Effects of the Earth’s Curvature on VLF Propagation

Nikolai G. Lehtinen, Linhai Qiu, Umran S. Inan and Morris B. Cohen

EE Department, Stanford University

April 18, 2012
Outline

1. Stanford Full-Wave Method (StanfordFWM) code

2. StanfordFWM in orthogonal curvilinear coordinates
   - Introduction
   - Validation in cylindrical coordinates
   - Earth-ionosphere waveguide modes

3. Conclusions
Capabilities and applications

Capabilities:

- Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (such as ionosphere)
- Arbitrary configuration of harmonically varying currents
- Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Applications:

- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on $D$-region disturbances
We work in Fourier (horizontal wave vector $k_\perp$) domain:

1. For each $k_\perp = const$ (Snell’s law) $\implies$ find $k_z$, $E$ and $H$ in each layer for each of 4 plane wave modes: 2 up ($u$) and 2 down ($d$)

2. Use continuity of $E_\perp$ and $H_\perp$ between layers to find reflection coefficients $R_{u,d}$ and mode amplitudes $u$, $d$
   - Recursion order $R_{k+1}^u \rightarrow R_k^u$ and $u_k \rightarrow u_{k+1}$ provides stability against “swamping” of solution by evanescent waves
   - Represent source currents as boundary conditions on $E_\perp$ and $H_\perp$ between layers

3. Inverse Fourier transform from $k_\perp$ to $r_\perp$
Example: VLF transmitter radiation

Features of VLF radiation from NPM ($f = 21.4$ kHz, $P = 424$ kW, $B = 34$ $\mu$T, $d = 38.4^\circ$):

- Mode interference (both on the ground and in space)
- Higher attenuation westward on the ground
- Radiation higher along $B$ into space
- West-East asymmetry for radiation into space

NPM: iri 00h 22–Jun–2010

Field on the ground

$\log_{10} B_{\perp}, T$

Upward power flux at $h=137.5$ km

$\log_{10} S_z, W/m^2$
Outline

1. Stanford Full-Wave Method (StanfordFWM) code

2. StanfordFWM in orthogonal curvilinear coordinates
   - Introduction
   - Validation in cylindrical coordinates
   - Earth-ionosphere waveguide modes

3. Conclusions
Motivation

- The mode height gains and therefore the attenuation coefficients may be different if we include Earth’s curvature.
- The previous models (LWPC) only included curvature in the isotropic part of the Earth-ionosphere waveguide and used cylindrical geometry.
- The curved stratification may be used for other problems, e.g., ducting of whistlers by a curved boundary of plasmasphere.

We would like to include curvatures in both directions \((x_1, x_2)\) in a medium which is translationally-symmetric in these directions:
Modifications to StanfordFWM

Operator \( \hat{L} \) transports the transverse \((\xi_1, \xi_2)\)-components of the E/M field \( \propto e^{-i\omega t} \) in \( \xi_3 \equiv Z \)-direction:

\[
\hat{L}F_\perp = \frac{1}{ik_0} \frac{\partial F_\perp}{\partial Z}, \quad F_\perp = \begin{pmatrix} E_\perp \\ H_\perp \end{pmatrix}, \quad k_0 = \frac{\omega}{c}
\]

Consider a curvilinear orthogonal coordinate system \( \{\xi_1, \xi_2, \xi_3\} \) with scale factors \( \{h_1, h_2, h_3\} \), such that

\[
h_{1,2} = 1 + \alpha_{1,2} \xi_3, \quad h_3 = 1, \quad \xi_3 \equiv Z
\]

where \( \alpha_{1,2} = \text{const} \) are curvatures. Examples:

- **cylindrical**: \( Z = \rho - R, \alpha_1 = 1/R, \alpha_2 = 0 \)
- **spherical**: \( Z = r - R, \alpha_1 = 1/R, \alpha_2 = 1/R \)

When \( h_i = 1 \) (i.e., at \( Z = 0 \)), we have

\[
\hat{L} = \hat{L}_\text{flat} - \frac{\alpha_1}{ik_0} \hat{I}_1 - \frac{\alpha_2}{ik_0} \hat{I}_2
\]

where \( \hat{I}_{1,2} \) are projection operators on \( \xi_{1,2} \).
Comparison with analytic solution

Perfect conductor at $r = a = 2$, the source current $|\hat{Z}|$ is at $r = b = 2.5$.

These FWM results coincide with analytic in the right panel. The × signs show the chosen stratification boundaries (i.e., in this case $\Delta r = 0.1$).
An example with shorter wavelength

Again, FWM is visibly different from the flat geometry case and gives good agreement with the analytic solution.
Error for different $\Delta r$

The error seems to stabilize as $\Delta r \to 0$. This may be due to the need to modify also the radiation condition at $r = 3$ (currently using flat geometry, in curvilinear this translates into a presence of a small reflected wave).
Error for different $\Delta r$
(for the second example using shorter wavelength)
Earth-ionosphere waveguide modes

Propagation in $x$-direction

Curvature in $y$-direction did not change the results appreciably (i.e., cylindrical Earth assumption gave correct results.)

![Graph showing Re(cosθ) and Im(cosθ) for f=21.4kHz; Vertical B=5e−5 T with and without curvature.](image1)

![Graph showing Attenuation, dB/Mm for strongest modes.](image2)
Outline

1. Stanford Full-Wave Method (StanfordFWM) code

2. StanfordFWM in orthogonal curvilinear coordinates
   - Introduction
   - Validation in cylindrical coordinates
   - Earth-ionosphere waveguide modes

3. Conclusions

Lehtinen et al (Stanford)
We have developed a theory to include curvature in full-wave method calculations. This method was validated by comparison with analytic solution for axially-symmetric TM mode in an isotropic cylindrically symmetric system. Analytic solutions for arbitrary axial number modes in a plasma with $\mathbf{B}_0 \parallel$ the axis are also known, and may be used for further validation in cylindrical coordinates. Preliminary results for Earth-ionosphere waveguide modes show that the curvature can have a significant effect on the mode attenuation and phase speed.