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Emission of ELF/VLF Waves by a Modulated Electrojet upwards into the Ionosphere and into the Earth-Ionosphere Waveguide

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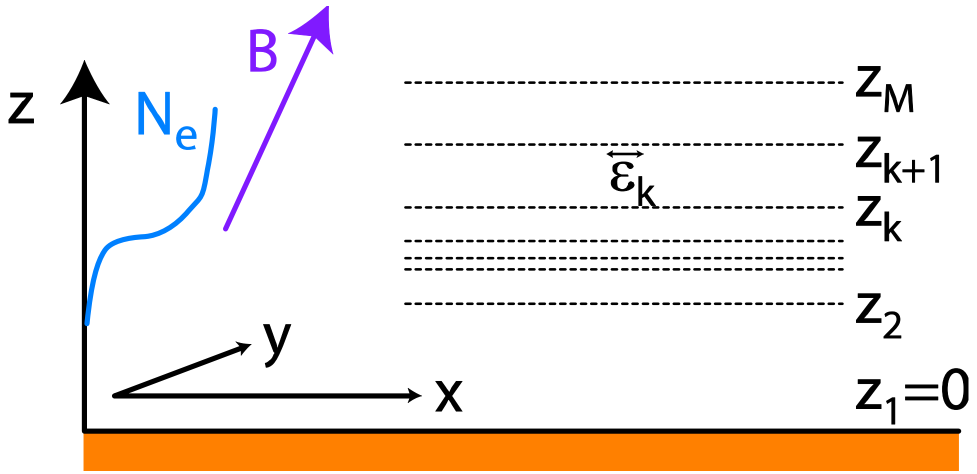
Abstract

The previously calculated modulation of the auroral electrojet by a ground-based ionosphere heating facility is used to calculate the emission of ELF/VLF waves both into ionosphere and into the Earth-ionosphere waveguide. For this purpose, we developed a finite element method of calculation of electromagnetic field in a horizontally-stratified ionosphere filled with magnetized plasma, with arbitrary harmonically-varying current distribution and the direction of geomagnetic field. This method is proven to be stable against the loss of precision due to "swamping" of useful modes by evanescent waves. The electromagnetic field is calculated both in the Earth-Ionosphere waveguide (at arbitrary horizontal distance and direction) and in the ionosphere (as a whistler mode). The presented method requires less computational resources than traditional FDFD and FDTD methods. The calculated values are compared to ground and satellite observations of electrojet emissions from modulation by existing ionosphere heating facilities. In particular, we find that the emission of the whistler waves upward into the ionosphere is contained in a relatively narrow channel, even in the absence of ducting, due to a substantial horizontal size of the emitting region.

VLF/ELF emission and propagation model

- Uses mode theory to solve Maxwell's equations in stratified medium, implemented in MATLAB
- Capabilities:
  - Full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
  - Magnetized plasma with arbitrary direction of geomagnetic field
  - Arbitrary configuration of harmonically varying currents
  - Both ionosphere and Earth-ionosphere waveguide
  - Stable against the "swamping" instability by evanescent waves
- Runs much faster than FDFD and FDTD models:
  - Cell size can be larger than the wavelength
  - Vertical cell size can be variable
  - Can be extended to satellite altitudes

Method description



1. Find k_z, E and H for all modes in each layer for each fixed horizontal wave vector $k_\perp$
2. Use continuity of $E_\perp$ and $H_\perp$ between layers to recursively find reflection coefficients and mode amplitudes, using Nygren's [1982] order of recursion
3. Represent source currents as thin sheets and introduce new boundary conditions at these sheets to find fields due to sources
4. Inverse Fourier transform from $k_\perp$ to $r_\perp$

1. Find the modes

- **Solve uniform Maxwell's equations for n_z, E, H (for ~$e^{-i\omega t}$)**

$$\mathbf{n}\times\mathbf{H} = -\overleftrightarrow{\varepsilon}\mathbf{E}$$

$$\mathbf{n}\times\mathbf{E} = \mathbf{H}$$

$$\mathbf{H} = Z_0\mathbf{H}_{SI},\, \mathbf{n} = \mathbf{k}/k_0,\, k_0 = \omega / c$$

- **u, d are downward and upward propagating (or evanescent) modes, Im(n_z) > 0**

$$\mathbf{u} = \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix}, \mathbf{d} = \begin{pmatrix} d^{(1)} \\ d^{(2)} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{pmatrix} = \mathsf{F}_k \begin{pmatrix} u_k^{(1)} e^{ik_0 n_z^{(u1)}(z-z_k)} \\ u_k^{(2)} e^{ik_0 n_z^{(u2)}(z-z_k)} \\ d_k^{(1)} e^{-ik_0 n_z^{(d1)}(z-z_k)} \\ d_k^{(1)} e^{-ik_0 n_z^{(d1)}(z-z_k)} \end{pmatrix} e^{ik_0 (\mathbf{n}_\perp \cdot \mathbf{r}_\perp)}$$

2. Find refraction coefficients and mode amplitudes

- **Above sources $d=R^u u$**
- **Below sources $u=R^d d$**
- **Use continuity of $E_\perp$ and $H_\perp$ between layers to find recursively**

  $R^u_{k+1} \to R^u_k$ and $R^d_k \to R^d_{k+1}$ with $R^u_M=0$ and $R^d_1=-I$

  $u_k \to u_{k+1}$ and $d_{k+1} \to d_k$ with initial values specified by sources

This order of recursion provides stability against “swamping” by evanescent waves and was suggested by Nygren [Planet. Space Sci., 30(4), p. 427, 1982]

3. Include sources

- **Boundary conditions for a sheet current J=Iδ(z):**

$$\Delta \mathbf{E}_{\perp} = \mathbf{n}_{\perp} \frac{Z_0 I_z}{\varepsilon_{zz}}$$

$$\Delta \mathbf{H}_{\perp} = \mathbf{I}_{\perp} \times \hat{z} - \frac{Z_0 \left(\vec{\varepsilon}_{\perp z} \times \hat{z}\right) I_z}{\varepsilon_{zz}}$$

- **Find Δu, Δd using the mode structure matrix F**
- **Find u immediately above the sheet and d below the sheet using known reflection coefficients, d+Δd=R^{u}u, u-Δu=R^{d}d**

4. "Reassemble" Fourier components

- **Obtain E, H from u,d**

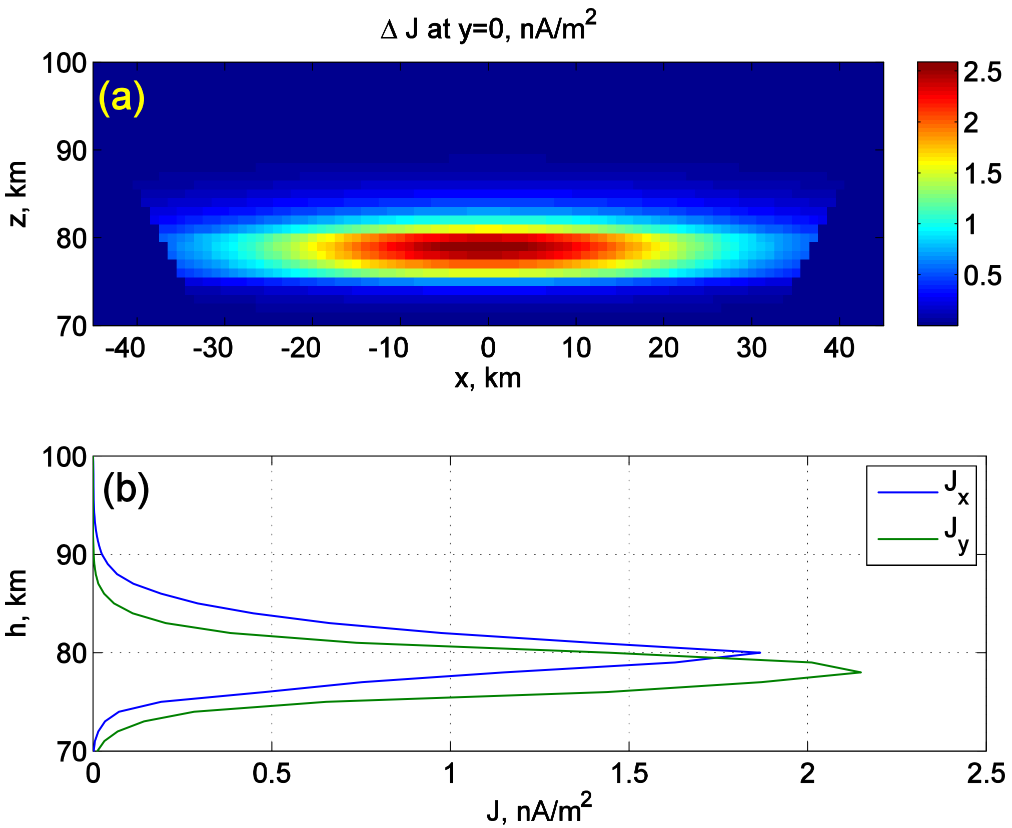
$$\begin{pmatrix} \mathbf{E}(\mathbf{n}_\perp) \\ \mathbf{H}(\mathbf{n}_\perp) \end{pmatrix} = \mathsf{F}_k(\mathbf{n}_\perp) \begin{pmatrix} \mathbf{u}(\mathbf{n}_\perp) \\ \mathbf{d}(\mathbf{n}_\perp) \end{pmatrix} \text{ in layer } k$$

- **Inverse Fourier transform from $\mathbf{k}_\perp$ to $\mathbf{r}_\perp$**

$$\{\mathbf{E},\mathbf{H}\}(\mathbf{r}_\perp) = \iint \{\mathbf{E},\mathbf{H}\}(k_0\mathbf{n}_\perp) e^{ik_0(\mathbf{n}_\perp \cdot \mathbf{r}_\perp)} \frac{k_0^2 d^2\mathbf{n}_\perp}{(2\pi)^2}, \quad \text{where } \mathbf{k}_\perp = k_0\mathbf{n}_\perp$$

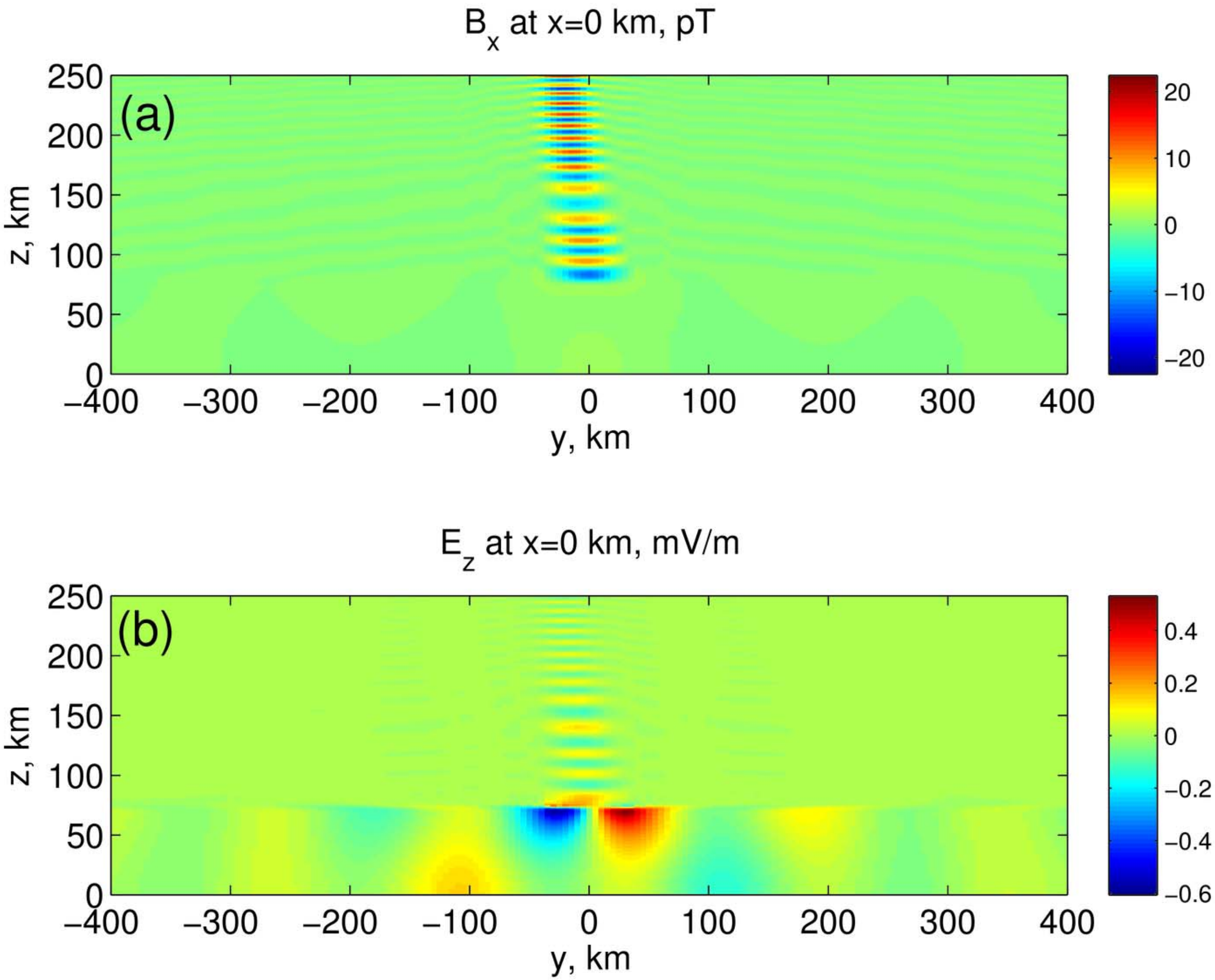
Application: Emission of ELF waves by HF-heated modulated electrojet

- Modulation frequency=1875 Hz



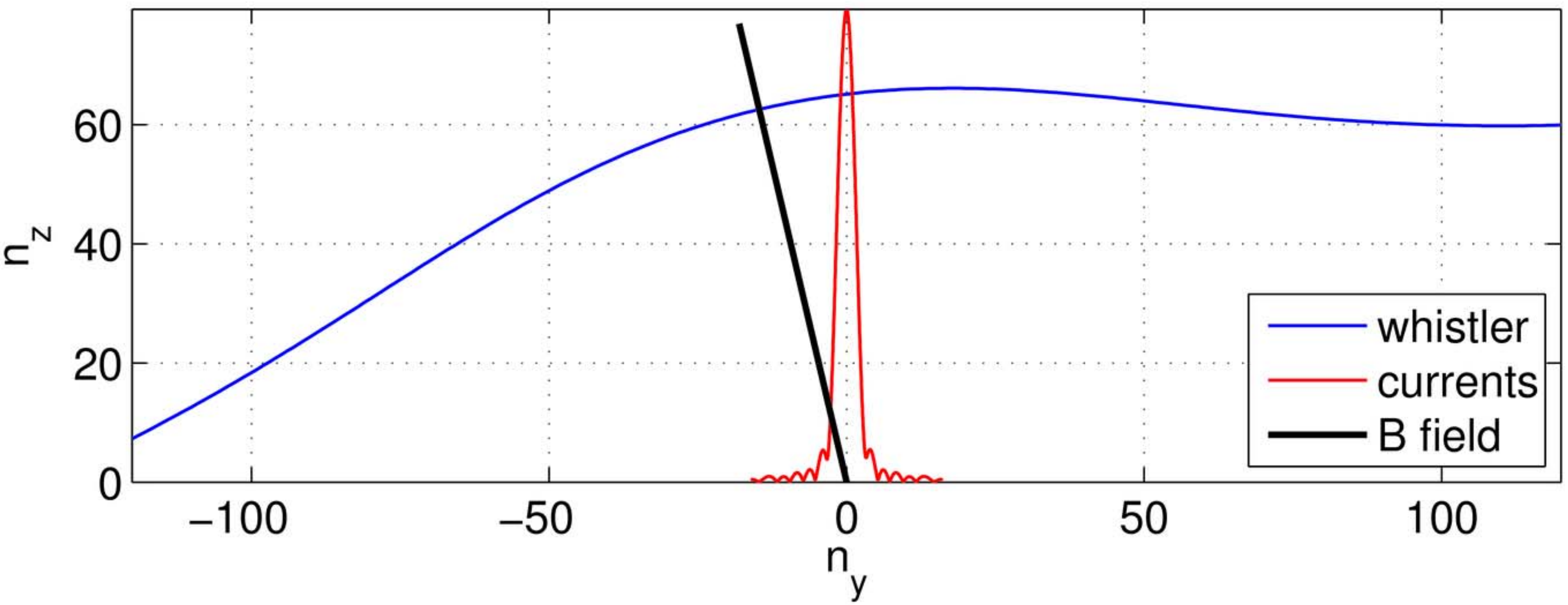
- Emission is caused by the change in electrojet current caused by HF heating
- The current structure was calculated by Payne et al [GRL, in review]. It has a horizontal pancake shape
- We use our method to find E and B fields

Collimated whistler beam
Wave in Earth-ionosphere waveguide

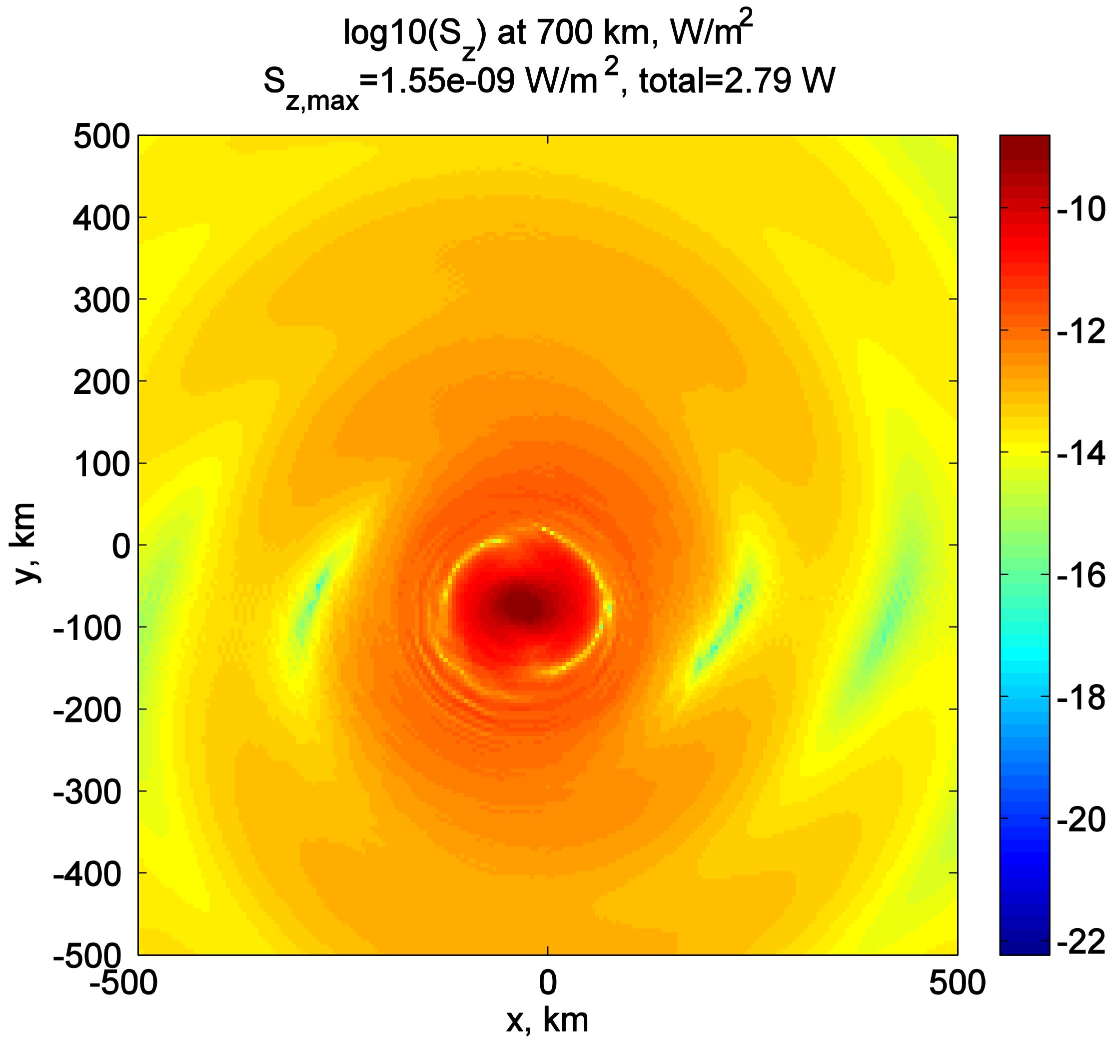


Why is the beam collimated?

- **The beam changes its shape, but insignificantly due to a large size of the emitting region (small perpendicular k-vector)**

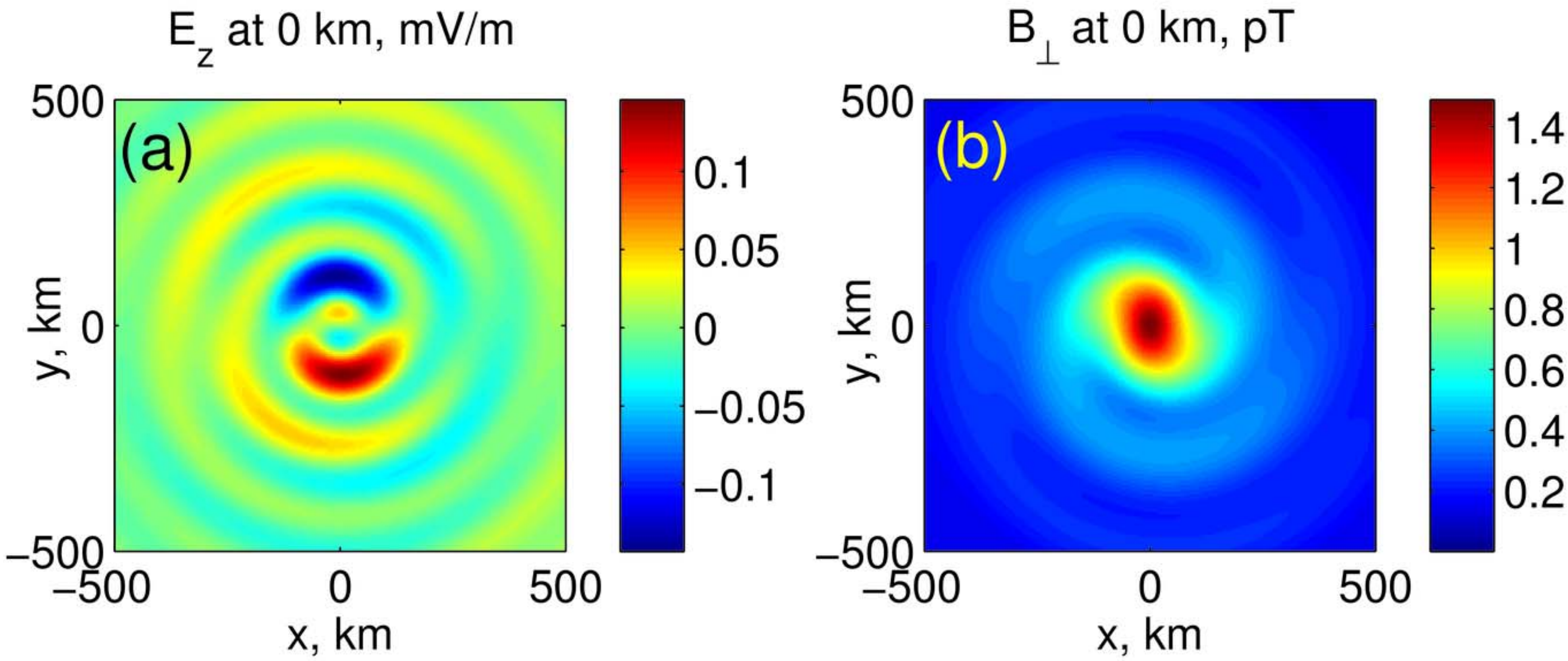


Poynting vector at satellite altitude (h=700 km)



Field on the ground

- **The observed field on the ground is due to both near field and propagating Earth-ionosphere waveguide modes.**



Summary

- We describe a stable method of calculations of fields in stratified media
- ELF emission by the HF-heated spot creates a collimated upward whistler beam
- Calculated emitted energy is ~3 W into magnetosphere, ~1 W into Earth-ionosphere waveguide