Saturation Effects in the VLF Scattering off Strongly Heated Ionosphere

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Outline

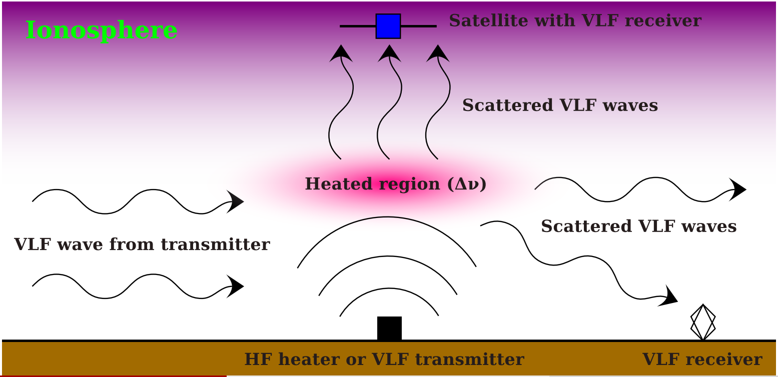
1. Overview
2. Stanford Full-Wave Method (SFWM) code
3. Scattering with the method of moments (MoM)
   - Model description
   - Results

VLF scattering on *D*-region disturbances

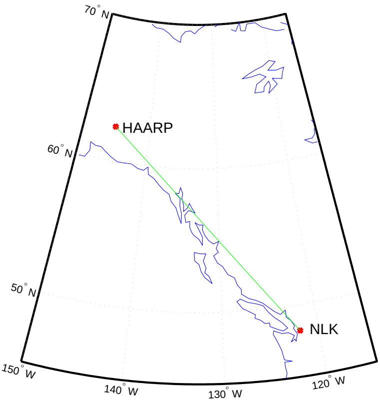
The VLF perturbations are caused by *D*-region disturbances due to HF heaters and can be calculated using Earth-ionosphere waveguide mode theory:

- with WKB and Born approximations [*Barr et al*, 1985; *Demirkol*, Ph. D thesis, 1999].
- with Born but no WKB [*Lehtinen et al*, 2011]
- neither Born nor WKB [*Foust et al*, 2011; present work]

We use Stanford Full-Wave Method (SFWM) together with the method of moments (MoM), which uses less computer resources than discontinuous Galerkin (DG) finite element method [*Foust et al*, 2011].



VLF scattering by an HF heater: NLK/HAARP



NLK VLF transmitter:

- Modelled as a ground-based vertical dipole
- $f = 24.8$ kHz
- $P = 250$ kW

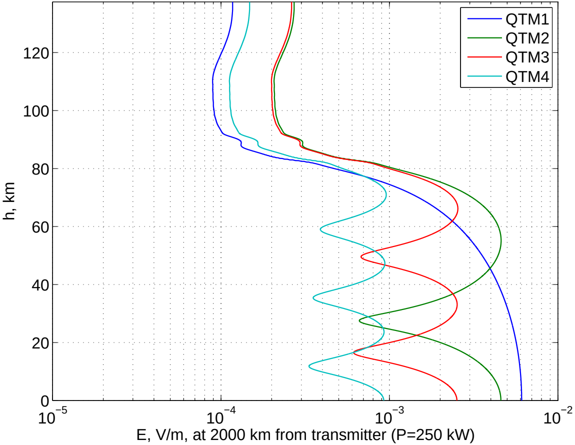
HAARP HF heater:

- $f_{\mathrm{HF}} = 5$ MHz
- $\mathrm{ERP} = 1$ GW
- Beam width ~23 km [*Payne et al*, 2007], we assume Gaussian horizontal shape
- ΔT_e and $\Delta\nu_e$ are found using kinetic equations

Incident VLF wave

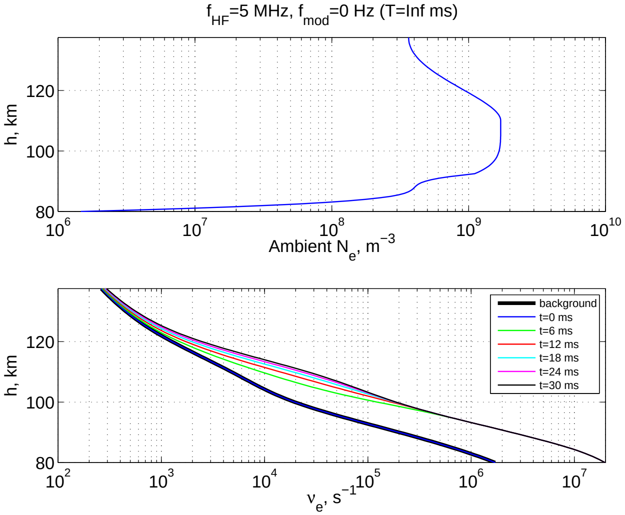
Strongest modes at $R_0 = 2000$ km (disturbance)

- Modes are calculated using SFWM using night-time ionosphere
- Attenuation is due to both absorption and radiation into ionosphere



Change in ν_e due to heating

Kinetic model results for steady heating starting at $t = 0$



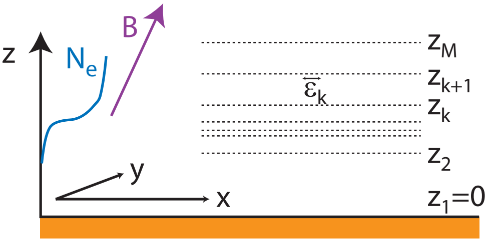
Stanford Full-Wave Method (SFWM) code

Capabilities:

- Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (such as ionosphere)
- Arbitrary configuration of harmonically varying currents
- Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Applications:

- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on *D*-region disturbances



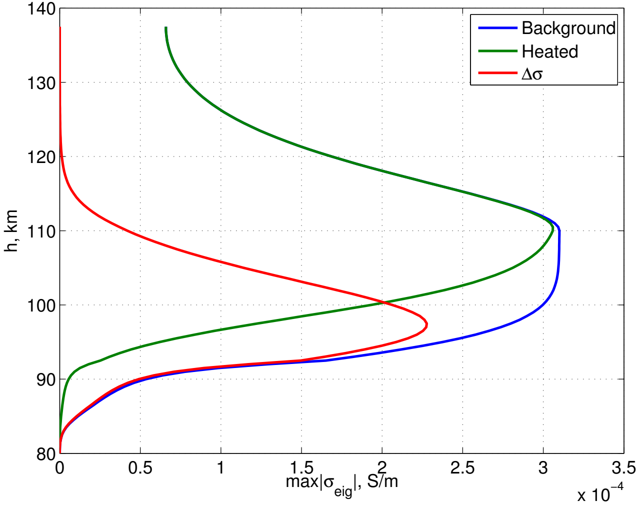
We work in Fourier (horizontal wave vector $\mathbf{k}_\perp$) domain:

1. For each $\mathbf{k}_\perp = const$ (Snell's law) $\Longrightarrow$ find k_z, E and H in each layer for each of 4 plane wave modes (2 up, 2 down)
2. Use continuity of $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers to find reflection coefficients $\mathrm{R}^{u,d}$ and mode amplitudes **u**, **d**
   - Recursion order $\mathrm{R}^{\mathrm{u}}_{k+1} \to \mathrm{R}^{\mathrm{u}}_{k}$ and $\mathbf{u}_k \to \mathbf{u}_{k+1}$ provides stability against "swamping" of solution by evanescent waves
   - Represent source currents as boundary conditions on $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers
3. Inverse Fourier transform from $\mathbf{k}_\perp$ to $\mathbf{r}_\perp$

Previously used Born approximation

- Neglect the scattered field $\mathbf{E}_s$ compared to the incident field $\mathbf{E}_0$ inside the perturbed region
- $\mathbf{E}_0$ acting together with the perturbation $\Delta\hat{\sigma}$ creates currents which radiate $\mathbf{E}_s$
- What if $\mathbf{E}_s$ is comparable to $\mathbf{E}_0$?

Motivation: $\Delta\hat{\sigma}$ may be large



Description of the method of moments (MoM)

- Green's function is a 3×3 matrix $\hat{G}$ with components

$$G_{ij}(\mathbf{r}_o, \mathbf{r}_s) \equiv E_i(\mathbf{r}_o) \text{ created by current } \mathbf{J}(\mathbf{r}) = \hat{x}_j \delta(\mathbf{r} - \mathbf{r}_s)$$

  $\mathbf{r}_s$ — source position
  $\mathbf{r}_o$ — observer position

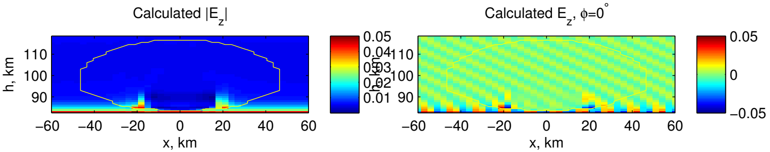
  In our case, Green's function is in the stratified medium, and currents $\mathbf{J} = \Delta\hat{\sigma}\mathbf{E}$ are due to conductivity perturbation.
- We have an integral equation for the scattered field $\mathbf{E}_s$:

$$\mathbf{E}_s(\mathbf{r}) = \int \hat{G}(\mathbf{r}, \mathbf{r}')\Delta\hat{\sigma}(\mathbf{r}') \left[\mathbf{E}_0(\mathbf{r}') + \mathbf{E}_s(\mathbf{r}')\right] \, d^3\mathbf{r}'$$

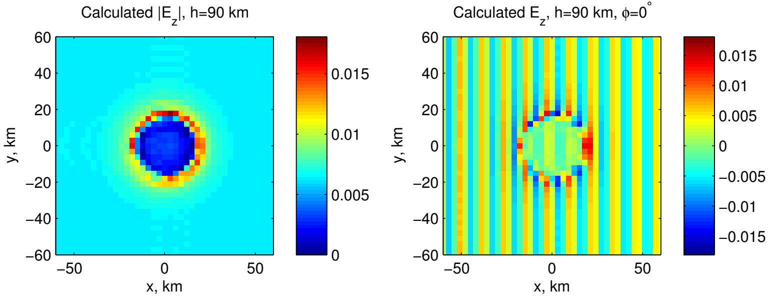
  where the integration is over the perturbed region ($\Delta\hat{\sigma} \neq 0$).
- MoM makes use of discretisation of **J**, **E**. Then the integral equation is solved numerically, and involves an inversion of a large matrix.

3D calculation of E_z from scattering of QTM1 mode

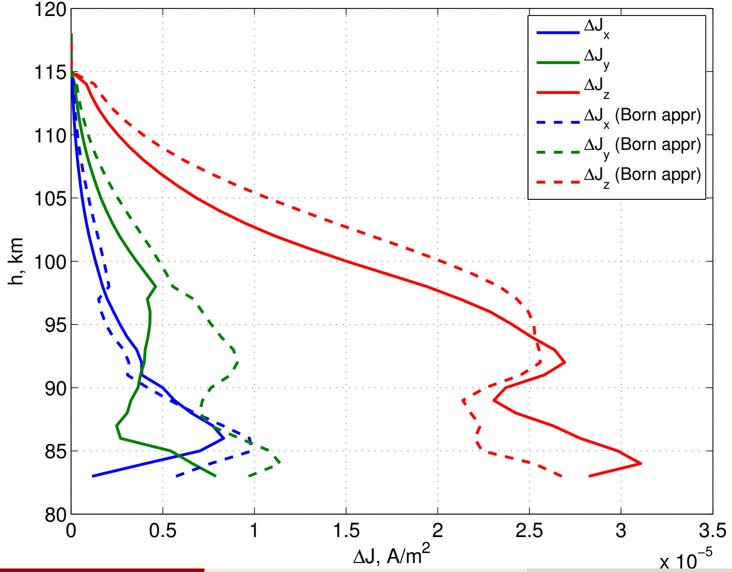
Vertical slice



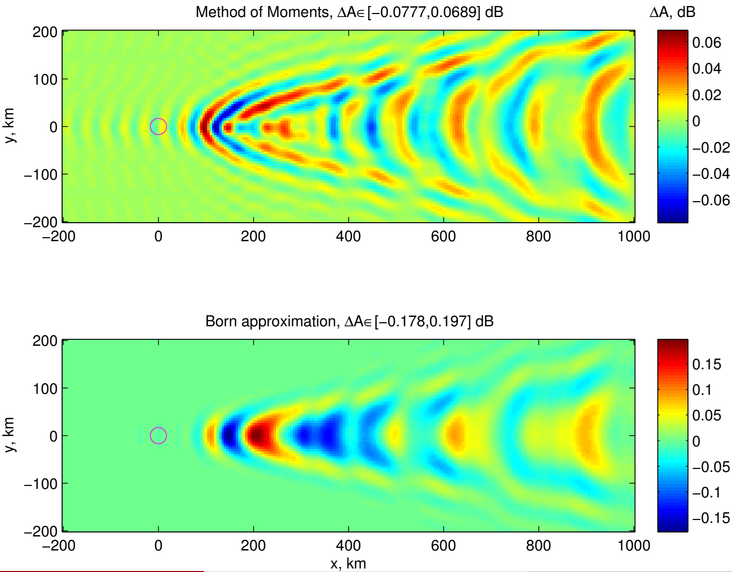
Horizontal slice



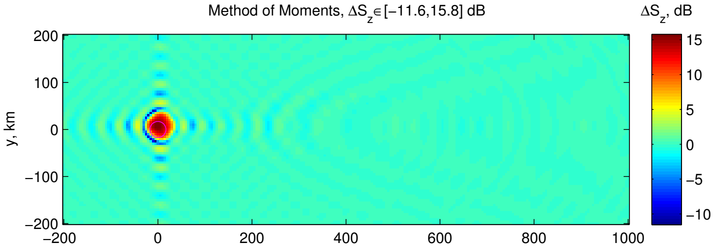
Error in $\Delta \mathbf{J} = \delta\hat{\sigma}\mathbf{E}$ due to Born approximation

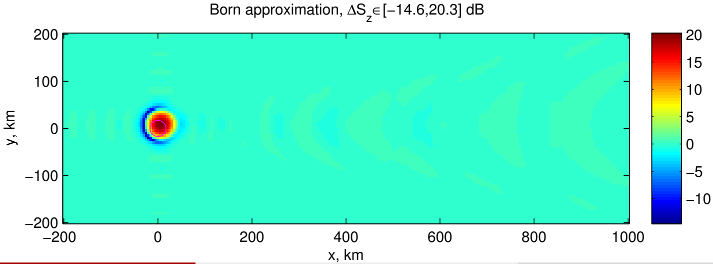


Amplitude change on the ground



Upward flux change at 137.5 km





Conclusions:

- The previously calculated scattering in Born approximation overestimates the effect of stronly heated ionosphere;
- Quantitatively the results are still of the same order.

Acknowledgements:

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