Saturation Effects in the VLF Scattering off HF Heated Ionosphere

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Outline

1. Overview

2. Stanford Full-Wave Method (SFWM) code

3. Scattering with the method of moments (MoM)
   - Model description
   - Results
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VLF scattering on *D*-region disturbances

The VLF perturbations are caused by *D*-region disturbances due to HF heaters and can be calculated using Earth-ionosphere waveguide mode theory:

- with WKB and Born approximations \([\text{Barr et al, 1985; Demirkol, Ph. D thesis, 1999}]\).
- with Born but no WKB \([\text{Lehtinen et al, 2011}]\)
- neither Born nor WKB \([\text{Foust et al, 2011; present work}]\)

We use Stanford Full-Wave Method (SFWM) together with the method of moments (MoM), which uses less computer resources than discontinuous Galerkin (DG) finite element method \([\text{Foust et al, 2011}]\).
VLF scattering by an HF heater: NLK/HAARP

**NLK** VLF transmitter:
- Modelled as a ground-based vertical dipole
- \( f = 24.8 \text{ kHz} \)
- \( P = 250 \text{ kW} \)

**HAARP** HF heater:
- \( f_{\text{HF}} = 5 \text{ MHz} \)
- ERP = 1 GW
- Beam width \( \sim 23 \text{ km} \) [Payne et al, 2007], we assume Gaussian horizontal shape
- \( \Delta T_e \) and \( \Delta v_e \) are found using kinetic equations
Incident VLF wave

Strongest modes at $R_0 = 2000$ km (disturbance)

- Modes are calculated using SFWM using night-time ionosphere
- Attenuation is due to both absorption and radiation into ionosphere

![Graph showing electric field strength vs. height for different QTM modes at 2000 km from transmitter (P=250 kW).]
Change in $\nu_e$ due to heating

Kinetic model results for steady heating starting at $t = 0$

$f_{HF} = 5\, \text{MHz}, f_{\text{mod}} = 0\, \text{Hz (T=Inf ms)}$

$N_e, \text{m}^{-3}$
$h, \text{km}$

$t=0\, \text{ms}$
$t=6\, \text{ms}$
$t=12\, \text{ms}$
$t=18\, \text{ms}$
$t=24\, \text{ms}$
$t=30\, \text{ms}$
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Stanford Full-Wave Method (SFWM) code

Capabilities:
- Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (such as ionosphere)
- Arbitrary configuration of harmonically varying currents
- Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Applications:
- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on $D$-region disturbances
We work in Fourier (horizontal wave vector $k_\perp$) domain:

1. For each $k_\perp = const$ (Snell’s law) $\implies$ find $k_z$, $E$ and $H$ in each layer for each of 4 plane wave modes (2 up, 2 down)

2. Use continuity of $E_\perp$ and $H_\perp$ between layers to find reflection coefficients $R_u,d$ and mode amplitudes $u,d$
   - Recursion order $R_u^{k+1} \to R_u^k$ and $u_k \to u_{k+1}$ provides stability against “swamping” of solution by evanescent waves
   - Represent source currents as boundary conditions on $E_\perp$ and $H_\perp$ between layers

3. Inverse Fourier transform from $k_\perp$ to $r_\perp$
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Previously used Born approximation

- Neglect the scattered field $E_s$ compared to the incident field $E_0$ inside the perturbed region
- $E_0$ acting together with the perturbation $\Delta \hat{\sigma}$ creates currents which radiate $E_s$
- What if $E_s$ is comparable to $E_0$?
Motivation: $\Delta \hat{\sigma}$ may be large
Description of the method of moments (MoM)

- Green’s function is a $3 \times 3$ matrix $\hat{G}$ with components
  
  $$G_{ij}(r_o, r_s) \equiv E_i(r_o)$$
  created by current $J(r) = \hat{x}_j \delta(r - r_s)$

  - $r_s$ — source position
  - $r_o$ — observer position

  In our case, Green’s function is in the stratified medium, and currents $J = \Delta \hat{\sigma} E$ are due to conductivity perturbation.

- We have an integral equation for the scattered field $E_s$:
  
  $$E_s(r) = \int \hat{G}(r, r') \Delta \hat{\sigma}(r') \left[ E_0(r') + E_s(r') \right] d^3r'$$

  where the integration is over the perturbed region ($\Delta \hat{\sigma} \neq 0$).

- MoM makes use of discretisation of $J, E$. Then the integral equation is solved numerically, and involves an inversion of a large matrix.
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3D calculation of $E_z$ from scattering of QTM1 mode

**Vertical slice**

Calculated $|E_z|$

Calculated $E_z$, $\phi=0^\circ$

**Horizontal slice**

Calculated $|E_z|$, $h=90$ km

Calculated $E_z$, $h=90$ km, $\phi=0^\circ$
Error in $\Delta J = \Delta \hat{E}$ due to Born approximation
Amplitude change on the ground

Method of Moments, $\Delta A \in [-0.00632, 0.00624]$ dB

Born approximation, $\Delta A \in [-0.0202, 0.0213]$ dB
Upward flux change at 137.5 km

Method of Moments, $\Delta S_z \in [-3.59, 0.0813]$ dB

Born approximation, $\Delta S_z \in [-11.8, 0.067]$ dB
Conclusions:
- The previously calculated scattering in Born approximation overestimates the effect of strongly heated ionosphere;
- Quantitatively the results are still of the same order.

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