H3-5 Mode conversion of downward-propagating Langmuir waves in the topside ionosphere

Nikolai G. Lehtinen, Nicholas L. Bunch, and Umran S. Inan

STAR Laboratory, Stanford University, Stanford, CA, U.S.A.
NRSM, Boulder, CO

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1 Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - Generalization to warm plasma

2 Comparisons with previous calculations

3 Conversion of Langmuir into electromagnetic waves in the ionosphere
   - Waves at the top and the bottom of a small $N_e$ ramp
   - Stanford FWM results
   - Explanation of the conversion efficiency

4 Conclusions
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4. Conclusions
StanfordFWM capabilities and applications

Capabilities:
- Arbitrary plane stratified anisotropic medium
- Arbitrary configuration of harmonically varying currents
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Previous applications (VLF waves in cold plasma):
- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on $D$-region disturbances
We work in Fourier (horizontal wave vector $k_\perp$) domain:

1. For each $k_\perp = const$ (Snell’s law) $\implies$ find $k_z$, $E$ and $H$ in each layer for each of 4 plane wave modes (2 up, 2 down)

2. Use continuity of $E_\perp$ and $H_\perp$ between layers to find reflection coefficients $\hat{R}^{u,d}$ ($2 \times 2$) and mode amplitudes $u$, $d$ (of length 2)
   - Recursion order $\hat{R}^{u}_{k+1} \rightarrow \hat{R}^{u}_{k}$ and $u_{k} \rightarrow u_{k+1}$ provides stability against “swamping” of solution by evanescent waves
   - Represent source currents as boundary conditions on $E_\perp$ and $H_\perp$ between layers

3. Inverse Fourier transform from $k_\perp$ to $r_\perp$
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Hydro-electro-dynamic equations

\[
\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + qn \mathbf{v} \quad q = -e
\]
\[
\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}
\]
\[
\rho n^{-\gamma} = \text{const} \quad \text{adiabatic, } \gamma = 3
\]
\[
m \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\frac{\nabla \rho}{n} + q \left[ \mathbf{E} + \mathbf{v} \times (\mathbf{B}_0 + \mu_0 \mathbf{H}) \right] - m \mathbf{v} \mathbf{v}
\]
\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{v}) = 0
\]

No ions \(\implies\) \(\omega \gg \omega_{LH}\) (HF range).

- Linearize for small disturbances \(\mathbf{E}, \mathbf{H}, \mathbf{v}, \rho \propto e^{-i\omega t}\);
- 6 components \(\mathbf{E}_\perp, \mathbf{H}_\perp, v_z\) and \(\rho\) are continuous between slabs;
- 3 “upward” and 3 “downward” mode amplitudes \(u\) and \(d\);
- Generalization of stable recursive calculation of reflection coefficients \(\hat{R}^{u,d}\) (3 \(\times\) 3) and amplitudes \(u\) and \(d\) is straightforward.
Important plasma parameters

Electron sound speed

\[ c_s = \sqrt{\frac{\gamma p_0}{m n_0}} \]

Dimensionless parameters

\[ X = \frac{q^2 n_0}{m \varepsilon_0 \omega^2}, \quad Y = \frac{q B_0}{m \omega}, \quad Z = \frac{\nu}{\omega}, \quad U = 1 + iZ, \quad \Gamma = \left( \frac{c_s}{c} \right)^2 \]

Note:

- \( X = \omega_p^2/\omega^2, \ |Y| = |\omega_H|/\omega \), where \( \omega_p \) and \( \omega_H \) are plasma and gyro frequencies of electrons;
- \( \Gamma = (2\gamma/3)(\varepsilon_{th}/\varepsilon_0) = 2\varepsilon_{th}/\varepsilon_0 \), where \( \varepsilon_{th} \) is the thermal energy and \( \varepsilon_0 = mc^2 \) is the rest energy of an electron. We consider non-relativistic plasma only, i.e. \( \Gamma \ll 1 \).
Comparisons with previous calculations

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Simulated ramp of $N_e$

- $B_0, \nu, T_e = \text{const} \implies Y, Z, \Gamma = \text{const}$.
- $N_e = N_e(z) \implies X = X(z)$.
- Important dimensionless parameter is $k_0 \Lambda$, where

$$\Lambda = \frac{X}{dX/dz}$$

- Gradient is simulated as a sinusoidal ramp of $X$
- $\Lambda$ is evaluated at $z = 0$
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Parameters used in the simulation

\[ f = 65 \text{ kHz}, \quad Y = 0.5, \quad Z = 10^{-5}, \quad \Gamma = 5 \times 10^{-7}, \quad \kappa_0 \Lambda = 53.5606, \quad \theta_B = 64.2^\circ \]

- *Budden and Jones [1987]* used a similar but unstable FWM approach
- Conversion of electrostatic *ES* mode incident onto a gradient of increasing \( N_e \) into extraordinary right-handed *RX* and ordinary left-handed *LO* modes

Path in the CMA diagram

- At \( X_1 \) (bottom): *LO*, *RX*, *ES*; at \( X_2 \) (top): no waves
- small attenuation of propagating waves \( \Rightarrow \) exact \( X_{1,2} \) are not very important.
We reproduced the features of Budden and Jones [1987, Fig 2], such as the effect of the radio window at $n_x = \sqrt{Y/(1 + Y)} \sin \theta_B = 0.520$. The backscattering $ES \rightarrow ES$ is lower than Budden and Jones [1987] result due to attenuation of $ES$ waves.
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Comparisons with previous calculations

- \textit{Mjølhus} [1990] used contour integration in the complex $k_z$-plane;
- Calculated attenuation $A(p)$ (dimensionless fraction of power) of LO wave when reflected from an upward ramp in $N_e$;
- Parameter is dimensionless factor $p = (k_0 \Lambda)^{1/3} Y^{1/2}$

\textit{Mjølhus} [1990, Fig 10]

Results are the same, except non-zero $A(p)$ at $p \to \infty$ due to collisions.
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Comparisons with previous calculations

- *Kim et al* [2008] used a fluid model;
- Calculated LO attenuation $A(p, q)$
- Parameters are dimensionless Mjølhus factors
  
  $$q = (k_0 \Lambda)^{1/3} n_x, \quad p = (k_0 \Lambda)^{1/3} \gamma^{1/2}$$

*Kim et al* [2008, Fig 6]

The peaks are in the same place!
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Consider a small density change around the resonance Path in the CMA diagram

At $X_1$ (bottom): $LO, Z, ES$; at $X_2$ (top): $LX$
Refractive index surfaces at $Y < 1$ near $X = 1$

Typical ionosphere, $f = 5$ MHz

$Z$ and $ES$ waves are actually the same, separated by the resonance cone. Also, $Z$ and $LX$ are the same for $\theta \neq 0$. This opens a possibility of conversion $ES \rightarrow Z \rightarrow LX$. 
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Efficency of conversion

The parameter is now $L = z_{\text{max}} - z_{\text{min}}$, the width of the ramp; $k_0 \lambda = \left( \frac{2L}{\lambda_0} \right) \frac{2X_0}{\Delta X}$.
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Double internal reflection

- Total internal reflection of upward $ES$ into downward $ZB$ (“backward”) mode on the resonance cone (with $S_z$ opposite to $k_z = k_0 q$).
- As $ZB$ mode propagates downwards to the lower density region, the resonance cone becomes wider and the mode gets further away from it.
- At some point with $q > 0$, the downward $ZB$ coalesces with upward $ZF$ (“forward”) mode, and converts to it in the total internal reflection process.
- The extraordinary $ZF$ mode becomes $LX$ mode above the ramp, without being affected by the resonance at $X = 1$. 

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- We generalized StanfordFWM to warm plasma
- Results compare well to previous workers’
- There is efficient conversion $ES \leftrightarrow EM$ due to total internal reflection mechanism