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Effects of Earth curvature on the sferic ground wave

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Outline

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2 StanfordFWM in orthogonal curvilinear coordinates
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   • Validation in cylindrical coordinates
   • Applications to propagation in the Earth-ionosphere waveguide (EIW)

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Capabilities and applications

Capabilities:

- Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (such as ionosphere)
- Arbitrary configuration of harmonically varying currents
- Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Applications:

- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on $D$-region disturbances
Algorithm

We work in Fourier (horizontal wave vector $k_\perp$) domain:

1. For each $k_\perp = \text{const}$ (Snell’s law) $\implies$ find $k_z$, $E$ and $H$ in each layer for each of 4 plane wave modes: 2 up ($u$) and 2 down ($d$)

2. Use continuity of $E_\perp$ and $H_\perp$ between layers to find reflection coefficients $R_{u,d}$ and mode amplitudes $u, d$
   - Recursion order $R_{u_{k+1}} \rightarrow R_{u_k}$ and $u_k \rightarrow u_{k+1}$ provides stability against “swamping” of solution by evanescent waves
   - Represent source currents as boundary conditions on $E_\perp$ and $H_\perp$ between layers

3. Inverse Fourier transform from $k_\perp$ to $r_\perp$
Booker equation: $k_z$ and $\{E, H\}$ in each layer

- From Maxwell’s equations, obtain the propagation equation along $z$ for “continuous” components $E_\perp, H_\perp$ [Clemmow and Heading, 1954, doi:10.1017/S030500410002939X]:

$$\frac{\partial}{ik_0} f = \hat{L} f, \quad f = \left(\begin{array}{c} E_\perp \\ Z_0 H_\perp \end{array}\right), \quad \hat{L}(k_\perp, \hat{K}) \text{ is a } 4 \times 4 \text{ matrix}$$

where $k_0 = \omega/c$ and $\hat{K} = \hat{K}(\omega)$ is the dimensionless constitutive tensor, connecting $(D/\varepsilon_0, cB)^T$ with $(E, Z_0 H)^T$

- Find $q = k_z/k_0$ and $(E_\perp, Z_0 H_\perp)^T$ as eigenvalues and eigenvectors of the above equation

- Express the remaining components in terms of the continuous components:

$$\left(\begin{array}{c} E_z \\ Z_0 H_z \end{array}\right) = \mathcal{L} f, \quad \mathcal{L}(k_\perp, \hat{K}) \text{ is a } 2 \times 4 \text{ matrix}$$
Motivation

- The mode height gains and therefore the attenuation coefficients may be different if we include Earth’s curvature.
- The previous models (LWPC) only included curvature in the isotropic part of the Earth-ionosphere waveguide and used cylindrical geometry.
- The curved stratification may be used for other problems, e.g., ducting of whistlers by a curved boundary of plasmasphere.

We would like to include curvatures in both directions \((x_1, x_2)\) in a medium which is translationally-symmetric in these directions:
Modifications to StanfordFWM

Operator $\hat{L}$ transports the transverse $(\xi_1, \xi_2)$-components of the E/M field $\propto e^{-i\omega t}$ in $\xi_3 \equiv Z$-direction:

$$\hat{L}f = \frac{1}{ik_0} \frac{\partial f}{\partial Z}, \quad f = \left( \begin{array}{c} E_{\perp} \\ H_{\perp} \end{array} \right), \quad k_0 = \frac{\omega}{c}$$

Consider a curvilinear orthogonal coordinate system $\{\xi_1, \xi_2, \xi_3\}$ with scale factors $\{h_1, h_2, h_3\}$, such that

$$h_{1,2} = 1 + \alpha_{1,2}\xi_3, \quad h_3 = 1, \quad \xi_3 \equiv Z$$

where $\alpha_{1,2} = \text{const}$ are curvatures. Examples:

- **cylindrical**: $Z = \rho - R$, $\alpha_1 = 1/R, \alpha_2 = 0$
- **spherical**: $Z = r - R$, $\alpha_1 = 1/R, \alpha_2 = 1/R$

When $h_i = 1$ (i.e., at $Z = 0$), we have

$$\hat{L} = \hat{L}_{\text{flat}} - \frac{\alpha_1}{ik_0} \hat{I}_1 - \frac{\alpha_2}{ik_0} \hat{I}_2$$

where $\hat{I}_{1,2}$ are projection operators on $\xi_{1,2}$. 

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Comparison with analytic solution

Perfect conductor at $r = a = 2$, the source current $\hat{z}$ is at $r = b = 2.5$

These FWM results coincide with analytic in the right panel. The × signs show the chosen stratification boundaries (i.e., in this case $\Delta r = 0.1$). Relative error $< 2 \times 10^{-3}$ and is due to imperfect radiation condition at $r = 3$ (currently using flat geometry, in curvilinear this translates into a presence of a small reflected wave).
Validation in anisotropic medium

- Magnetized plasma at $0.2 < r < 5$ with Stix parameters $S(r) = 1 + r$, $D(r) = r/2$ and $P(r) = 1 - r$
- Source $J_e(r) = 2\delta(r - r_s)\exp(ik_0\beta z)$, at $r_s = 1.2$, with $k_0 = 0.5$, $m = 4$ and $\beta = 0.1$.

Max error drops fast: $\delta = \max \Delta\{E, H\} = 1.3 \times 10^{-2}$ at $\Delta r = 0.2$ to $\delta = 3.3 \times 10^{-3}$ at $\Delta r = 0.1$, and to $\delta = 9.3 \times 10^{-4}$ at $\Delta r = 0.05$. 
Earth-ionosphere waveguide modes

Propagation in $x$-direction

Curvature in $y$-direction did not change the results appreciably (i.e., cylindrical Earth assumption gave correct results.)
Ground wave amplitude

At $f = 20 \ kHz$, the effects of curvature for up to 200 km are insignificant.
Summary

We have developed and used StanfordFWM (Stanford full-wave method) code which:

- calculates VLF propagation in ionosphere and the Earth-ionosphere waveguide
- has been generalized to curvilinear coordinates

Calculation results indicate that:

- the Earth’s curvature contributes significantly to attenuation of modes
- at VLF frequencies, the Earth curvature does not affect the ground wave at least at distances modelled by FWM
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