Full-wave method of calculation of electromagnetic fields in stratified media

Nikolai G. Lehtinen, Timothy F. Bell, Umran S. Inan

STAR Laboratory, Stanford University, Stanford, CA, U.S.A.
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Outline

1. Stanford Full-Wave Method (SFWM) code
   - Capabilities
   - Description

2. Applications of SFWM
   - Wave propagation through ionosphere
   - Modulated electrojet VLF radiation
   - VLF transmitter radiation
   - Radiation from lightning
   - Earth-ionosphere waveguide modes
   - Scattering on ionospheric disturbances

3. Conclusions

4. Extra slides
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Capabilities:

- Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (such as ionosphere)
- Arbitrary configuration of harmonically varying currents
- Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Applications:

- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on $D$-region disturbances
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We work in Fourier (horizontal wave vector $k_\perp$) domain:

1. For each $k_\perp = \text{const}$ (Snell’s law) $\implies$ find $k_z$, $E$ and $H$ in each layer for each of 4 plane wave modes: 2 up ($u$) and 2 down ($d$)

2. Use continuity of $E_\perp$ and $H_\perp$ between layers to find reflection coefficients $R_{u,d}$ and mode amplitudes $u, d$
   - Recursion order $R_{u,k+1} \rightarrow R_{u,k}$ and $u_k \rightarrow u_{k+1}$ provides stability against “swamping” of solution by evanescent waves
   - Represent source currents as boundary conditions on $E_\perp$ and $H_\perp$ between layers

3. Inverse Fourier transform from $k_\perp$ to $r_\perp$
### Full-wave method background

General description of waves in stratified media and reviews:


Methods using the same recursion order as the one described here:


Other methods:

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TE and TM modes
Highly-collisional plasma (low altitude)

\[ h = 73 \text{ km} \]
\[ X = \frac{\omega_p^2}{\omega^2} = 11.201, \quad Y = \frac{\omega_H}{\omega} = 139.96, \quad Z = \frac{\nu}{\omega} = 68.509 \]

- **Solid** = \( \Re(n_Z) \)
- **Dashed** = \( \Im(n_Z) \)
- \( Z \gg X \)
- The modes at the same frequency in the lower D-region are (almost) like in vacuum, with \(|n| = 1|\)}
Waves in magnetized plasma (high altitude)

h = 300 km

$X = \frac{\omega_p^2}{\omega^2} = 3.6738 \times 10^5$, $Y = \frac{\omega_H}{\omega} = 139.96$, $Z = \frac{\nu}{\omega} = 6.2761 \times 10^{-5}$

- Solid = Re($n_z$)
- Dashed = Im($n_z$)
- In VLF/ELF frequency range, the 4 solutions in ionosphere and magnetosphere are upward and downward whistler and evanescent waves.

UP: $n_1$ (evanescent)
UP: $n_2$ (whistler)
DOWN: $n_3$ (whistler)
DOWN: $n_4$ (evanescent)

Geomagnetic field

Solid = Re($n_z$)
Dashed = Im($n_z$)
In VLF/ELF frequency range, the 4 solutions in ionosphere and magnetosphere are upward and downward whistler and evanescent waves.
Comparison with absorption from Helliwell [1965]

Total losses (including absorption and reflection) for TE (circles) and TM (crosses) incident at nighttime ionosphere at \( \theta = 45^\circ \) with vertical in the plane of \( B_0 \).

- TE and TM modes behave differently
- Reflection losses \(~4 \text{ dB (not taken into account by Helliwell [1965])}\)
- The transmitted energy is maximized parallel to \( B_0 \)

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Schematics

The HF radiation (~3–10 MHz) modulates the polar electrojet at ELF/VLF frequencies ($f_{\text{mod}} \sim 300$ Hz–3 kHz)
Vertical slice of fields [Lehtinen and Inan, 2008]

HAARP HF beam at 3.2 MHz, ERP = 24 MW, $f_{\text{mod}} = 1875$ Hz.

Upper: $B_x$, formation of upward whistler “column” (∼3 W)
Lower: $E_z$, radiation into the Earth-ionosphere waveguide (∼1 W)
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Schematics
Waves are launched both into ionosphere and into the Earth-ionosphere waveguide.
SFWM results

Features of VLF radiation from NPM ($f = 21.4$ kHz, $P = 424$ kW, $B = 34$ $\mu$T, $d = 38.4^\circ$):

- Mode interference (both on the ground and in space)
- Higher attenuation westward on the ground
- Radiation higher along $B$ into space
- West-East asymmetry for radiation into space

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N. Lehtinen (Stanford)
Comparison with satellite data

DEMETER pass over NWC transmitter on October 24, 2006, starting at 14:50:40 UT:
(a) $N_e$; (b) VLF energy flux at 700 km; (c) data and SFWM results for two profiles of
electron-neutral collision rate $\nu_e$ [Lehtinen and Inan, 2009].

The deficit of measured VLF energy points to scattering on irregularities in the ionosphere
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V- and U-shaped streaks in satellite spectrograms

DEMETER observations [Parrot et al., 2008]

⇐ On path: V-shaped

Off-path: U-shaped ⇒
Calculated VLF “streaks” on and off-path of a satellite

\[ \log_{10}(S_z) \text{ at } 120 \text{ km, } \frac{W}{m^2/Hz} \]

\[ P_0 = 1 \text{ W/Hz, } \Delta y = 0 \text{ km} \]

\[ \log_{10}(S_z) \text{ at } 120 \text{ km, } \frac{W}{m^2/Hz} \]

\[ P_0 = 1 \text{ W/Hz, } \Delta y = 300 \text{ km} \]
Spectrogram of $E$ field on the ground

$R^{1/2}E_z$ at 0 km, dB re $(V/m-m^{1/2})^2$/Hz, for $I^2 = 1 (A-m)^2$/Hz
Time-domain sferic waveform calculation

- Apply an inverse Fourier transform: $\omega \rightarrow t$ to the field calculated for a set of $\omega$
- Validated using the sferic database [Said, personal communication, 2010]
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- Modal (dispersion) relation: \( \text{det}(1 - R^d R^u) = 0 \)
- Attenuation is due to both absorption and radiation into ionosphere

**Strongest modes at** \( R_0 = 2000 \text{ km} \)
Waveguide radiation leaking into ionosphere

Waveguide leakage into ionosphere, at $R_0=2000$ km

$S_z$, dB re W/m$^2$

$x=(R-R_0)$, km

QTM1
QTM2
QTM3
QTM4
Role of whistler radiation in the total attenuation

![Diagram showing attenuation and whistler radiation](image-url)

- Total
- Due to whistler radiation
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Schematics

ELVES are transient luminous events caused by lightning EMP (electromagnetic pulse)
Scattering in Born approximation

- **Born approximation**: neglect the scattered field $E_s$ compared to the incident field $E_0$ inside the scattering region.
- $E_0$ acting together with the perturbation $\Delta \sigma$ creates currents which radiate $E_s$. 
Scattered VLF wave ($\Delta A$) on the ground

[Lehtinen et al., 2010]

- VLF wave propagates from left to right ($x = R - R_0$)
- scattering region is indicated by a circle at $x = 0$, $y = 0$

$$\int \Delta N_e \, dz, \, m^{-2}$$

![Vert 20V/m ×1](a)

![Horiz 5V/m ×60](b)
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Future work

- Method of moments instead of Born approximation (more accurate)
- Effects of the Earth’s curvature
- Long-distance propagation by using segmented path of a VLF signal
Summary

We have developed a full-wave method (Stanford FMW) which is stable against “swamping” and easily parallelized. It has been applied to calculate:

- Plane wave transport through ionosphere
- Modulated electrojet current radiation
- Radiation from ground-based transmitters
- Radiation from lightning
- Earth-ionosphere waveguide modes
- Scattering on ionospheric disturbances
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Amplitudes $u$, $d$ and reflection coefficients $R^u$, $R^d$

- Separation into $u$ and $d$:
  \[
  \text{Im} \, k_z \geq 0 \Rightarrow \text{upward mode,} \quad \text{Im} \, k_z \leq 0 \Rightarrow \text{downward mode}
  \]

- Total electromagnetic field $\propto e^{i(k_\perp \cdot r_\perp)}$ is a linear combination
  \[
  \begin{pmatrix}
  E(z) \\
  H(z)
  \end{pmatrix}
  = \mathcal{F}
  \begin{pmatrix}
  u(z) \\
  d(z)
  \end{pmatrix},
  \quad \mathcal{F} \text{ is a } 6 \times 4 \text{ matrix}
  \]

- Propagation up or down within a uniform layer:
  \[
  u(z > 0) = P^u(z)u(0), \quad d(z < 0) = P^d(z)d(0)
  \]
  \[
  P^u(z) = \begin{pmatrix}
  e^{ik^u_{z_1}z} & 0 \\
  0 & e^{ik^u_{z_2}z}
  \end{pmatrix}, \quad P^d(z) = \begin{pmatrix}
  e^{ik^d_{z_1}z} & 0 \\
  0 & e^{ik^d_{z_2}z}
  \end{pmatrix}
  \]

- $\|P^u\| \leq 1$, $\|P^d\| \leq 1 \Rightarrow$ numerical stability

- Reflection coefficients “from above” $R^u$ and “from below” $R^d$ (2 × 2 matrices):
  \[
  d = R^u u \quad u = R^d d
  \]

- Transporting the reflection coefficients through a layer of thickness $h$:
  \[
  R^u(z < 0) = P^d(z)R^u(0)P^u(z), \quad R^d(z > 0) = P^u(z)R^d(0)P^d(z)
  \]

- $u$, $d$ are transported “forward”; $R^u$, $R^d$ are transported “backward.”
At the boundaries between layers

- $\Delta E_\perp = 0$ and $\Delta H_\perp = 0$ at each layer boundary
- We find $R^{u'}$ in terms of $R^u$ and $R^{d'}$ in terms of $R^d$
- We also find $u' = Uu$ and $d' = Dd$

The source currents are assumed to be flowing in thin layers $\Rightarrow$ they give $\Delta E_\perp \neq 0$ and $\Delta H_\perp \neq 0$. 
Upward flux in $k_\perp$-space

Earth-ionosphere waveguide modes manifest as **maxima in $k_\perp$-space**

- $k_\perp = \{k_x, k_y\}$
- $r_\perp = \{x, y\}$

\[ k_\perp \leftrightarrow r_\perp \]

by Fourier transform.

\[
P_{up} = \int\int S_z(k_\perp) \frac{d^2k_\perp}{(2\pi)^2}
\]

– gives a more accurate result than

\[
\int\int S_z(r_\perp) \ d^2r_\perp:
\]

\[
\frac{P_{up}}{P} = 13\%
\]
Scattering in Born approximation

We must solve the wave equation:

$$\nabla \times (\nabla \times E) - k_0^2 \hat{\varepsilon} E = 0 \quad (k_0 = \omega / c)$$

where $E = E_0 + E_s$, $\hat{\varepsilon} = \hat{\varepsilon}_0 + \Delta \hat{\varepsilon}$,

- $E_0$ — incoming wave (e.g., a waveguide mode in stratified $\hat{\varepsilon}_0$);
- $E_s$ — scattered wave;
- $\Delta \hat{\varepsilon}$ — inhomogeneous change in the dielectric permittivity tensor.

In a stratified waveguide $\nabla \times (\nabla \times E_0) - k_0^2 \hat{\varepsilon}_0 E_0 = 0 \quad \Rightarrow \quad \nabla \times (\nabla \times E_s) - k_0^2 \hat{\varepsilon}_0 E_s = k_0^2 \Delta \hat{\varepsilon} (E_0 + E_s)$

- **Born approximation**: neglect $E_s$ compared to $E_0$ inside the scattering region (rhs).
- rhs gives the source currents for scattered waves:

  $$\nabla \times (\nabla \times E_s) - k_0^2 \hat{\varepsilon}_0 E_s \approx k_0^2 \Delta \hat{\varepsilon} E_0 = i k_0 Z_0 \Delta J$$

  where $Z_0$ is the impedance of free space.