AE13A-0338

Effects of Earth curvature on the sferic ground wave

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2013 AGU Fall Meeting, San Francisco

December 9, 2013

Outline

1. Stanford Full-Wave Method (StanfordFWM) code
   - Description
2. StanfordFWM in orthogonal curvilinear coordinates
   - Introduction
   - Validation in cylindrical coordinates
   - Applications to propagation in the Earth-ionosphere waveguide (EIW)
3. Conclusions

Capabilities and applications

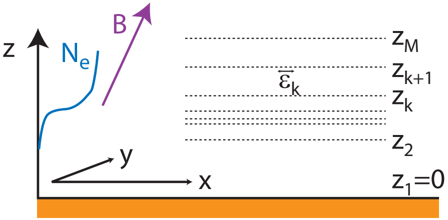
Capabilities:

- Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (such as ionosphere)
- Arbitrary configuration of harmonically varying currents
- Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Applications:

- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on *D*-region disturbances

Algorithm



We work in Fourier (horizontal wave vector $\mathbf{k}_\perp$) domain:

1. For each $\mathbf{k}_\perp = const$ (Snell's law) $\Longrightarrow$ find k_z, **E** and **H** in each layer for each of 4 plane wave modes: 2 up (**u**) and 2 down (**d**)
2. Use continuity of $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers to find reflection coefficients $R^{u,d}$ and mode amplitudes **u**, **d**
   - Recursion order $R^u_{k+1} \to R^u_k$ and $\mathbf{u}_k \to \mathbf{u}_{k+1}$ provides stability against "swamping" of solution by evanescent waves
   - Represent source currents as boundary conditions on $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers
3. Inverse Fourier transform from $\mathbf{k}_\perp$ to $\mathbf{r}_\perp$

Booker equation: k_z and $\{\mathbf{E}, \mathbf{H}\}$ in each layer

- From Maxwell's equations, obtain the propagation equation along z for "continuous" components $\mathbf{E}_\perp$, $\mathbf{H}_\perp$ [*Clemmow and Heading*, 1954, doi:10.1017/S030500410002939X]:

$$\frac{\partial_z}{ik_0}\mathbf{f} = \hat{\mathrm{L}}\mathbf{f}, \quad \mathbf{f} = \begin{pmatrix} \mathbf{E}_\perp \\ Z_0\mathbf{H}_\perp \end{pmatrix}, \quad \hat{\mathrm{L}}(\mathbf{k}_\perp, \hat{\mathrm{K}}) \text{ is a } 4 \times 4 \text{ matrix}$$

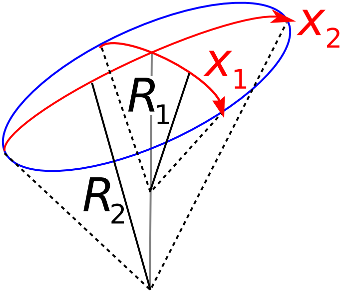
  where $k_0 = \omega/c$ and $\hat{\mathrm{K}} = \hat{\mathrm{K}}(\omega)$ is the dimensionless constitutive tensor, connecting $(\mathbf{D}/\varepsilon_0, c\mathbf{B})^T$ with $(\mathbf{E}, Z_0\mathbf{H})^T$
- Find $q = k_z/k_0$ and $(\mathbf{E}_\perp, Z_0\mathbf{H}_\perp)^T$ as eigenvalues and eigenvectors of the above equation
- Express the remaining components in terms of the continuous components:

$$\begin{pmatrix} E_z \\ Z_0 H_z \end{pmatrix} = \mathcal{L}\mathbf{f}, \quad \mathcal{L}(\mathbf{k}_\perp, \hat{\mathrm{K}}) \text{ is a } 2 \times 4 \text{ matrix}$$

Motivation

- The mode height gains and therefore the attenuation coefficients may be different if we include Earth's curvature
- The previous models (LWPC) only included curvature in the isotropic part of the Earth-ionosphere waveguide and used cylindrical geometry
- The curved stratification may be used for other problems, e.g., ducting of whistlers by a curved boundary of plasmasphere.

We would like to include curvatures in both directions (x_1, x_2) in a medium which is translationally-symmetric in these direcions:



Modifications to StanfordFWM

Operator $\hat{\mathrm{L}}$ transports the transverse (ξ_1, ξ_2)-components of the E/M field $\propto e^{-i\omega t}$ in $\xi_3 \equiv Z$-direction:

$$\hat{\mathrm{L}}\mathbf{f} = \frac{1}{ik_0}\frac{\partial \mathbf{f}}{\partial Z}, \qquad \mathbf{f} = \begin{pmatrix} \mathbf{E}_\perp \\ \mathbf{H}_\perp \end{pmatrix}, \qquad k_0 = \frac{\omega}{c}$$

Consider a curvilinear orthogonal coordinate system $\{\xi_1, \xi_2, \xi_3\}$ with scale factors $\{h_1, h_2, h_3\}$, such that

$$h_{1,2} = 1 + \alpha_{1,2}\xi_3, \quad h_3 = 1, \quad \xi_3 \equiv Z$$

where $\alpha_{1,2} = const$ are curvatures. Examples:

cylindrical: $Z = \rho - R$, $\alpha_1 = 1/R$, $\alpha_2 = 0$

spherical: $Z = r - R$, $\alpha_1 = 1/R$, $\alpha_2 = 1/R$

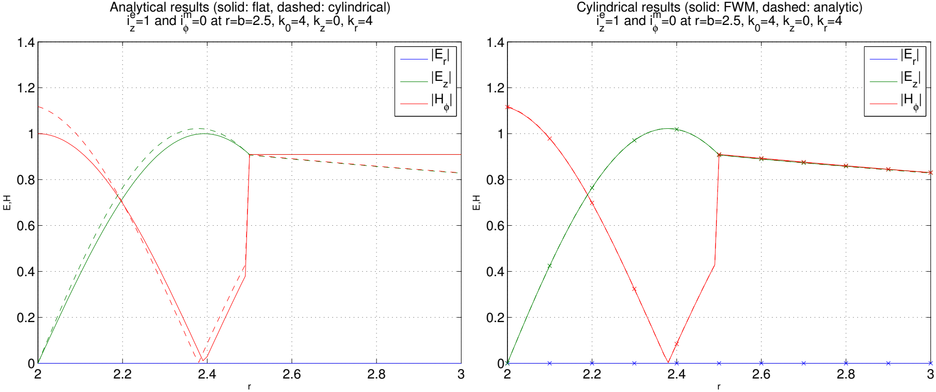
When $h_i = 1$ (i.e., at $Z = 0$), we have

$$\hat{\mathrm{L}} = \hat{\mathrm{L}}_{\mathrm{flat}} - \frac{\alpha_1}{ik_0}\hat{\mathrm{I}}_1 - \frac{\alpha_2}{ik_0}\hat{\mathrm{I}}_2$$

where $\hat{\mathrm{I}}_{1,2}$ are projection operators on $\xi_{1,2}$.

Comparison with analytic solution

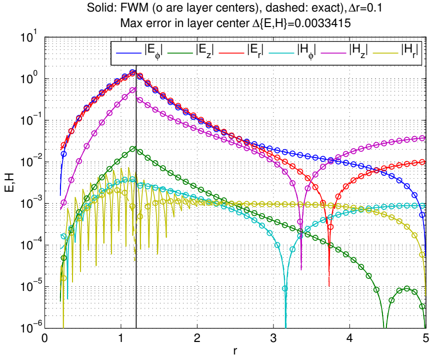
Perfect conductor at $r = a = 2$, the source current $\|\hat{z}$ is at $r = b = 2.5$



These FWM results coincide with analytic in the right panel. The $\times$ signs show the chosen stratification boundaries (i.e., in this case $\Delta r = 0.1$). Relative error $< 2 \times 10^{-3}$ and is due to imperfect radiation condition at $r = 3$ (currently using flat geometry, in curvilinear this translates into a presence of a small reflected wave).

Validation in anisotropic medium

- Magnetized plasma at $0.2 < r < 5$ with Stix parameters $S(r) = 1 + r$, $D(r) = r/2$ and $P(r) = 1 - r$
- Source $\mathbf{J}_e(r) = \hat{z}\delta(r - r_s)\exp(im\phi + ik_0\beta z)$, at $r_s = 1.2$, with $k_0 = 0.5$, $m = 4$ and $\beta = 0.1$.

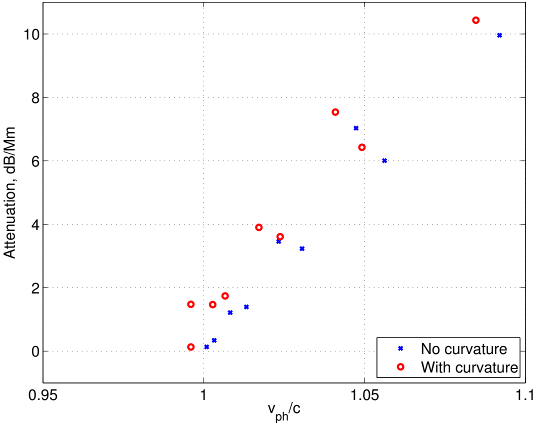


Max error drops fast: $\delta = \max\Delta\{E, H\} = 1.3 \times 10^{-2}$ at $\Delta r = 0.2$ to $\delta = 3.3 \times 10^{-3}$ at $\Delta r = 0.1$, and to $\delta = 9.3 \times 10^{-4}$ at $\Delta r = 0.05$.

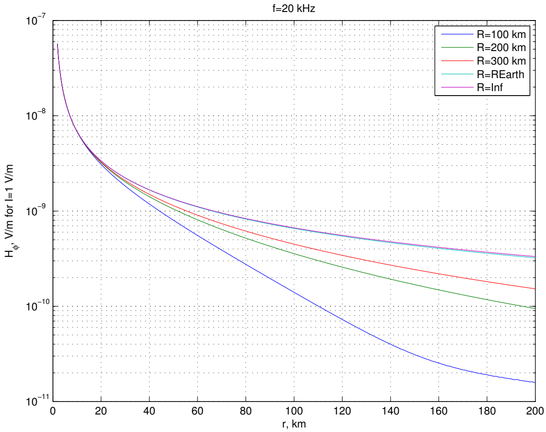
Earth-ionosphere waveguide modes

Propagation in *x*-direction

Curvature in *y*-direction did not change the results appreciably (i.e., cylindrical Earth assumption gave correct results.



Ground wave amplitude



At $f = 20\ kHz$, the effects of curvature for up to 200 km are insignificant.

Summary

We have developed and used StanfordFWM (Stanford full-wave method) code which

- calculates VLF propagation in ionosphere and the Earth-ionosphere waveguide
- has been generalized to curvilinear coordinates

Calculation results indicate that

- the Earth's curvature contributes significantly to attenuation of modes
- at VLF frequencies, the Earth curvature does not affect the ground wave at least at distances modelled by FWM

Acknowledgments

This work was supported by the following grants to Stanford University:

- AFRL FA9453-11-C0011
- DARPA HR0011-10-1-0058
- DTRA HDTRA1-10-1-0115