Comparison of non-Maxwellian and Maxwellian models of ionospheric heating by HF radiation

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Outline

- Introduction
- Kinetic equation
- Comparison with Maxwellian model
- Self-consistent HF propagation
- DC conductivity
- Electrojet current modulation
- ELF/VLF wave generation
HF heating of ionosphere and ELF/VLF radiation

120 km
70 km

modulated electrojet current ($\Delta J$)
3 MHz HF emission modulated at ELF/VLF

modulated heating ($\Delta \sigma$)

Earth-ionosphere waveguide

injected VLF wave

heating facility

geomagnetically conjugate point
Dynamic friction force

\[ F = \sum N_i \sigma_i(v) \Delta \varepsilon_i \]

\[ \frac{E}{N} = \sum \frac{N_i}{N} \sigma_i(v) \frac{\Delta \varepsilon_i}{e} \]

Inelastic processes:
- Rotational
- Vibrational
- Electronic level excitations
- Dissociative losses
- Ionization

\[(E/N)_{br} = 130 \text{ Td where } 1 \text{ Td} = 10^{-21} \text{ V-m}^2\]
Boltzmann equation in spherical harmonic expansion

\[
\frac{\partial n}{\partial t} = \frac{\partial}{\partial \varepsilon} \left( (D + \nu_e T) \varepsilon^{3/2} \frac{\partial}{\partial \varepsilon} \left( \frac{n}{\sqrt{\varepsilon}} + \nu_e \varepsilon n \right) \right) + \left( \frac{\partial n}{\partial t} \right)_{\text{inel}} + \left( \frac{\partial n}{\partial t} \right)_{\text{e-e}}
\]

\[
n(\varepsilon) = C \sqrt{\varepsilon} f_0(\varepsilon)
\]

\[
D = \frac{2e^2 E_{RMS}^2}{3m} \frac{\nu_m}{(\omega + \omega_H)^2 + \nu_m^2}, \quad E_{RMS} = \frac{|E|}{\sqrt{2}}
\]

\[
\left( \frac{\partial n}{\partial t} \right)_{\text{inel}} = \sum_s N_s \left\{ \nu(\varepsilon + \varepsilon_s) \sigma(\varepsilon + \varepsilon_s) n(\varepsilon + \varepsilon_s) - \nu(\varepsilon) \sigma(\varepsilon) n(\varepsilon) \right\}
\]

\[
\overline{f_1} = \frac{e}{m} \overline{E} \frac{1}{\nu_m - i(\omega + \omega_H)} \frac{\partial f_0}{\partial \nu}
\]

- ordinary; - extraordinary
- includes inelastic collisions
- By inspection of D we see that the effect of E is reduced at high frequencies
Calculated electron distributions

Electron distributions for various RMS E/N (in Td). f>0 corresponds to extraordinary wave (f_H=1 MHz, h=91 km)

- Effective electric field is smaller than in DC case:

\[ E_{\text{eff}} = \frac{E}{\sqrt{1 + \left( \frac{\omega_{\text{eff}}}{v_{m,\text{eff}}} \right)^2}} \]

\[ \nu_{m,\text{eff}} / N = 2 \times 10^{-13} \text{ s}^{-1} \text{ m}^3 \]

\[ \omega_{\text{eff}} = \omega \pm \omega_H \]

+ ordinary
- extraordinary
Self-consistent HF wave propagation

- Power flux (1D), including losses:

\[
\frac{dS}{dz} = -\left(2\frac{\omega}{c} \mathrm{Im} n_r + \frac{2}{R}\right) S(z, t) \quad n_r(z, t) = \sqrt{1 + \frac{i\sigma_{\text{HF}}(z, t)}{\omega\varepsilon_0}}
\]

- HF conductivity (ordinary/extraordinary)

\[
\sigma_{\text{HF}\{o,x\}} = -\frac{2e^2}{3m} \int \frac{\varepsilon^{3/2}}{\nu_m - i(\omega \pm \omega_H)} \frac{\partial}{\partial \varepsilon} \left(\frac{n(\varepsilon)}{\varepsilon^{1/2}}\right) d\varepsilon
\]

- \(n(\varepsilon)\) is calculated from Boltzmann equation, with rms E field

\[
E = \sqrt{Z_0 S / \text{Re}(n_r)}
\]
Electric field at current HAARP level (f=3.1MHz, x-mode)

- Electric field decreases due to self-absorption
Maxwellian model

- Electron distribution has a fixed shape determined by a single parameter $T$:
  \[ n(\varepsilon) = 2\sqrt{\frac{\varepsilon}{\pi T^3}} e^{-\varepsilon/T} \]

- From kinetic equation, obtain time dependence of $\langle \varepsilon \rangle = 3T/2$:
  \[
  \frac{d\langle \varepsilon \rangle}{dt} = -\frac{T-T_0}{T} \langle \nu \varepsilon \rangle - \sum_s N_s \langle \nu \sigma_s \rangle \varepsilon e^{-\varepsilon_s/T_0} \left( e^{\varepsilon_s/T_0} - e^{\varepsilon_s/T} \right) + \frac{|E|^2}{2} \text{Re} \frac{\sigma_{HF}}{N_e}
  \]
**Effective temperature**

- Current HAARP power is high enough for the changes to be both nonlinear and non-Maxwellian (results for f=3.1MHz, x-mode)
Conductivity changes

Dashed lines correspond to Maxwellian distribution

Linear ($\Delta\sigma \sim E^2$)
Electron distribution modified by HAARP, with $E=.5\ \text{V/m}$

- Clearly seen the effect of N2 vibrational barrier
Square-modulated heating

- Absolute change and harmonic contents of conductivity depends on non-thermal effects at current HAARP power level.
Square-modulated heating (2)

- The effect is higher if the power is increased


- **DC Conductivity tensor**

- **Conductivity changes due to modification of electron distribution**

- **Pedersen (transverse)**

\[ \sigma_p = -\frac{2e^2}{3m} \int \frac{\nu_m \varepsilon^{3/2}}{\omega_H^2 + \nu_m^2} \frac{\partial}{\partial \varepsilon} \frac{n}{\varepsilon^{1/2}} d\varepsilon \approx \frac{N_e e^2}{m} \left\langle \frac{\nu_m}{\omega_H^2 + \nu_m^2} \right\rangle \]

- **Hall (off-diagonal)**

\[ \sigma_h = -\frac{2e^2}{3m} \int \frac{\omega_H \varepsilon^{3/2}}{\omega_H^2 + \nu_m^2} \frac{\partial}{\partial \varepsilon} \frac{n}{\varepsilon^{1/2}} d\varepsilon \approx \frac{N_e e^2}{m} \left\langle \frac{\omega_H}{\omega_H^2 + \nu_m^2} \right\rangle \]

- **Parallel**

\[ \sigma_z = -\frac{2e^2}{3m} \int \frac{\varepsilon^{3/2}}{\nu_m} \frac{\partial}{\partial \varepsilon} \frac{n}{\varepsilon^{1/2}} d\varepsilon \approx \frac{N_e e^2}{m} \left\langle \frac{1}{\nu_m} \right\rangle \]
Electrojet current modulation calculations

- We assume static current, i.e.

\[ \vec{J} = -\sigma \nabla \varphi \]

\[ \nabla \cdot \vec{J} = 0 \]

- This is justified because the conductivity time scale \( \sigma/\varepsilon_0 >> f \)
- Vertical B (z axis)
- Ambient E = 25 mV/m is along x axis
- 3D calculations
Conductivity time scales
(for justification of steady state)
3D stationary $\Delta J$: vertical profile

First harmonic current amplitude

- Pedersen current
- Hall current
3D stationary $\Delta J$: horizontal slices

- Vertical $B$ (z axis)
- Ambient $E = 25$ mV/m is along x axis

Hall current (80 km)  
Pedersen current (90 km)
3D stationary $\Delta J$: vertical slices

- Closing vertical currents

**Hall currents**

**Pedersen currents**
ELF/VLF radiation: application of mode theory

- $E_x, E_y$ are continuous at each boundary between horizontal layers
- Change in $H_x, H_y$ is proportional to horizontal current
- Boundary conditions: $E_{x,y} = 0$ at $h=0$, radiation at $h_{\text{max}}$
- The source current is represented as a linear combination of its horizontal Fourier components
- Current limitation: no $J_z$, but it is located higher in ionosphere and should not affect at least the Earth-ionosphere waveguide results
f=1 kHz radiation into Earth-ionosphere waveguide

- Only QTEM mode => no nulls
f=3 kHz radiation into Earth-ionosphere waveguide

- QTEM mode ($\lambda=105$ km) and QTE1 mode ($\lambda=120$ km) => nulls
Radiation of the whistler mode into ionosphere: energy flux

\( f_{\text{mod}} = 1 \text{ kHz:} \)

Total = 55.3692 W

\( f_{\text{mod}} = 3 \text{ kHz:} \)

Total = 5.4351 W