SA12A-03
Conversion of ionospheric heater HF waves into electron acoustic waves in warm ionospheric plasma

Nikolai G. Lehtinen, Umran S. Inan and Nicholas L. Bunch

STAR Laboratory, Stanford University, Stanford, CA, U.S.A.
AGU Fall Meeting, San Francisco

December 3, 2012
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
StanfordFWM capabilities and applications

Capabilities:
- Arbitrary plane stratified anisotropic medium
- Arbitrary configuration of harmonically varying currents
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Previous applications (VLF waves in cold plasma):
- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on $D$-region disturbances
We work in Fourier (horizontal wave vector $k_\perp$) domain:

1. For each $k_\perp = \text{const}$ (Snell’s law) $\implies$ find $k_z$, $\mathbf{E}$ and $\mathbf{H}$ in each layer for each of 4 plane wave modes (2 up, 2 down)

2. Use continuity of $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers to find reflection coefficients $\hat{R}^{u,d}$ ($2 \times 2$) and mode amplitudes $u, d$ (of length 2)
   - Recursion order $\hat{R}_{k+1}^u \rightarrow \hat{R}_k^u$ and $u_k \rightarrow u_{k+1}$ provides stability against “swamping” of solution by evanescent waves
   - Represent source currents as boundary conditions on $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers

3. Inverse Fourier transform from $k_\perp$ to $r_\perp$
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
In $k_\perp$-space

The O-mode is converted into upward or downward Z-mode in the upward ($n_x = -0.14$) and downward ($n_x = +0.14$) Ellis radio windows, respectively. Note that there are no Langmuire waves in cold plasma.
In $r_\perp$-space

**Upward window**

- $f=2f_H$ RF window (P)
- $h_{\text{attr}} = 228.8355$ km; $h_{\text{refl}} = 229.2894$ km

**Downward window**

- $f=2f_H$ Downward RF window (Q)
- $h_{\text{attr}} = 228.8355$ km; $h_{\text{refl}} = 229.2894$ km

Budden “The propagation of Radio Waves”, 1985, Fig. 10.8

$$X = \frac{1 - Y^2}{1 - Y^2 \cos^2 \theta_0}$$

- $X = 1$
- $X = 1 - Y^2$
- Base of ionosphere

Lehtinen et al (Stanford) Wave conversion in warm plasma December 3, 2012
Outline

1 Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2 Comparisons with previous calculations

3 Conversion of HF radio into Langmuir waves in the ionosphere

4 Conclusions
Hydro-electro-dynamic equations

\[
\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + qn\mathbf{v} \quad q = -e
\]

\[
\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}
\]

\[
 pn^{-\gamma} = \text{const} \quad \text{adiabatic, } \gamma = 3
\]

\[
m \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = -\frac{\nabla p}{n} + q [\mathbf{E} + \mathbf{v} \times (\mathbf{B}_0 + \mu_0 \mathbf{H})] - m\mathbf{v} \mathbf{v}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0
\]

No ions \(\Rightarrow\) \(\omega \gg \omega_{LH}\) (HF range).

- Linearize for small disturbances \(\mathbf{E}, \mathbf{H}, \mathbf{v}, \rho \propto e^{-i\omega t}\);
- 6 components \(\mathbf{E}_\perp, \mathbf{H}_\perp, v_z\) and \(\rho\) are continuous between slabs;
- 3 “upward” and 3 “downward” mode amplitudes \(\mathbf{u}\) and \(\mathbf{d}\);
- Generalization of stable recursive calculation of reflection coefficients \(\hat{R}_{u,d}\) (3 \times 3) and amplitudes \(\mathbf{u}\) and \(\mathbf{d}\) is straightforward.
Important plasma parameters

Electron sound speed

\[ c_s = \sqrt{\frac{\gamma p_0}{mn_0}} \]

Dimensionless parameters

\[ X = \frac{q^2 n_0}{m \varepsilon_0 \omega^2}, \quad Y = \frac{q B_0}{m \omega}, \quad Z = \frac{\gamma}{\omega}, \quad U = 1 + iZ, \quad \Gamma = \left( \frac{c_s}{c} \right)^2 \]

Note:

- \( X = \frac{\omega_p^2}{\omega^2}, \ |Y| = \frac{\omega_H}{\omega}, \) where \( \omega_p \) and \( \omega_H \) are plasma and gyro frequencies of electrons;
- \( \Gamma = \left( \frac{2 \gamma}{3} \right) \left( \frac{\varepsilon_{th}}{\varepsilon_0} \right) = 2 \varepsilon_{th}/\varepsilon_0, \) where \( \varepsilon_{th} \) is the thermal energy and \( \varepsilon_0 = mc^2 \) is the rest energy of an electron. We consider non-relativistic plasma only, i.e. \( \Gamma \ll 1. \)
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
Simulated ramp of $N_e$

- $B_0, \nu, T_e = const \implies Y, Z, \Gamma = \text{const}$.
- $N_e = N_e(z) \implies X = X(z)$.
- Important dimensionless parameter is $k_0 \Lambda$, where
  \[ \Lambda = \frac{X}{dX/dz} \]
- Gradient is simulated as a sinusoidal ramp of $X$
- $\Lambda$ is evaluated at $z = 0$
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
Comparisons with previous calculations

**Parameters used in the simulation**

\[ f = 65 \text{ kHz}, \quad Y = 0.5, \quad Z = 10^{-5}, \quad \Gamma = 5 \times 10^{-7}, \quad k_0 \Lambda = 53.5606, \quad \theta_B = 64.2^\circ \]

- *Budden and Jones [1987]* used a similar but unstable FWM approach
- Conversion of electrostatic *ES* mode incident onto a gradient of increasing \( N_e \) into extraordinary right-handed *RX* and ordinary left-handed *LO* modes

**Path in the CMA diagram**

- At \( X_1 \) (bottom): *LO*, *RX*, *ES*; at \( X_2 \) (top): no waves
- Small attenuation of propagating waves \( \Rightarrow \) exact \( X_{1,2} \) are not very important.
Comparisons with previous calculations

**Comparison**

*Budden and Jones [1987, Fig 2]*

We reproduced the features of *Budden and Jones [1987, Fig 2]*, such as the effect of the radio window at

\[ n_x = \sqrt{\frac{Y}{1 + Y}} \sin \theta_B = 0.520. \]

The backscattering \( ES \rightarrow ES \) is lower than *Budden and Jones [1987]* result due to attenuation of ES waves.
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
Comparisons with previous calculations

- **Mjølhus [1990]** used contour integration in the complex $k_z$-plane;
- Calculated attenuation $A(p)$ (dimensionless fraction of power) of LO wave when reflected from an upward ramp in $N_e$;
- Parameter is dimensionless factor $p = (k_0\Lambda)^{1/3}Y^{1/2}$

---

**Mjølhus [1990, Fig 10]**

Results are the same, except non-zero $A(p)$ at $p \to \infty$ due to collisions.
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
- Kim et al [2008] used a fluid model;
- Calculated LO attenuation $A(p, q)$
- Parameters are dimensionless Mjølhus factors

\[ q = (k_0 \Lambda)^{1/3} n_x, \quad p = (k_0 \Lambda)^{1/3} Y^{1/2} \]

Kim et al [2008, Fig 6]

The peaks are in the same place!
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
Other mode conversion results for Budden and Jones [1987] conditions

- $ES \rightarrow RX$ more efficient in the upward radio window at $n_x = +0.520$ than the downward ($n_x = -0.520$)
- $RX \rightarrow ES$ very inefficient, less efficient in the upward radio window
- $LO \rightarrow ES$ very efficient conversion in both radio windows
Refractive index surfaces at $Y < 1$ near $X = 1$

Typical ionosphere, $f = 5$ MHz

Conversion of $LO$ into $ES$ for $X < 1$ is explained by

1. radio window conversion $LO \rightarrow Z$
2. resonance cone $Z$ transformation into $ES$ because they are on the same surface.

Note: $Z$ mode for $\theta \neq 0$ is continuous through $X = 1$ because it is extraordinary.
Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - HF propagation with “cold” StanfordFWM
   - Generalization to warm plasma

2. Comparisons with previous calculations

3. Conversion of HF radio into Langmuir waves in the ionosphere

4. Conclusions
Summary

- We generalized StanfordFWM to warm plasma
- Results compare well to previous workers’
- There is efficient conversion $LO \rightarrow ES$ due to $LO \rightarrow Z \rightarrow ES$
- The absorption of the $ES$ mode near plasma resonance contributes to ionosphere $F$-layer electron heating
- This is a linear mechanism, which may contribute together with nonlinear ones such as parametric decay absorption