- Higher charge removal needed
- Energetic electrons are lost quickly due to moving out of the high E field region
Altitude above which B becomes important

- For $f_{H0} = 1 \text{ MHz}$
- Elastic: $\nu_m = \omega_H$ ($\nu_m$ is the momentum transfer rate)
- Inelastic: $F_D/\rho = \omega_H$ ($F_D$ is the dynamic friction)
Energy of avalanche-producing electrons

- Low energy electrons are stopped by friction
- Avalanche continues only if $E > E_{\text{min}}$
- $E_{\text{min}} \sim mc^2/R_0$
- For a uniform avalanche, there are more low-energy electrons $\Rightarrow E_{\text{min}}$ is the important energy scale
Gamma Ray Emissivity at the Equator

\[ \varepsilon = N N_m \nu \frac{d\sigma}{dE_{\text{ph}}} \]

- \[ \frac{d\sigma}{dE_{\text{ph}}} \] – Heitler’s bremsstrahlung cross-section
- Avalanche is suppressed at high altitudes (when \( B > 2E/c \))
The power produced in gamma rays at the equator

Integrate over volume, assume ~10km transverse size of the electron beam
Conclusions

- The number of relativistic electrons produced in the avalanche depends very nonlinearly on the electric field.
- At the equator, the avalanche has to take place below 40 km.