Metamaterial Waveguide Model of a Return Stroke Channel

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Outline

Introduction

The return stroke (RS) is the most energetic process in a lightning flash [Rakov and Rachidi, 2009]. It is studied using electromagnetic and transmission-line (TL) models. We introduce a new waveguide (electromagnetic) model of the return-stroke channel and discuss its connection with TL model.

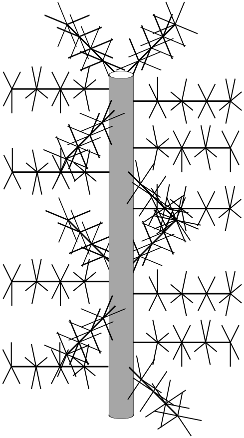
Some of the previous waveguide models include:

- conducting cylindrical core [Volland, 1981]
- cylindrical core with inclusion of the effects of the conducting Earth [Baba and Rakov, 2005]
- highly-conducting thin core surrounded by a perfectly-conducting cylindrical shell corona shield, which may be further developed a multi-fractal approach [Wang, 2010]
- wire immersed in an artificial isotropic dielectric medium [Moini et al., 2000]
- ...

Motivation

We propose that space around the RS channel has high radial and low axial conductivity

- The time-domain fractal lightning (TDFL) model [Carlson et al., 2011; Liang et al., 2010] and observations [Lebedev et al., 2007] have short leaders branching away from the main channel.
- Explanation for the relatively low speed of the return stroke of $c/3$–$c/2$ [Rakov, 2007].
- Incorporation for use with TDFL: (1) efficient calculation of branches; (2) effects of corona shield.



Outline

“Metamaterial” waveguide

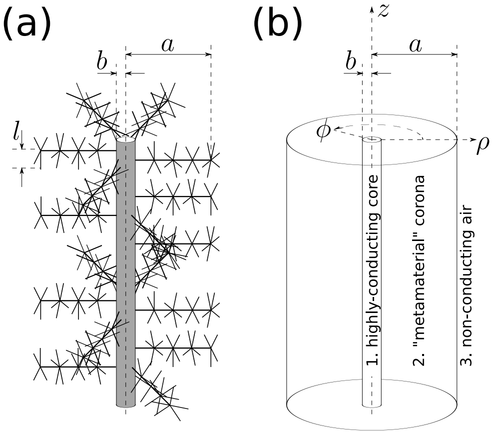
We model the RS channel as a three layer cylindrical waveguide:

1. highly conducting thin core channel
2. “metamaterial” corona, i.e., corona with an effective $\hat{\varepsilon}$ due to the branches:

$$\hat{\varepsilon}(\omega) = \hat{K} + \frac{i\hat{\sigma}}{\varepsilon_0\omega}$$

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\rho & 0 & 0 \\ 0 & \varepsilon_\phi & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

3. surrounding non-conducting air



Axially symmetric TM mode

We assume

- axial symmetry $\partial/\partial\phi \equiv 0$
- TM mode: H_ϕ, E_ρ, and E_z (no TE modes at low f due to a cutoff)
- excitation of wave solutions $\propto e^{-i\omega t+ik_z z}$
- external source current densities $j_z^e \parallel \hat{z}$

Solution is obtained in terms of Hankel functions $H_{0,1}^{(1,2)}$ (i.e., outgoing and incoming waves). This is a stratified medium propagation problem, so methods similar to flat geometry may be used [Lehtinen and Inan, 2008].

Solution (for the brave and mathematically inclined)

$$\begin{aligned} H_\phi &= \Psi/Z_0 \\ E_z &= \frac{i}{k_0 \varepsilon_z \rho}\left(\frac{\partial(\rho\Psi)}{\partial\rho} - j_z^e\right) \\ E_\rho &= \frac{k_z}{k_0 \varepsilon_\rho}\Psi \end{aligned}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$, $k_0 = \omega/c$. In uniform medium ($\partial\varepsilon/\partial\rho = 0$, $\partial j_z/\partial\rho = 0$) for Ψ we have:

$$\frac{\partial}{\partial\rho}\left[\frac{1}{\rho}\frac{\partial(\rho\Psi)}{\partial\rho}\right] + k_\rho^2\Psi = 0$$

with $k_\rho = \sqrt{\varepsilon_z(k_0^2 - k_\rho^2/\varepsilon_\rho)}$, chosen so that $\Im\mathfrak{m}\, k_\rho > 0$. In each waveguide layer

$$\Psi(\rho) = A_k H_1^{(1)}(k_{\rho,k}\rho) + B_k H_1^{(2)}(k_{\rho,k}\rho)$$

and E_z is expressed in terms of $H_0^{(1,2)}$. The coefficients A_k, B_k are found from boundary conditions:

- H_ϕ and E_z are continuous at the boundaries between layers,
- In outer layer: $B_3 = 0$ (radiating),
- In inner layer, $A_1 = B_1$ (finite at $\rho = 0$).

Time- and space-domain formulation

The Earth and the cloud are modelled as perfectly conducting planes at $z = 0$ and $z = L$. This restricts $k_{z,n} = \pi n/L$, where $n = -\infty \ldots \infty$.

Solution steps at fixed ω:

1. Fourier transform external currents $z \to k_{z,n}$:

$$j_z^e(\rho, k_{z,n}) = \frac{1}{L}\int_0^L j_z^e(\rho, z) \cos k_{z,n} z \, dz$$

2. Calculate $H_\phi(\rho)$ and $E_z(\rho)$ in a wave $\propto e^{-i\omega t + k_{z,n} z}$
3. Inverse Fourier transform $k_{z,n} \to z$

$$H_\phi(\rho, z) = \sum_n H_\phi(\rho, k_{z,n}) e^{ik_{z,n} z}$$

Time-domain solutions

Fourier transform $\omega \to t$.

Excitation at the ground

(due to attachment)

- external current flows only in the core (j_z^e uniform at $\rho < b$)
- localized just above the ground (at $z = +0$), i.e.,

$$j_z^e(z,t) = \frac{\iota(t)}{\pi b^2}\delta(z-0), \quad \rho < b$$

where $\iota(t)$ is the time profile of the external current moment

- induced conductivity current j_z partially cancels j_z^e
- source current moment $\iota(t)$ is found by fixing the total core current flowing through $z = 0$ with an assumed known time profile

$$I_0(t) = I(z=0,t) = \int_0^b (j_z + j_z^e)\, 2\pi\rho\, d\rho$$

Outline

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1. Introduction
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3. Results
   - The current wavefront propagation
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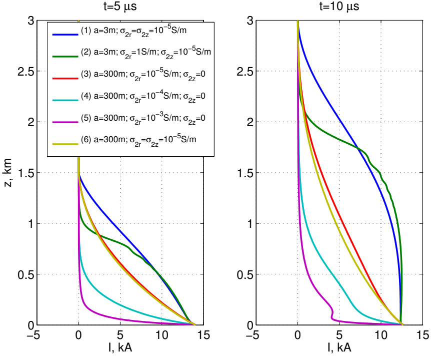
Time- and space-domain calculations

Current:

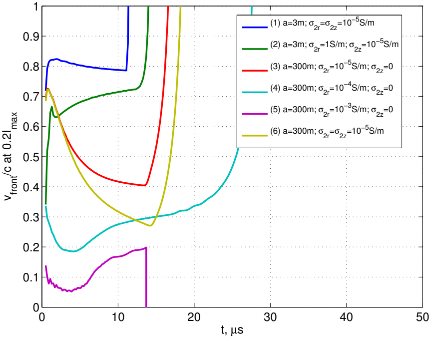
$$I_0(t) = I_m(e^{-at} - e^{-bt}), \qquad t > 0$$

where $I_m = 20$ kA, $a = 4.4 \times 10^4$ s^{-1}, $b = 4.6 \times 10^5$ s^{-1}

Channel length: $L = 3$ km.



Current in the channel driven by the Bruce-Golde model current at $z = 0$, at two selected moments of time: $t = 5$ µs (left) and $t = 10$ µs (right)



The speed of the wave front measured at the level of $0.2I_{\max}$

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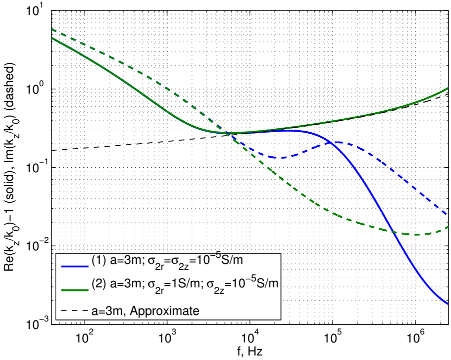
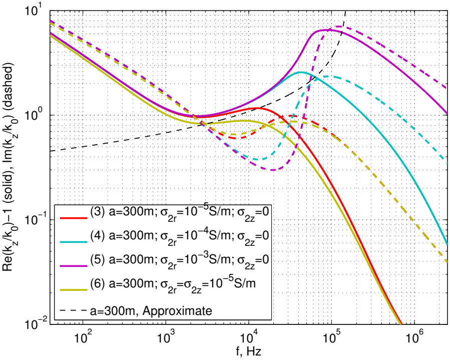
Phase velocity slowdown factor

for various sets of parameters of the metamaterial corona

TL model gives the slowdown factor (labelled as “Approximate”)

$$\frac{c}{v_{ph}} = \Re\mathfrak{e}\{k_z/k_0\} = \sqrt{\frac{\log[1/(k_0 b)]}{\log[1/(k_0 a)]}}, \qquad k_0 = \omega/c$$





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TL (transmission line) model result

- The outer electrode of the TL is taken at large distance $\rho_0 \gg a, b$
- ρ_0 is the evanescense scale $\sim 1/k_0$, where $k_0 = \omega/c$ is the vacuum wave number
- C — capacitance per unit length
  Charge is accumulated on the outer boundary of the corona, at $\rho = a$.

$$C = \frac{2\pi\varepsilon_0}{\log[\rho_0/a]} \approx \frac{2\pi\varepsilon_0}{\log[1/(k_0 a)]}$$

- L — inductance per unit length
  All the current flows in the highly-conducting core, at $\rho < b$

$$L = \frac{\mu_0}{2\pi}\log[\rho_0/b] \approx \frac{\mu_0}{2\pi}\log[1/(k_0 b)]$$

- Dispersion equation and v_{ph} — phase velocity

$$\left(\frac{\omega}{k_z}\right)^2 = v_{ph}^2 = \frac{1}{LC} \approx c^2 \frac{\log[1/(k_0 a)]}{\log[1/(k_0 b)]}$$

Model assumptions and parameters

- Streamer conductivity is only of the order of 0.1 S/m [Raizer, 1991, p. 434], thus the modelled RS slowdown occurs only when radial conductivity is due to leaders, not streamers
- Recent optical observations [Lebedev et al., 2007] demonstrated the presence of faint leader branches of 1–3 m length
- These leaders had rather chaotic direction outward from the channel with bending along the channel axis in both directions. Small bending may be included in “metamaterial” properties by increasing the effective dielectric permittivity of the corona $\Re\mathfrak{e}\,\varepsilon_z$
- A corona with a radius of 300 m [Cases (3)–(6)] may be considered as representation of the long branches
- The increased capacitive loading by arbitrarily long branches may explain the abrupt dropping of the return stroke speed after passing each major branch [Baba and Rakov, 2009, Section 1.2]

Summary

- We have presented a “metamaterial” corona model of the return stroke channel, which takes into account the effect of the contribution of the branch leaders of the corona of the return stroke channel into its electrodynamic properties, without detailed modeling of each branch
- See Lehtinen [2012] for more details
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Outline

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