H3-5 Mode conversion of downward-propagating Langmuir waves in the topside ionosphere

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Outline

1. Stanford Full-Wave Method (StanfordFWM)
   - Algorithm description (cold plasma)
   - Generalization to warm plasma
2. Comparisons with previous calculations
   - *Budden and Jones* [1987, doi:10.1098/rspa.1987.0077]
   - *Mjølhus* [1990, doi:10.1029/RS025i006p01321]
   - *Kim et al* [2008, doi:10.1063/1.2994719]
3. Conversion of Langmuir into electromagnetic waves in the ionosphere
   - Waves at the top and the bottom of a small N_e ramp
   - Stanford FWM results
   - Explanation of the conversion efficiency
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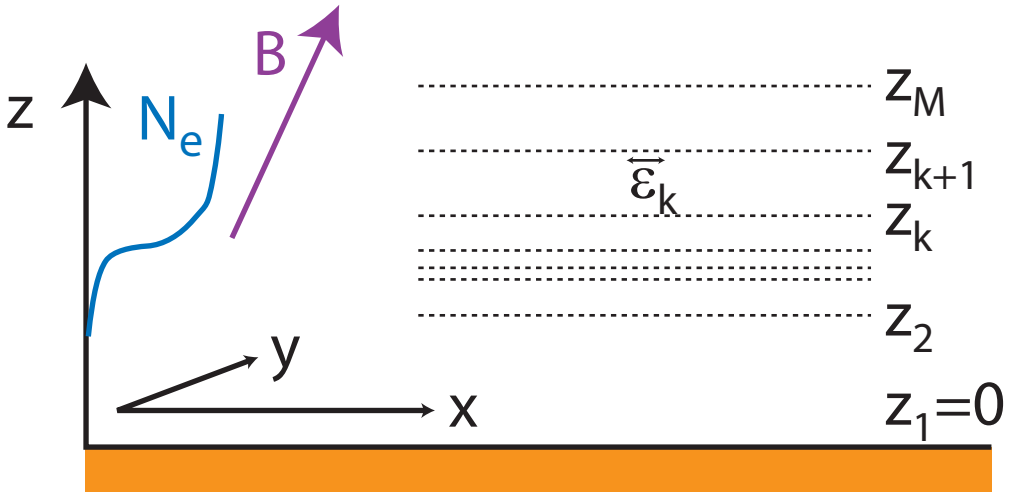
StanfordFWM capabilities and applications

Capabilities:

- Arbitrary plane stratified anisotropic medium
- Arbitrary configuration of harmonically varying currents
- Stable against the “swamping” instability by evanescent waves
- Efficient use of the computer resources, easily parallelized

Previous applications (VLF waves in cold plasma):

- Trans-ionospheric propagation
- Earth-ionosphere waveguide propagation
- Scattering on *D*-region disturbances



We work in Fourier (horizontal wave vector $\mathbf{k}_\perp$) domain:

1. For each $\mathbf{k}_\perp = const$ (Snell's law) $\Longrightarrow$ find k_z, $\mathbf{E}$ and $\mathbf{H}$ in each layer for each of 4 plane wave modes (2 up, 2 down)
2. Use continuity of $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers to find reflection coefficients $\hat{\mathrm{R}}^{\mathrm{u,d}}$ (2×2) and mode amplitudes $\mathbf{u}$, $\mathbf{d}$ (of length 2)
   - Recursion order $\hat{\mathrm{R}}^{\mathrm{u}}_{k+1} \rightarrow \hat{\mathrm{R}}^{\mathrm{u}}_{k}$ and $\mathbf{u}_k \rightarrow \mathbf{u}_{k+1}$ provides stability against "swamping" of solution by evanescent waves
   - Represent source currents as boundary conditions on $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ between layers
3. Inverse Fourier transform from $\mathbf{k}_\perp$ to $\mathbf{r}_\perp$

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Hydro-electro-dynamic equations

$$
\begin{aligned}
\nabla \times \mathbf{H} &= \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + qn\mathbf{v} \qquad q = -e \\
\nabla \times \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \\
pn^{-\gamma} &= const \qquad \text{adiabatic, } \gamma = 3 \\
m\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] &= -\frac{\nabla p}{n} + q\left[\mathbf{E} + \mathbf{v}\times(\mathbf{B}_0 + \mu_0\mathbf{H})\right] - m\nu\mathbf{v} \\
\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) &= 0
\end{aligned}
$$

No ions $\Longrightarrow \omega \gg \omega_{LH}$ (HF range).

- Linearize for small disturbances $\mathbf{E}$, $\mathbf{H}$, $\mathbf{v}$, $p \propto e^{-i\omega t}$;
- 6 components $\mathbf{E}_\perp$, $\mathbf{H}_\perp$, v_z and p are continuous between slabs;
- 3 “upward” and 3 “downward” mode amplitudes $\mathbf{u}$ and $\mathbf{d}$;
- Generalization of stable recursive calculation of reflection coefficients $\hat{\mathrm{R}}^{\mathrm{u,d}}$ (3×3) and amplitudes $\mathbf{u}$ and $\mathbf{d}$ is staightforward.

Important plasma parameters

Electron sound speed

$$c_s = \sqrt{\frac{\gamma p_0}{m n_0}}$$

Dimensionless parameters

$$X = \frac{q^2 n_0}{m \varepsilon_0 \omega^2}, \quad \mathbf{Y} = \frac{q \mathbf{B}_0}{m \omega}, \quad Z = \frac{\nu}{\omega}, \quad U = 1 + iZ, \quad \Gamma = \left(\frac{c_s}{c}\right)^2$$

Note:

- $X = \omega_p^2/\omega^2$, $|\mathbf{Y}| = |\omega_H|/\omega$, where ω_p and ω_H are plasma and gyro frequencies of electrons;
- $\Gamma = (2\gamma/3)(\mathcal{E}_{th}/\mathcal{E}_0) = 2\mathcal{E}_{th}/\mathcal{E}_0$, where $\mathcal{E}_{th}$ is the thermal energy and $\mathcal{E}_0 = mc^2$ is the rest energy of an electron. We consider non-relativistic plasma only, i.e. $\Gamma \ll 1$.

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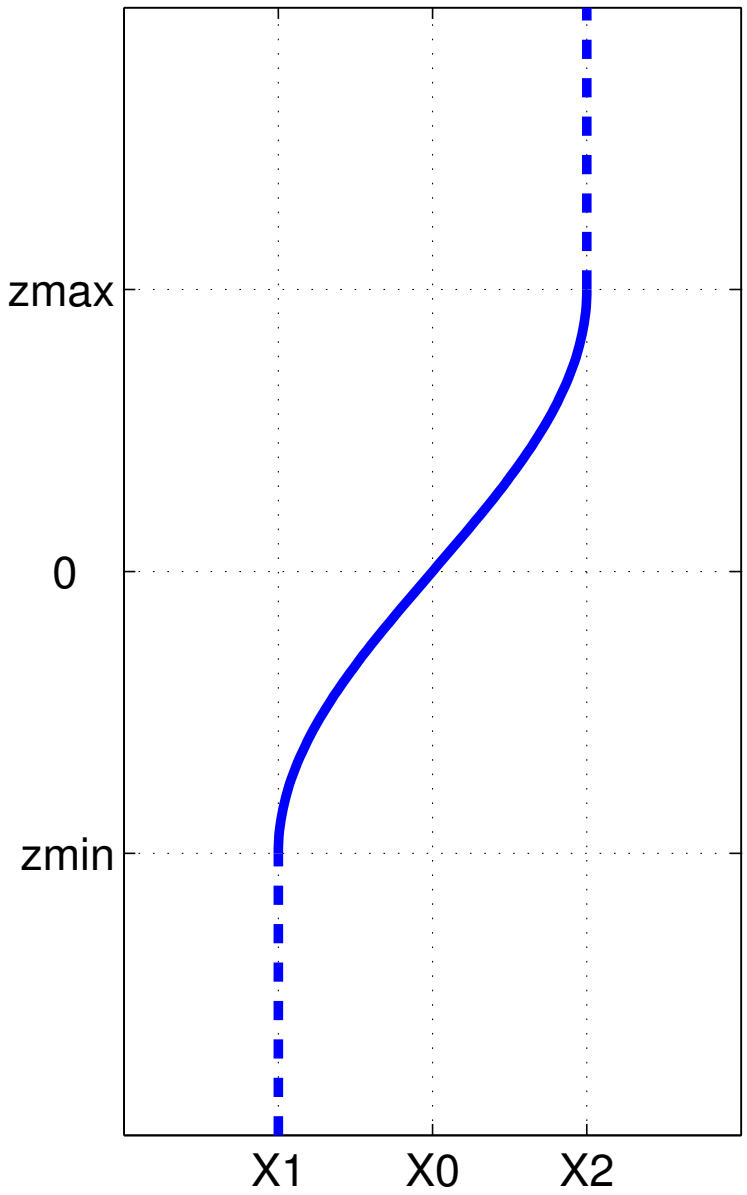
Simulated ramp of N_e

- $\mathbf{B}_0, \nu, T_e = const \implies \mathbf{Y}, Z, \Gamma = const$.
- $N_e = N_e(z) \implies X = X(z)$.
- Important dimensionless parameter is $k_0\Lambda$, where

$$\Lambda = \frac{X}{dX/dz}$$

- Gradient is simulated as a sinusoidal ramp of X
- Λ is evaluated at $z = 0$



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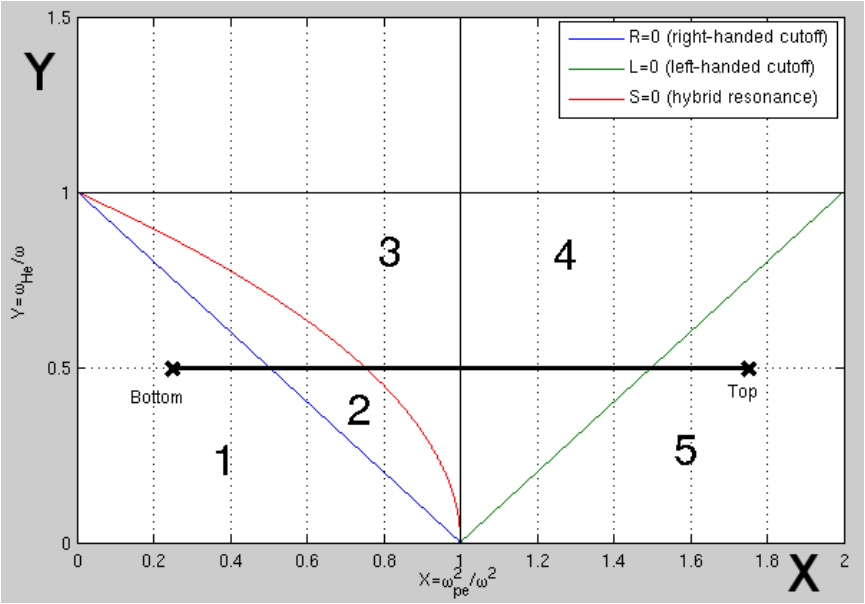
Parameters used in the simulation

$f = 65$ kHz, $Y = 0.5$, $Z = 10^{-5}$, $\Gamma = 5 \times 10^{-7}$, $k_0\Lambda = 53.5606$, $\theta_B = 64.2^\circ$

- *Budden and Jones* [1987] used a similar but unstable FWM approach
- Conversion of electrostatic *ES* mode incident onto a gradient of increasing N_e into extraordinary right-handed *RX* and ordinary left-handed *LO* modes

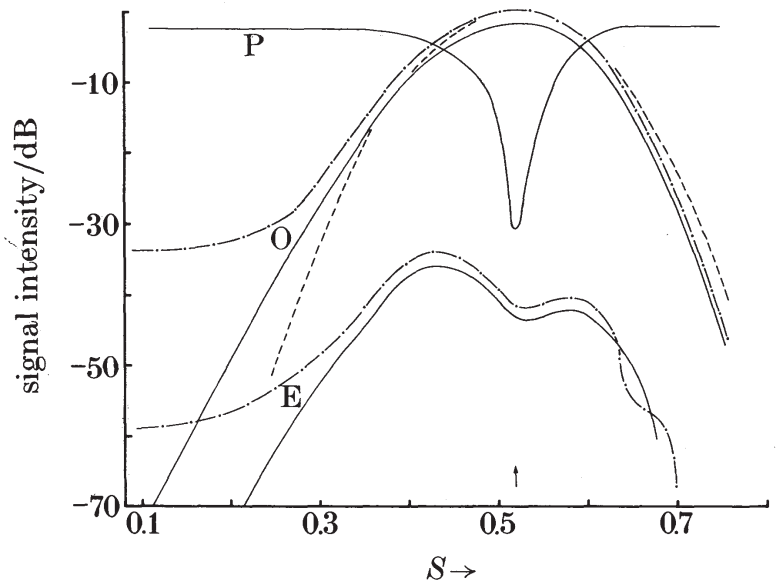
Path in the CMA diagram



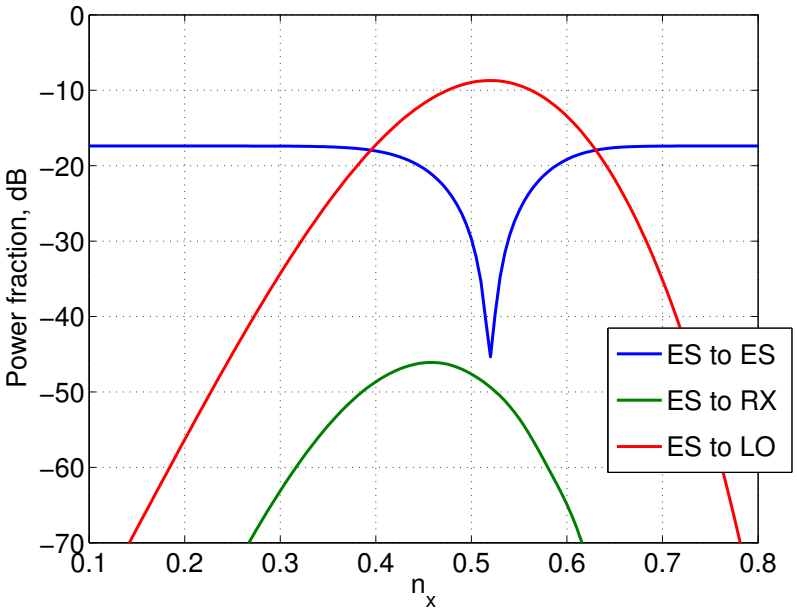
- At X_1 (bottom): *LO*, *RX*, *ES*; at X_2 (top): no waves
- small attenuation of propagating waves $\Rightarrow$ exact $X_{1,2}$ are not very important.

Comparison

Budden and Jones [1987, Fig 2]



Present work



We reproduced the features of *Budden and Jones* [1987, Fig 2], such as the effect of the radio window at $n_x = \sqrt{Y/(1+Y)}\sin\theta_B = 0.520$. The backscattering $ES \to ES$ is lower than *Budden and Jones* [1987] result due to attenuation of *ES* waves.

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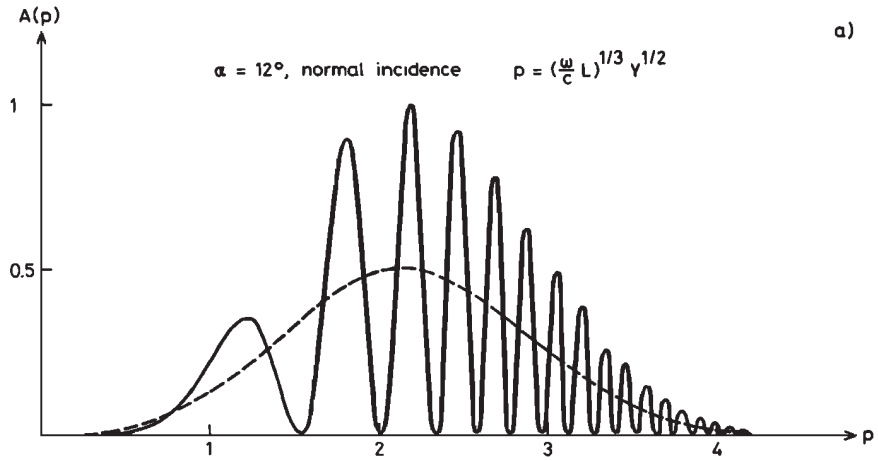
- *Mjølhus* [1990] used contour integration in the complex k_z-plane;
- Calculated attenuation $A(p)$ (dimensionless fraction of power) of *LO* wave when reflected from an upward ramp in N_e
- Parameter is dimensionless factor $p = (k_0\Lambda)^{1/3} Y^{1/2}$

Mjølhus [1990, Fig 10]



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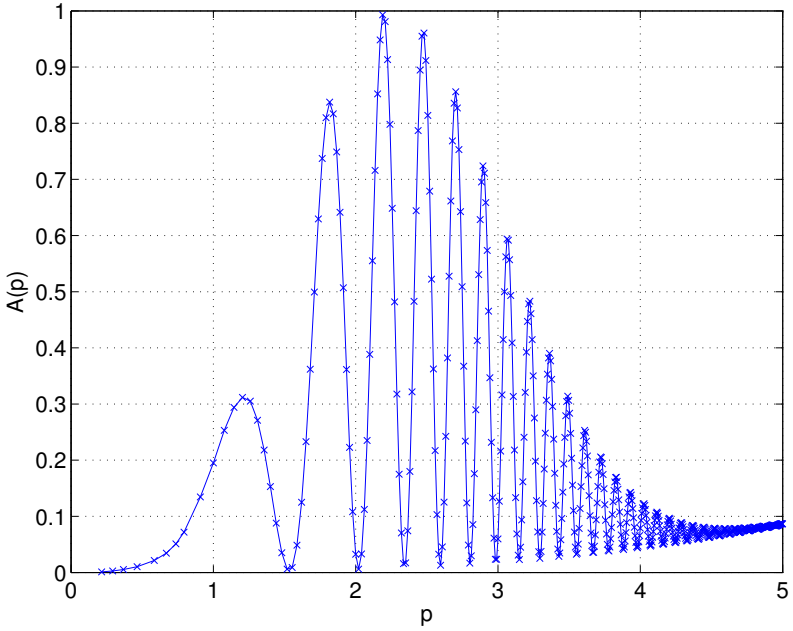

Results are the same, except non-zero $A(p)$ at $p \to \infty$ due to collisions.

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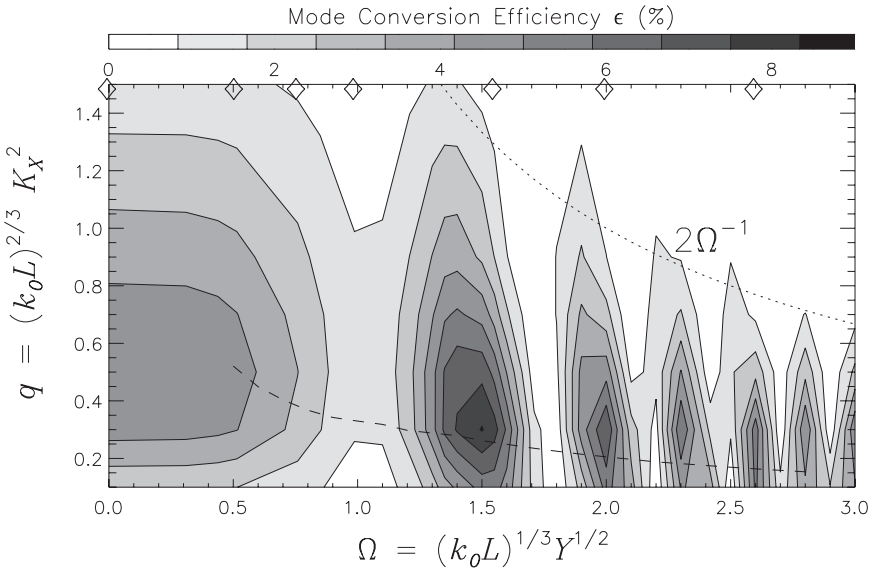
- *Kim et al* [2008] used a fluid model;
- Calculated *LO* attenuation $A(p, q)$
- Parameters are dimensionless Mjølhus factors

$$q = (k_0\Lambda)^{1/3} n_x, \qquad p = (k_0\Lambda)^{1/3} Y^{1/2}$$

Kim et al [2008, Fig 6]



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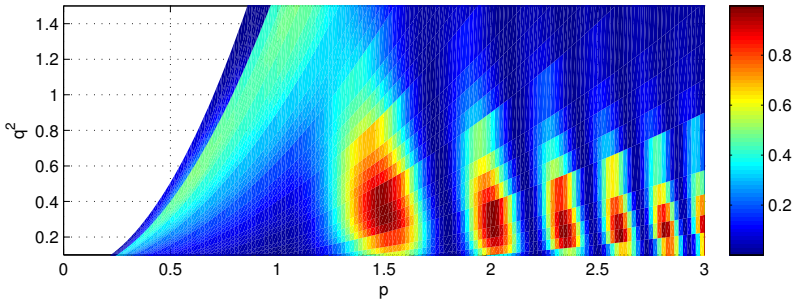

The peaks are in the same place!

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Waves

Consider a small density change around the resonance

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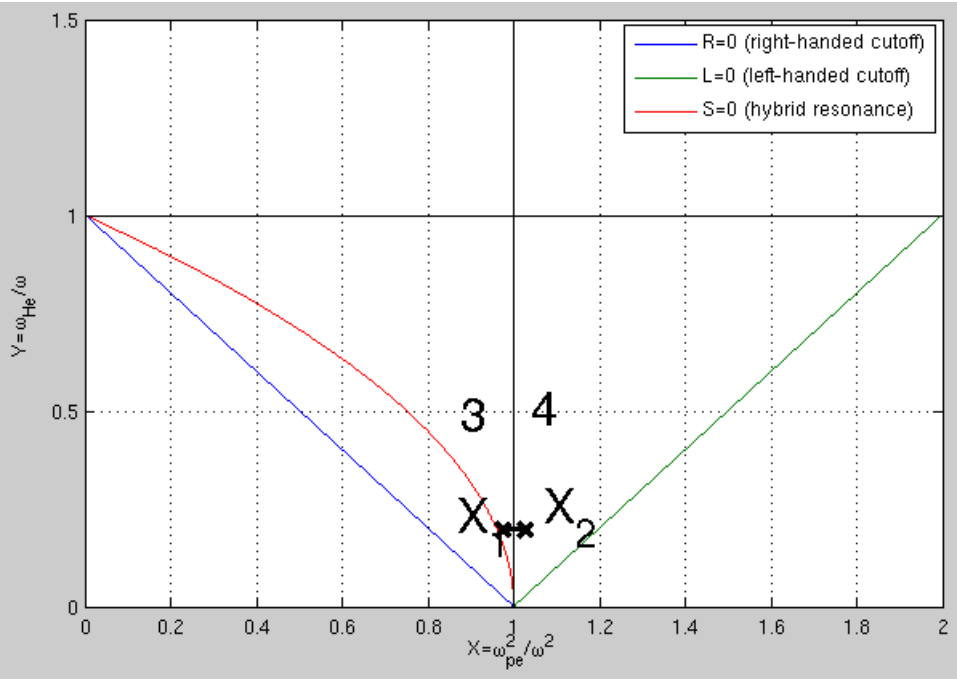
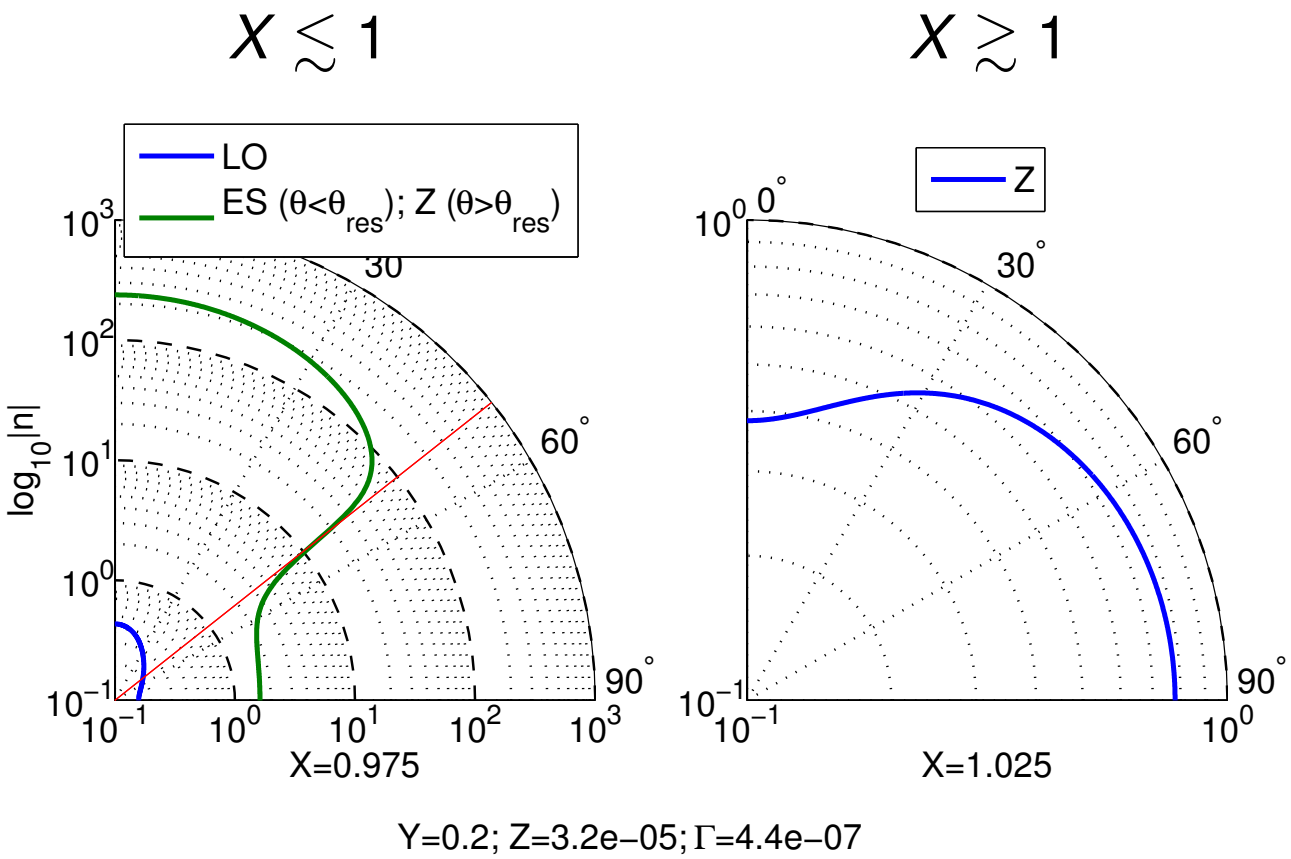


At X_1 (bottom): *LO*, *Z*, *ES*; at X_2 (top): *LX*

Refractive index surfaces at $Y < 1$ near $X = 1$

Typical ionosphere, $f = 5$ MHz



Z and ES waves are actually the same, separated by the resonance cone. Also, Z and LX are the same for $\theta \neq 0$. This opens a possibility of conversion $ES \rightarrow Z \rightarrow LX$.

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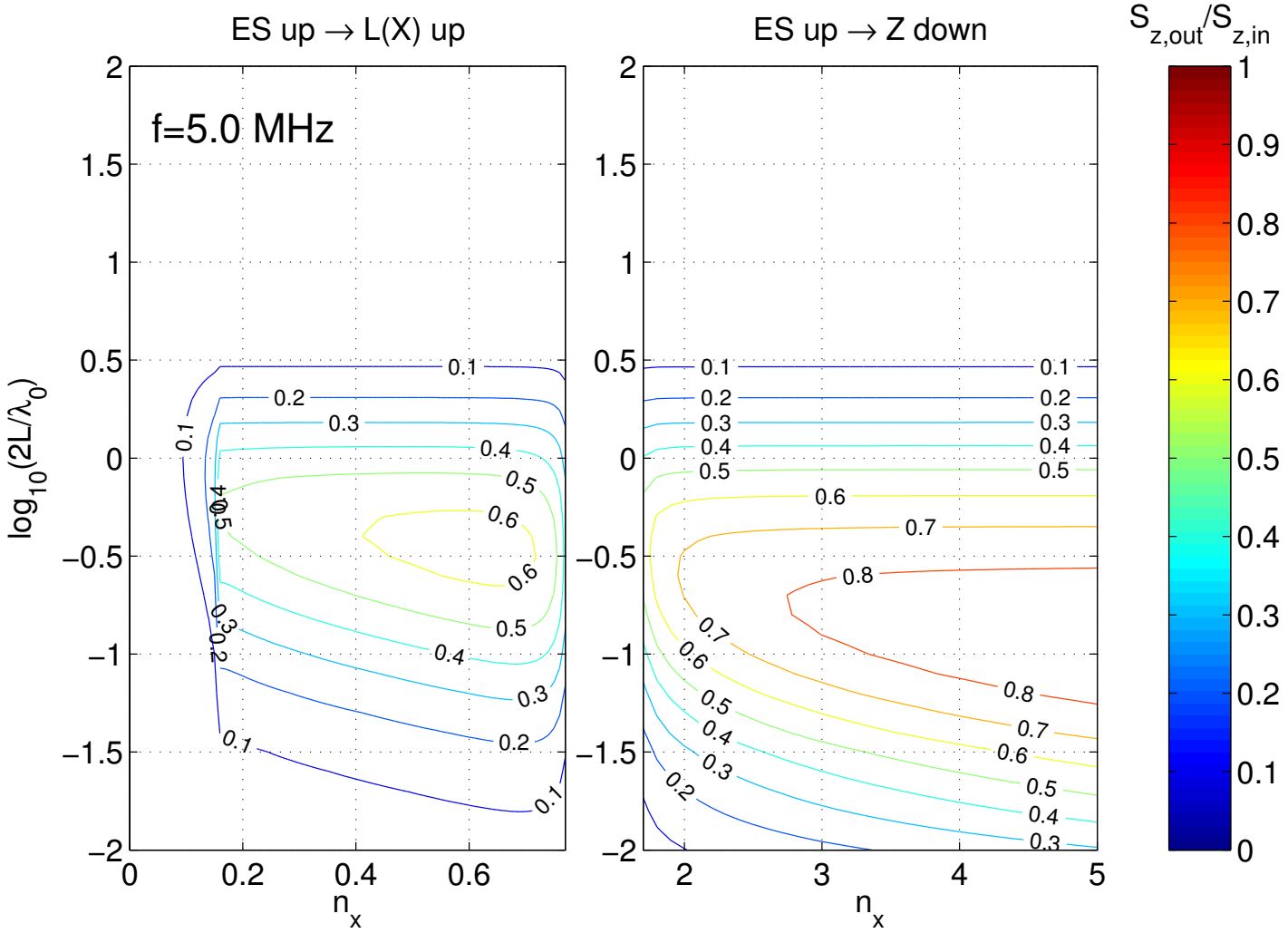
Efficency of conversion

The parameter is now $L = z_{\max} - z_{\min}$, the width of the ramp; $k_0\Lambda = \left(\frac{2L}{\lambda_0}\right)\frac{2X_0}{\Delta X}$



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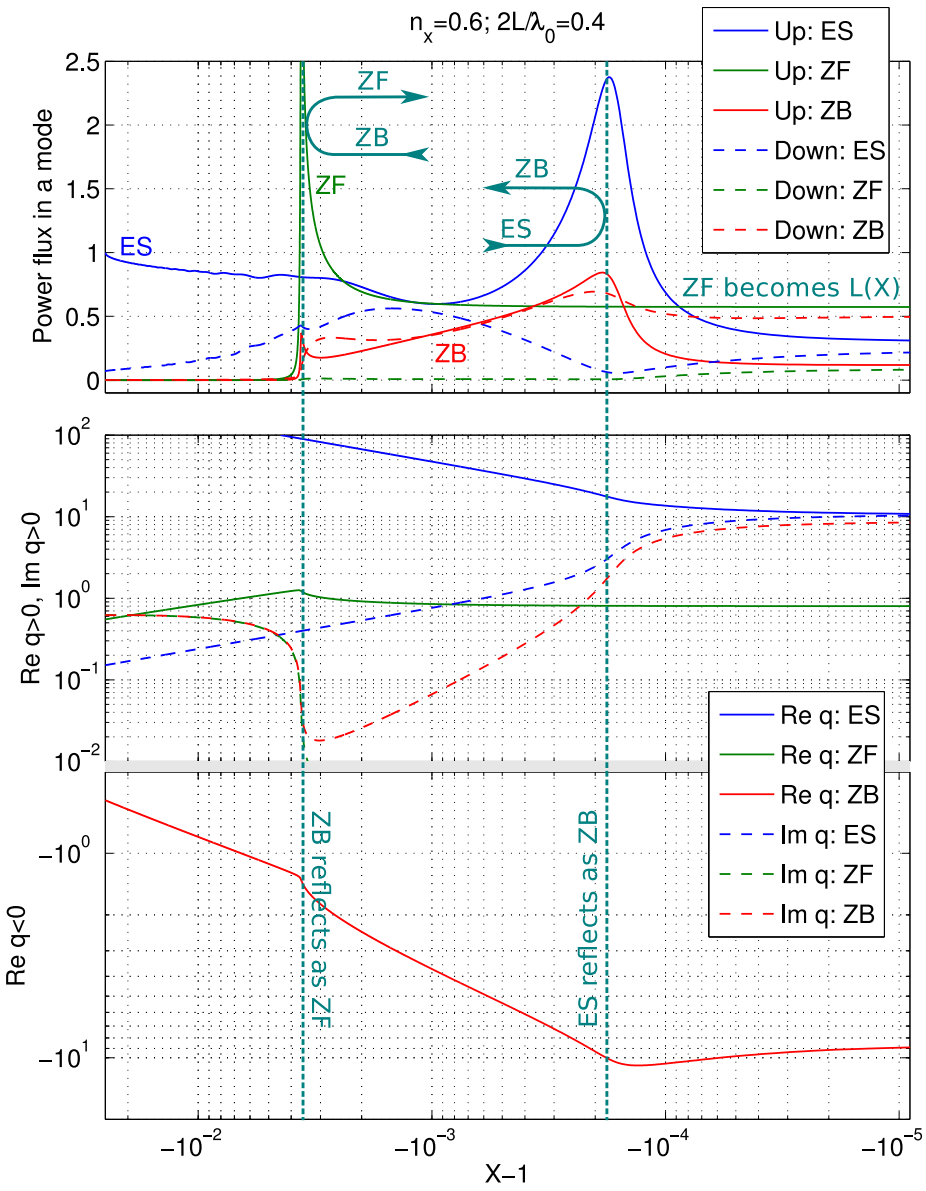
Double internal reflection



- Total internal reflection of upward *ES* into downward *ZB* ("backward") mode on the resonance cone (with S_z opposite to $k_z = k_0 q$)
- As *ZB* mode propagates downwards to the lower density region, the resonance cone becomes wider and the mode gets further away from it.
- At some point with $q > 0$, the downward *ZB* coalesces with upward *ZF* ("forward") mode, and converts to it in the total internal reflection process.
- The extraordinary *ZF* mode becomes *LX* mode above the ramp, without being affected by the resonance at $X = 1$.

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- We generalized StanfordFWM to warm plasma
- Results compare well to previous workers'
- There is efficient conversion $ES \leftrightarrow EM$ due to total internal reflection mechanism