Saturation Effects in the VLF Scattering off Strongly Heated Ionosphere

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AGU 2011 Fall Meeting

December 8, 2011
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VLF scattering on $D$-region disturbances

The VLF perturbations are caused by $D$-region disturbances due to HF heaters and can be calculated using Earth-ionosphere waveguide mode theory:

- with Born but no WKB [Lehtinen et al, 2011]
- neither Born nor WKB [Foust et al, 2011; present work]

We use Stanford Full-Wave Method (SFWM) together with the method of moments (MoM), which uses less computer resources than discontinuous Galerkin (DG) finite element method [Foust et al, 2011].
VLF scattering by an HF heater: NLK/HAARP

**NLK** VLF transmitter:
- Modelled as a ground-based vertical dipole
- $f = 24.8$ kHz
- $P = 250$ kW

**HAARP** HF heater:
- $f_{\text{HF}} = 5$ MHz
- ERP = 1 GW
- Beam width $\sim 23$ km [Payne et al, 2007], we assume Gaussian horizontal shape
- $\Delta T_e$ and $\Delta v_e$ are found using kinetic equations
Incident VLF wave

Strongest modes at $R_0 = 2000$ km (disturbance)

- Modes are calculated using SFWM using night-time ionosphere
- Attenuation is due to both absorption and radiation into ionosphere
Change in $\nu_e$ due to heating
Kinetic model results for steady heating starting at $t = 0$

$f_{HF}=5$ MHz, $f_{mod}=0$ Hz ($T=\text{Inf ms}$)
Stanford Full-Wave Method (SFWM) code

Capabilities:

- Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (such as ionosphere)
- Arbitrary configuration of harmonically varying currents
- Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Applications:

- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on $D$-region disturbances
We work in Fourier (horizontal wave vector $k_\perp$) domain:

1. For each $k_\perp = \text{const}$ (Snell’s law) $\implies$ find $k_z$, $E$ and $H$ in each layer for each of 4 plane wave modes (2 up, 2 down)

2. Use continuity of $E_\perp$ and $H_\perp$ between layers to find reflection coefficients $R_{u,d}$ and mode amplitudes $u$, $d$
   - Recursion order $R_{u,k+1} \rightarrow R_{u,k}$ and $u_k \rightarrow u_{k+1}$ provides stability against “swamping” of solution by evanescent waves
   - Represent source currents as boundary conditions on $E_\perp$ and $H_\perp$ between layers

3. Inverse Fourier transform from $k_\perp$ to $r_\perp$
Previously used Born approximation

- Neglect the scattered field $E_s$ compared to the incident field $E_0$ inside the perturbed region
- $E_0$ acting together with the perturbation $\Delta \sigma$ creates currents which radiate $E_s$
- What if $E_s$ is comparable to $E_0$?
Motivation: $\Delta \hat{\sigma}$ may be large
Description of the method of moments (MoM)

- Green’s function is a $3 \times 3$ matrix $\hat{G}$ with components

\[ G_{ij}(r_o, r_s) \equiv E_i(r_o) \text{ created by current } J(r) = \hat{x}_j \delta(r - r_s) \]

- $r_s$ — source position
- $r_o$ — observer position

In our case, Green’s function is in the stratified medium, and currents $J = \Delta \hat{o} \mathbf{E}$ are due to conductivity perturbation.

- We have an integral equation for the scattered field $\mathbf{E}_s$:

\[ \mathbf{E}_s(r) = \int \hat{G}(r, r') \Delta \hat{o}(r') \left[ \mathbf{E}_0(r') + \mathbf{E}_s(r') \right] d^3r' \]

where the integration is over the perturbed region ($\Delta \hat{o} \neq 0$).

- MoM makes use of discretisation of $\mathbf{J}$, $\mathbf{E}$. Then the integral equation is solved numerically, and involves an inversion of a large matrix.
3D calculation of $E_z$ from scattering of QTM1 mode

**Vertical slice**

- **Calculated $|E_z|$**
- **Calculated $E_z$, $\phi=0^\circ$**

**Horizontal slice**

- **Calculated $|E_z|$, $h=90$ km**
- **Calculated $E_z$, $h=90$ km, $\phi=0^\circ$**

N. Lehtinen (Stanford)
Error in $\Delta J = \delta \hat{E}$ due to Born approximation
Amplitude change on the ground

Method of Moments, $\Delta A \in [-0.0777, 0.0689]$ dB

Born approximation, $\Delta A \in [-0.178, 0.197]$ dB
Upward flux change at 137.5 km

Method of Moments, $\Delta S_z \in [-11.6, 15.8]$ dB

Born approximation, $\Delta S_z \in [-14.6, 20.3]$ dB
Conclusions:

- The previously calculated scattering in Born approximation overestimates the effect of strongly heated ionosphere;
- Quantitatively the results are still of the same order.

Acknowledgements:
This work was supported by Dept. of Air Force grant FA9453-11-C0011 and DTRA grant HDTRA1-10-1-0115 to Stanford University.