SA53B-1379: Non-Maxwellian model of ionospheric heating by HF radiation

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Outline

- Introduction
- Kinetic equation
- Comparison with Maxwellian model
- Self-consistent HF propagation
- DC conductivity
- Electrojet current modulation
- ELF/VLF wave generation
HF heating of ionosphere and ELF/VLF radiation
HF heating facilities

Effective Radiated Power (ERP) dBW vs. Frequency (MHz)

- HAARP Feb 2007
- TROMSØ, Norway
- SURA, Russia
- HAARP November 2003
- HIPAS, Alaska
- ARECIBO, Puerto Rico
Dynamic friction force

\[ F = \sum N_i \sigma_i(v) \Delta \varepsilon_i \]

\[ \frac{E}{N} = \sum \frac{N_i}{N} \sigma_i(v) \frac{\Delta \varepsilon_i}{e} \]

Inelastic processes:
- Rotational
- Vibrational
- Electronic level excitations
- Dissociative losses
- Ionization

\[ (E/N)_{br} = 130 \text{ Td where } 1 \text{ Td} = 10^{-21} \text{ V-m}^2 \]
Boltzmann equation in spherical harmonic expansion

\[
\frac{\partial n}{\partial t} = \frac{\partial}{\partial \epsilon} \left( D + \frac{2m}{M} v_m T \right) \epsilon^{3/2} \frac{\partial}{\partial \epsilon} \left( \frac{n}{\sqrt{\epsilon}} \right) + \frac{2m}{M} v_m \epsilon n + \left( \frac{\partial n}{\partial t} \right)_{\text{inel}} + \left( \frac{\partial n}{\partial t} \right)_{e-e}
\]

\[n(\epsilon) = C \sqrt{\epsilon} f_0(\epsilon)\]

\[D = \frac{2e^2 E_{\text{RMS}}^2}{3m} \frac{v_m}{(\omega + \omega_H)^2 + v_m^2}, \quad E_{\text{RMS}} = \left| \bar{E} \right| / \sqrt{2}\]

\[\left( \frac{\partial n}{\partial t} \right)_{\text{inel}} = \sum_s N_s \left\{ v(\epsilon + \epsilon_s) \sigma(\epsilon + \epsilon_s)n(\epsilon + \epsilon_s) - v(\epsilon)\sigma(\epsilon)n(\epsilon) \right\}\]

\[\bar{f}_1 = \frac{e}{m} \frac{\bar{E}}{v_m - i(\omega \pm \omega_H)} \frac{1}{\partial \nu} \frac{\partial f_0}{\partial \epsilon}\]

- ordinary; - extraordinary
- includes inelastic collisions
- By inspection of D we see that the effect of E is reduced at high frequencies
Calculated electron distributions

Electron distributions for various RMS E/N (in Td). \( f>0 \) corresponds to extraordinary wave \( (f_H=1 \text{ MHz, } h=91 \text{ km}) \)

Effective electric field is smaller than in DC case:

\[
E_{\text{eff}} = \sqrt{\frac{E}{1 + \left( \frac{\omega_{\text{eff}}}{v_{m,\text{eff}}} \right)^2}}
\]

\[
v_{m,\text{eff}} / N = 2 \times 10^{-13} s^{-1} m^3
\]

\[
\omega_{\text{eff}} = \omega \pm \omega_H
\]

+ ordinary
- extraordinary
Maxwellian vs non-Maxwellian

- Steady-state heating by HF wave produces different effective temperatures for different electron distributions.
- HF and DC conductivity are both dependent on electron distribution (shown DC conductivity).
Self-consistent HF wave propagation

- Power flux (1D), including losses:

\[
\frac{dS}{dz} = -\left(2 \frac{\omega}{c} \text{Im} n_r + \frac{2}{R}\right)S(z,t) \quad n_r(z,t) = \sqrt{1 + \frac{i\sigma_{HF}(z,t)}{\omega\varepsilon_0}}
\]

- HF conductivity (ordinary/extraordinary)

\[
\sigma_{HF\{o,x\}} = -\frac{2e^2}{3m} \int \frac{\varepsilon^{3/2}}{v_m - i(\omega \pm \omega_H)} \frac{\partial}{\partial \varepsilon} \left(\frac{n(\varepsilon)}{\varepsilon^{1/2}}\right) d\varepsilon
\]

- \(n(\varepsilon)\) is calculated from Boltzmann equation, with rms E field

\[
E = \sqrt{Z_0 S / \text{Re}(n_r)}
\]
Time scales of various processes

- **Black**: energy losses to molecules and ions
- **Blue**: electron-electron collisions (maxwellize the distribution, but do not change Teff)
- **Red**: E-field (example for upgraded HAARP)
Dynamics of self-absorption

- 1 kHz square-modulated wave
- Electric field is not constant during the heating half-cycle
Steady-state modulated E field (ratio to breakdown field $E_{br}$)

- The peak $E/E_{br}$ in steady-state can be several times lower than initially (at the same HAARP power) due to established self-absorption.
For the generation of $\Delta J$, the difference between “heated” and “cooled” states is important.
DC Conductivity tensor

- Conductivity changes due to modification of electron distribution
  - Pedersen (transverse)
    \[
    \sigma_p = -\frac{2e^2}{3m} \int \frac{\nu_m \varepsilon^{3/2}}{\omega_H^2 + \nu_m^2} \frac{\partial}{\partial \varepsilon} \varepsilon^{1/2} \frac{n}{\varepsilon} \, d\varepsilon \approx \frac{N_e e^2}{m} \left( \frac{\nu_m}{\omega_H^2 + \nu_m^2} \right)
    \]
  - Hall (off-diagonal)
    \[
    \sigma_h = -\frac{2e^2}{3m} \int \frac{\omega_H \varepsilon^{3/2}}{\omega_H^2 + \nu_m^2} \frac{\partial}{\partial \varepsilon} \varepsilon^{1/2} \frac{n}{\varepsilon} \, d\varepsilon \approx \frac{N_e e^2}{m} \left( \frac{\omega_H}{\omega_H^2 + \nu_m^2} \right)
    \]
  - Parallel
    \[
    \sigma_z = -\frac{2e^2}{3m} \int \frac{\varepsilon^{3/2}}{\nu_m} \frac{\partial}{\partial \varepsilon} \varepsilon^{1/2} \frac{n}{\varepsilon} \, d\varepsilon \approx \frac{N_e e^2}{m} \left( \frac{1}{\nu_m} \right)
    \]
Steady heating conductivity modification

- Pedersen conductivity is increased
- Parallel conductivity is decreased
Time dynamics of conductivity changes

- Time scales to change the steady-state determined by energy exchange between electrons and neutrals (ions)
Electrojet current modulation calculations

- We assume static current, i.e.
  \[ \vec{J} = -\bar{\sigma} \nabla \phi \]
  \[ \nabla \cdot \vec{J} = 0 \]

- This is justified because the conductivity time scale \( \sigma/\varepsilon_0 \gg f \)
- Vertical B (z axis)
- Ambient E = 25 mV/m is along x axis
- 3D calculations
Conductivity time scales
(for justification of steady state)
3D stationary $\Delta J$: vertical profile

First harmonic current amplitude

- Pedersen current
- Hall current

$\Delta J$, nA/m²

$h$, km

$\text{mod} = 3 \text{ kHz}$
$\text{mod} = 2 \text{ kHz}$
$\text{mod} = 1 \text{ kHz}$
3D stationary $\Delta J$: horizontal slices

- Vertical B (z axis)
- Ambient $E = 25 \text{ mV/m}$ is along x axis

Hall current (80 km)  
Pedersen current (90 km)
3D stationary $\Delta J$: vertical slices

- Closing vertical currents

Hall currents

Pedersen currents
ELF/VLF radiation: application of mode theory

- $E_x$, $E_y$ are continuous at each boundary between horizontal layers
- Change in $H_x$, $H_y$ is proportional to horizontal current
- Boundary conditions: $E_{x,y} = 0$ at $h=0$, radiation at $h_{\text{max}}$
- The source current is represented as a linear combination of its horizontal Fourier components
- Current limitation: no $J_z$, but it is located higher in ionosphere and should not affect at least the Earth-ionosphere waveguide results
f=1 kHz radiation into Earth-ionosphere waveguide

- Only QTEM mode => no nulls
f=3 kHz radiation into Earth-ionosphere waveguide

- QTEM mode ($\lambda=105$ km) and QTE1 mode ($\lambda=120$ km) => nulls
Radiation of the whistler mode into ionosphere: energy flux

$f_{mod} = 1$ kHz:
Total = 55.3692 W

$f_{mod} = 3$ kHz:
Total = 5.4351 W