Conversion of formulae and quantities between unit systems

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1 cgs→SI

SI system has additional constants ε_0 and μ_0, which satisfy $c^2\varepsilon_0\mu_0 = 1$ and $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2} by definition. It is "rationalized" in the sense that there are no factors 4π or c in Maxwell's equations.

1.1 Convert all quantities

To convert a cgs formula to SI:

Physical quantity	Symbol	**multiplied by**	$L^aT^bM^cQ^d$
Distance	r	10^2	1 0 0 0
Time	t	unchanged	0 1 0 0
Mass	m	10^3	0 0 1 0
Force	F	10^5	1 -2 1 0
Energy	$\mathcal{E}$	10^7	2 -2 1 0
Charge	Q	$10c$	0 0 0 1
Electric field	E	$10^4/c$	1 -2 1 -1
Magnetic induction	B	10^4	-1
Conductivity	σ	$10^{-7}c^2$	2

After this, the appearing factors of 10 are changed to $(4\pi c^2\varepsilon_0)^{1/7}$ or $(4\pi/\mu_0)^{1/7}$.

Note that to convert *values*, one should *divide* by the value in the third column, using c in SI system of units. E.g., electron charge $e = 4.8 \times 10^{-10}$ ecgs $= 4.8 \times 10^{-10}/(10 \times 3 \times 10^8)$ C $= 1.6 \times 10^{-19}$ C.

1.2 Mechanical quantities don't change

We use the fact that ε_0 contains the unit of charge (Coulomb) in 2 power and μ_0 contains it in (-2) power. Then in formulas for quantities containing p-th power of Coulomb and n-th power of c in the above conversion table, change them to same multiplied by $\sqrt{\mu_0/(4\pi)}$ to p-th power, then multiply by c to n-th power. E.g., $Q \to Qc\sqrt{\mu_0/(4\pi)}$. One can also change c to $(\varepsilon_0\mu_0)^{-1/2}$. Then in the above example we have $Q \to Q/\sqrt{4\pi\varepsilon_0}$. The result is Table 3 on page 819 in Jackson, "Classical Electrodynamics", 2nd edition.

2 SI→cgs

To convert an SI formula to cgs:

- Change ε_0 to $1/(4\pi \times 10^{-3}c^2)$ and μ_0 to $4\pi \times 10^{-7}$. The "strange" factor $4\pi \times 10^{-3}c^2$ is just $4\pi \times 10^{-7}c^2$ in SI system.
- Change the physical quantities to the same divided by the value in the third column of the table. Note that c present in the table value must be first converted to cgs, i.e. $c \to 10^2 c$.

The factors of 10 should go away after this procedure (if the units in the formula match), therefore it is not necessary to consider them from the very beginning. Thus do the same as above but drop all powers of 10. This leads to the following shortcuts:

- Mechanical units (not involving Coulombs) don't change
- Charge/current units are divided by c
- $E \to Ec$; B stays the same
- $Q_1Q_2/(4\pi\varepsilon_0) \to Q_1Q_2$.
- $Q/(4\pi c\varepsilon_0) \to Q$

To convert values, multiply by the value in the third column of the table in Seciton 1, taking c in SI system ($\approx 3 \times 10^8$).

3 SI→natural

The natural system of units is such in which $c = 1$ and $\hbar = 1$. Otherwise, it is like SI, except that ε_0 and μ_0 should also be changed to 1. There are no factors 4π in Maxwell's equation, like in SI. The conversion is therefore very easy.

4 cgs→natural

First, convert to SI and then to natural as in previous section. One can just convert to SI and simultaneously put $c = \hbar = \varepsilon_0 = \mu_0 = 1$.

5 natural→SI

Quantities of dimensionality $L^a T^b M^c Q^d$ (they have dimensionality T^{a+b-c} in the natural system, since in it $L = T$, $M = 1/T$, and $Q = 1$) are divided by $c^{a-2c+d/2} \hbar^{c+d/2} \varepsilon_0^{d/2}$. Examples:

1. Energy $[\mathcal{E}] = L^2 T^{-2} M^1 Q^0$, therefore it is divided by $c^0 \hbar^1 \varepsilon_0^0 = \hbar$. From $\mathcal{E} = \omega$ we get $\mathcal{E} = \hbar\omega$.

2. Q is divided by $\sqrt{\varepsilon_0 \hbar c}$. Thus the fine structure constant $e^2/(4\pi)$ becomes $e^2/(4\pi\hbar c)$.

6 natural→cgs

First convert to SI, then to cgs.