Comparison of non-Maxwellian and Maxwellian models of ionospheric heating by HF radiation

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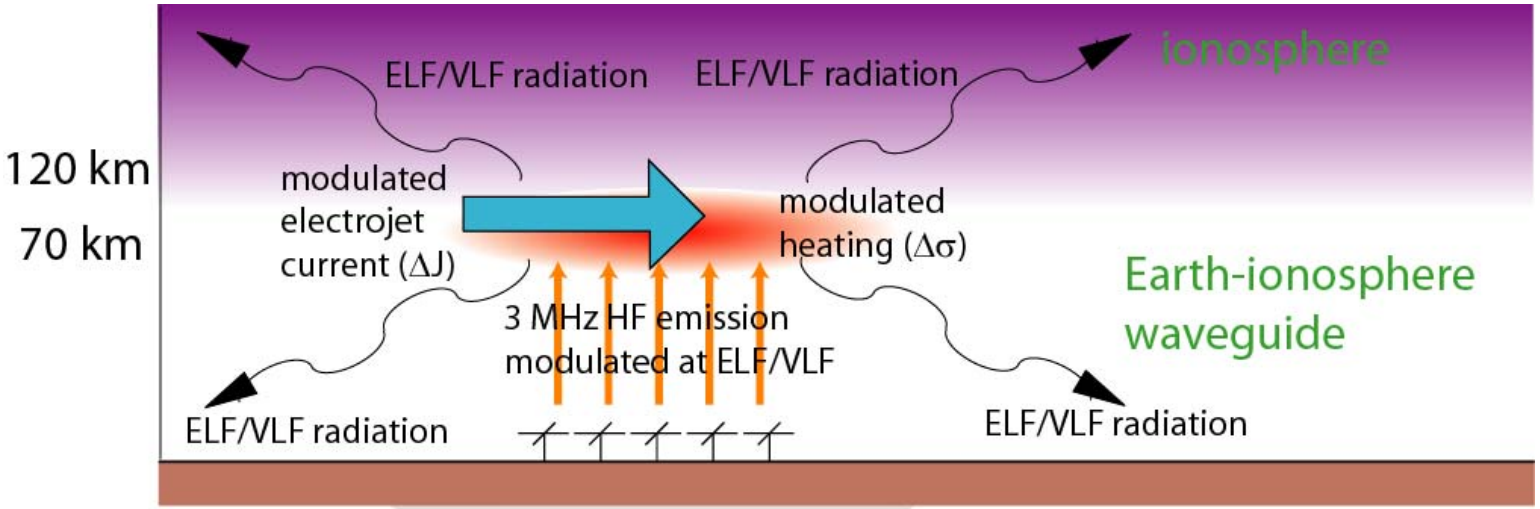
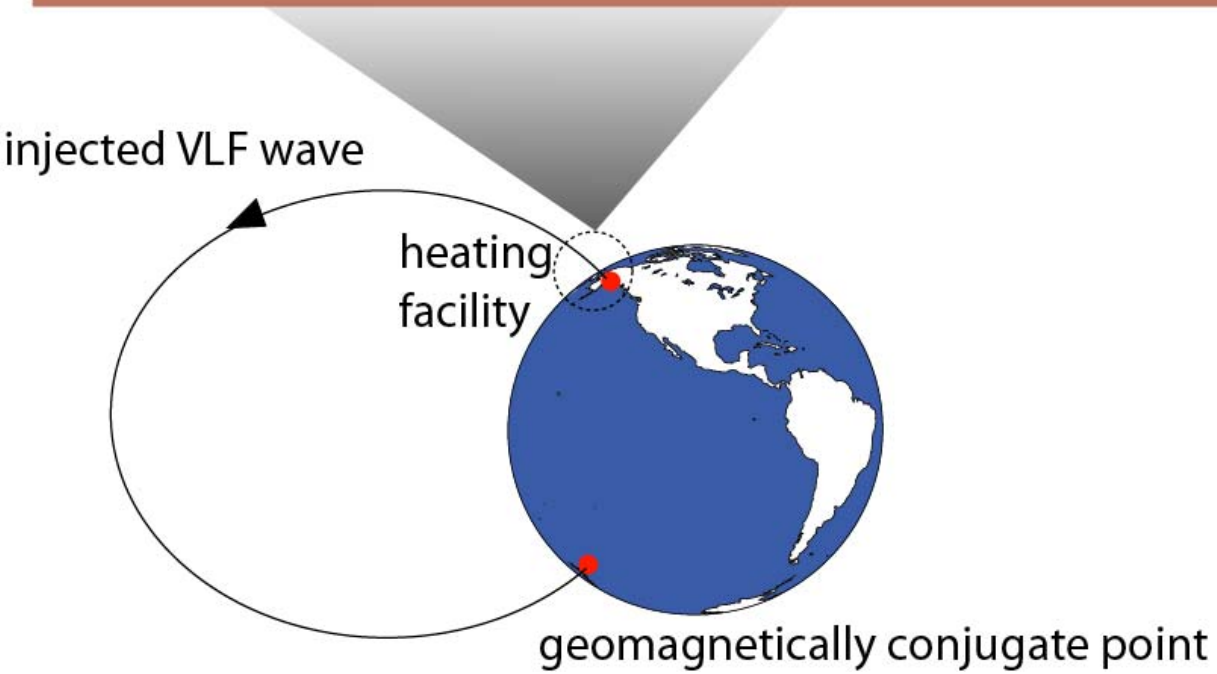
Outline

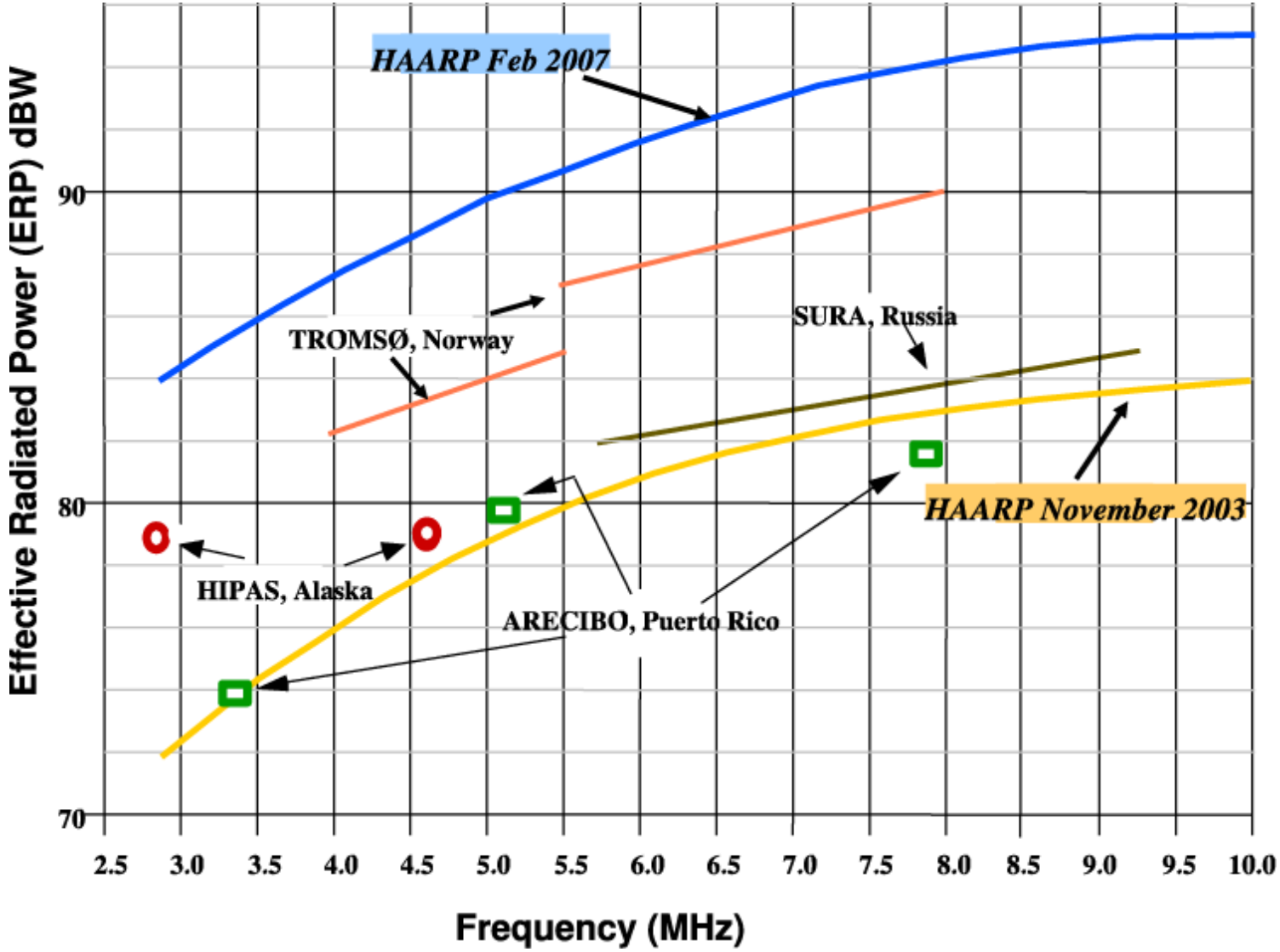
- Introduction
- Kinetic equation
- Comparison with Maxwellian model
- Self-consistent HF propagation
- DC conductivity
- Electrojet current modulation
- ELF/VLF wave generation

HF heating of ionosphere and ELF/VLF radiation

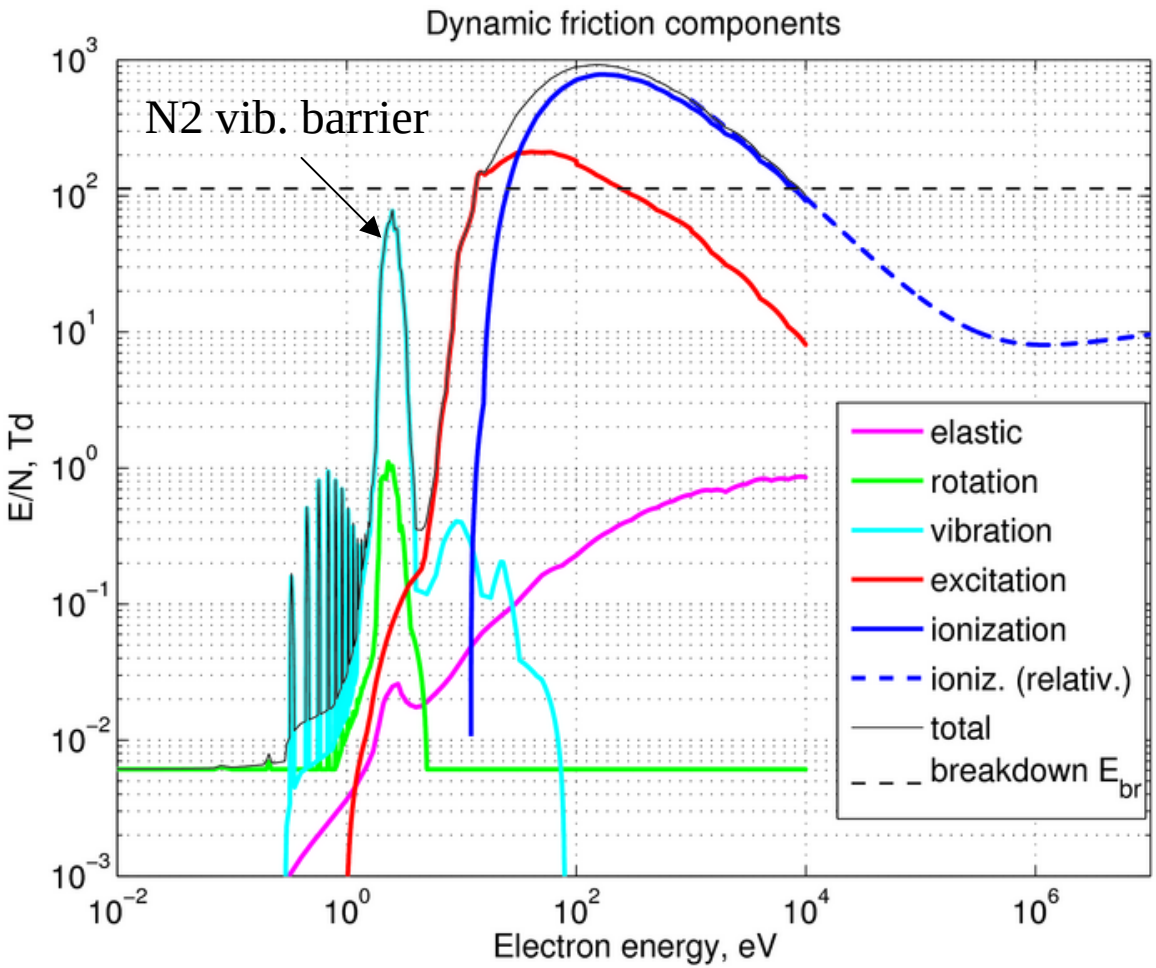




HF heating facilities



Dynamic friction force



$$F = \sum N_i \sigma_i(v) \Delta\varepsilon_i$$

$$\frac{E}{N} = \sum \frac{N_i}{N} \sigma_i(v) \frac{\Delta\varepsilon_i}{e}$$

Inelastic processes:

- **Rotational**
- **Vibrational**
- **Electronic level excitations**
- **Dissociative losses**
- **Ionization**

$(E/N)_{br}$=130 Td where 1 Td = 10^{-21} V-m^2

Boltzmann equation in spherical harmonic expansion

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial \varepsilon}\left(\left(D + \nu_\varepsilon T\right)\varepsilon^{3/2} \frac{\partial}{\partial \varepsilon}\left(\frac{n}{\sqrt{\varepsilon}}\right) + \nu_\varepsilon \varepsilon n \right) + \left(\frac{\partial n}{\partial t}\right)_{inel} + \left(\frac{\partial n}{\partial t}\right)_{e-e}$$

$$n(\varepsilon) = C\sqrt{\varepsilon}\, f_0(\varepsilon)$$

$$D = \frac{2e^2 E_{RMS}^2}{3m} \frac{\nu_m}{\left(\omega \pm \omega_H\right)^2 + \nu_m^2}, \quad E_{RMS} = \left|\vec{E}\right| / \sqrt{2}$$

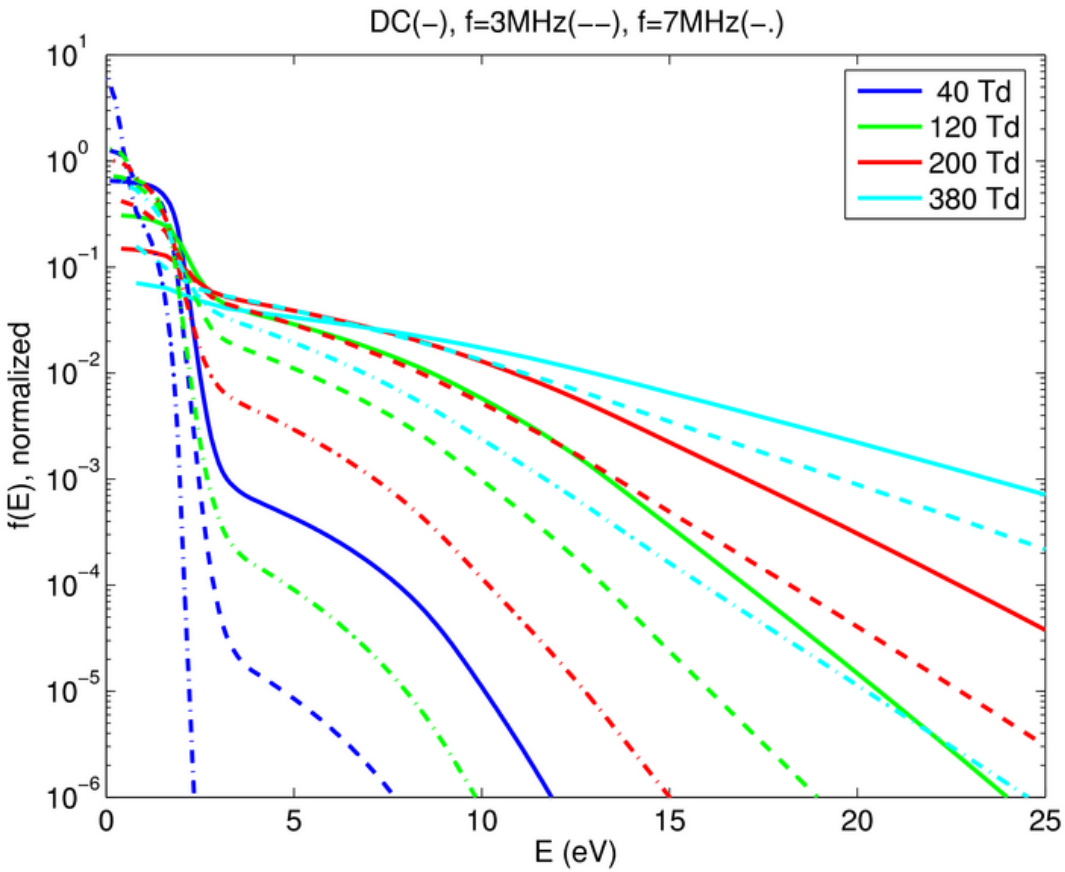
$$\left(\frac{\partial n}{\partial t}\right)_{inel} = \sum_s N_s \left\{ v(\varepsilon + \varepsilon_s)\sigma(\varepsilon + \varepsilon_s) n(\varepsilon + \varepsilon_s) - v(\varepsilon)\sigma(\varepsilon) n(\varepsilon) \right\}$$

$$\vec{f_1} = \frac{e}{m}\vec{E} \frac{1}{\nu_m - i\left(\omega \pm \omega_H\right)} \frac{\partial f_0}{\partial v}$$

- **+ ordinary; - extraordinary**
- **includes inelastic collisions**
- **By inspection of D we see that the effect of E is reduced at high frequencies**

Calculated electron distributions

Electron distributions for various RMS E/N (in Td). f>0 corresponds to extaordinary wave (f_H=1 MHz, h=91 km)



- **Effective electric field is smaller than in DC case:**

$$E_{eff} = \frac{E}{\sqrt{1+\left(\frac{\omega_{eff}}{\nu_{m,eff}}\right)^2}}$$

$$\nu_{m,eff} / N = 2\times10^{-13} s^{-1} m^3$$

$$\omega_{eff} = \omega \pm \omega_H$$

+ ordinary
- extraordinary

Self-consistent HF wave propagation

- **Power flux (1D), including losses:**

$$\frac{dS}{dz} = -\left(2\frac{\omega}{c}\operatorname{Im} n_r + \frac{2}{R}\right)S(z,t) \qquad n_r(z,t) = \sqrt{1 + \frac{i\sigma_{HF}(z,t)}{\omega\varepsilon_0}}$$

- **HF conductivity (ordinary/extaordinary)**

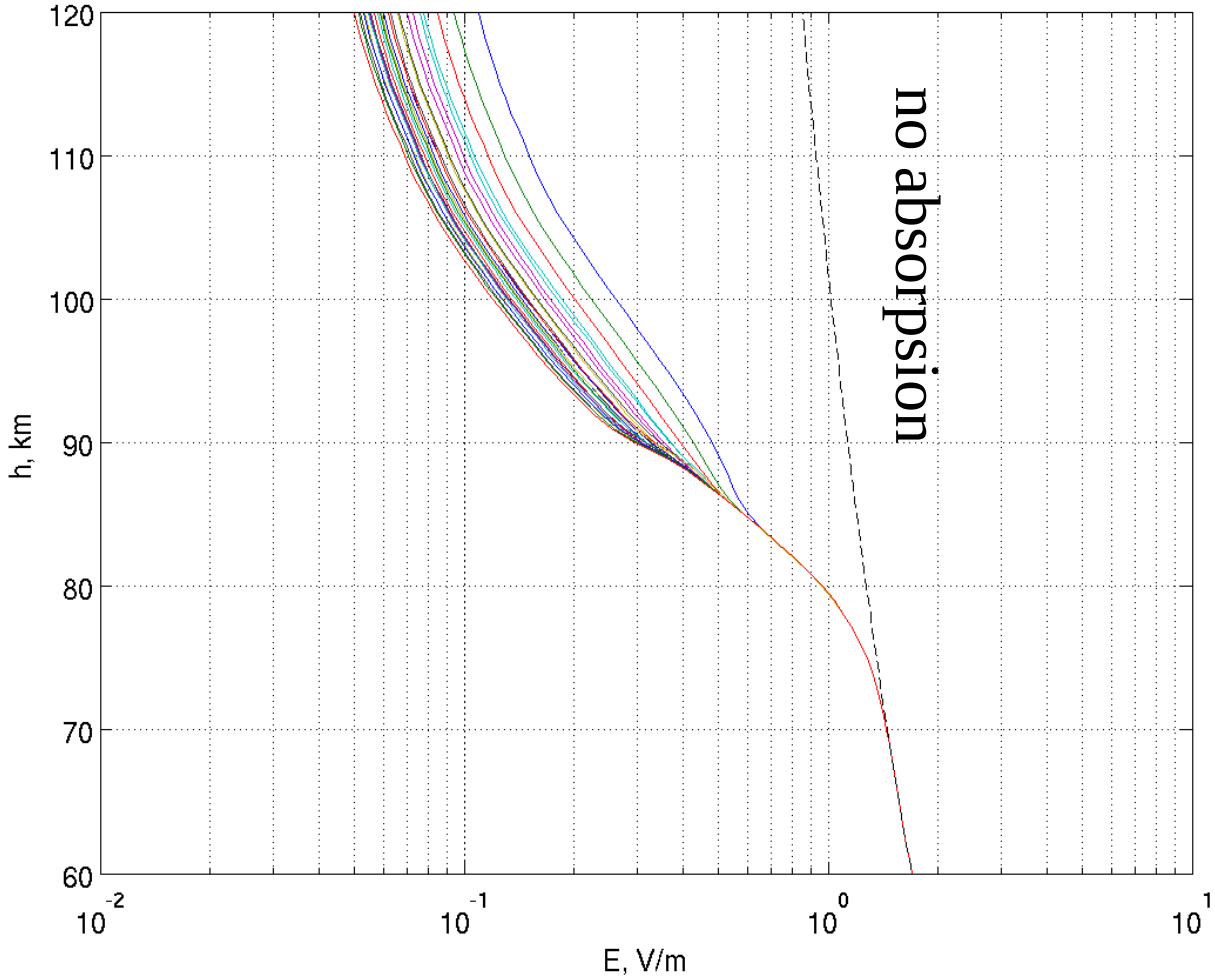
$$\sigma_{HF\{o,x\}} = -\frac{2e^2}{3m}\int \frac{\varepsilon^{3/2}}{\nu_m - i(\omega \pm \omega_H)}\frac{\partial}{\partial\varepsilon}\left(\frac{n(\varepsilon)}{\varepsilon^{1/2}}\right)d\varepsilon$$

- **n(ε) is calculated from Boltzmann equation, with rms E field**

$$E = \sqrt{Z_0 S / \operatorname{Re}(n_r)}$$

Electric field at current HAARP level (f=3.1MHz, x-mode)

- **Electric field decreases due to self-absorption**



Maxwellian model

- **Electron distribution has a fixed shape determined by a single parameter T:**

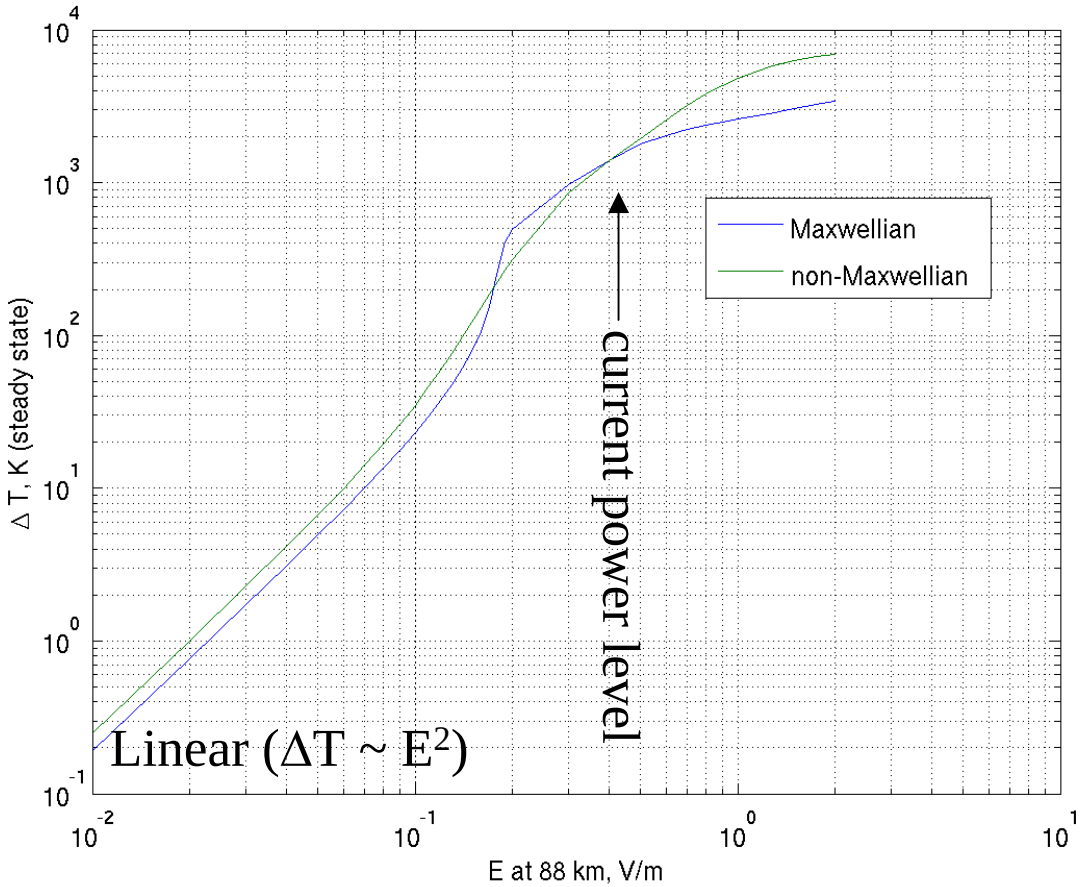
$$n(\varepsilon) = 2\sqrt{\frac{\varepsilon}{\pi T^3}}e^{-\varepsilon/T}$$

- **From kinetic equation, obtain time dependence of <ε>=3T/2:**

$$\frac{d\langle\varepsilon\rangle}{dt} = -\frac{T-T_0}{T}\langle\nu_\varepsilon\varepsilon\rangle - \sum_s N_s\langle v\sigma_s\rangle\varepsilon_s e^{-\varepsilon_s/T_0}\left(e^{\varepsilon_s/T_0} - e^{\varepsilon_s/T}\right) + \frac{|E|^2}{2}\mathrm{Re}\frac{\sigma_{HF}}{N_e}$$

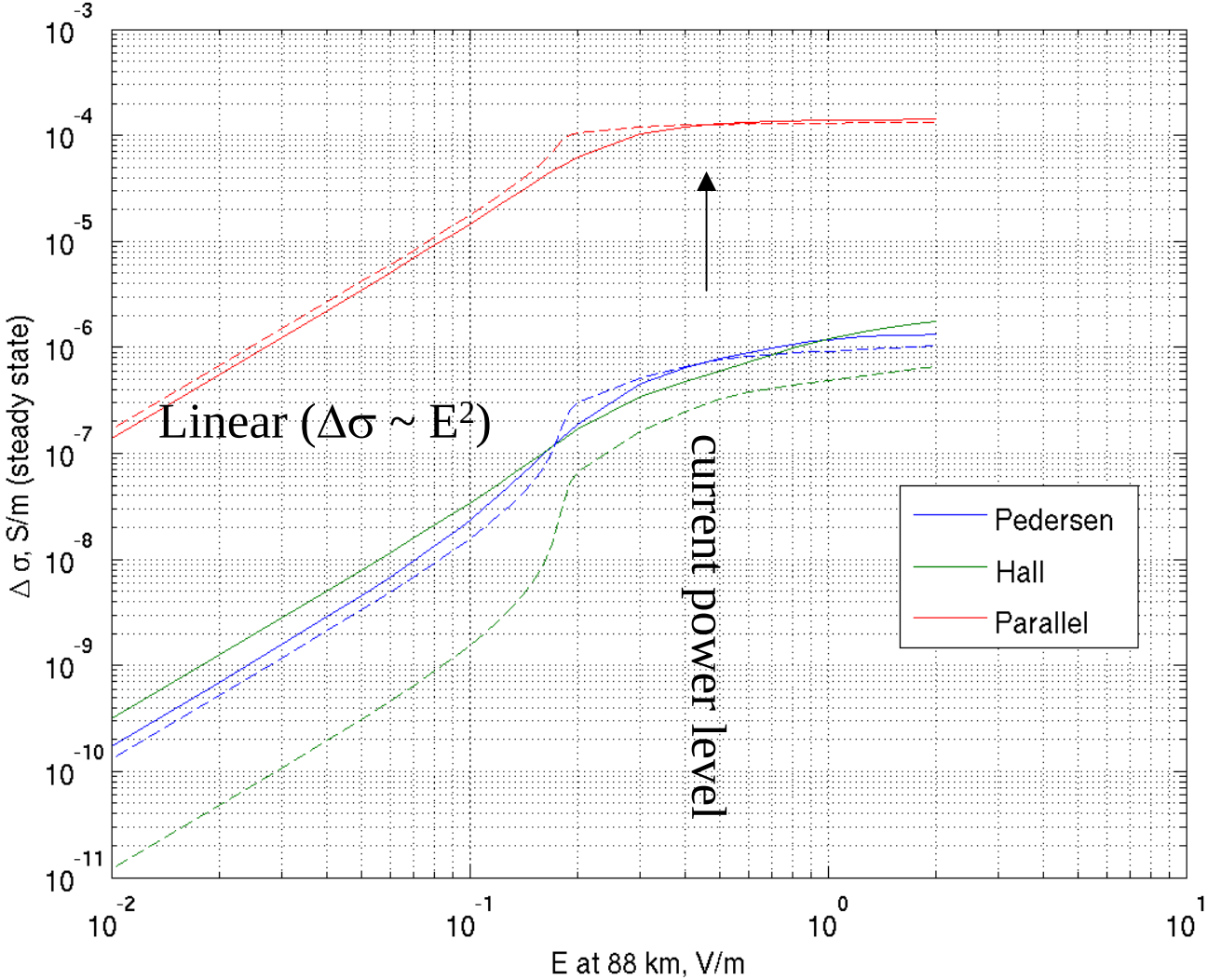
Effective temperature

- **Current HAARP power is high enough for the changes to be both nonlinear and non-Maxwellian (results for f=3.1MHz, x-mode)**



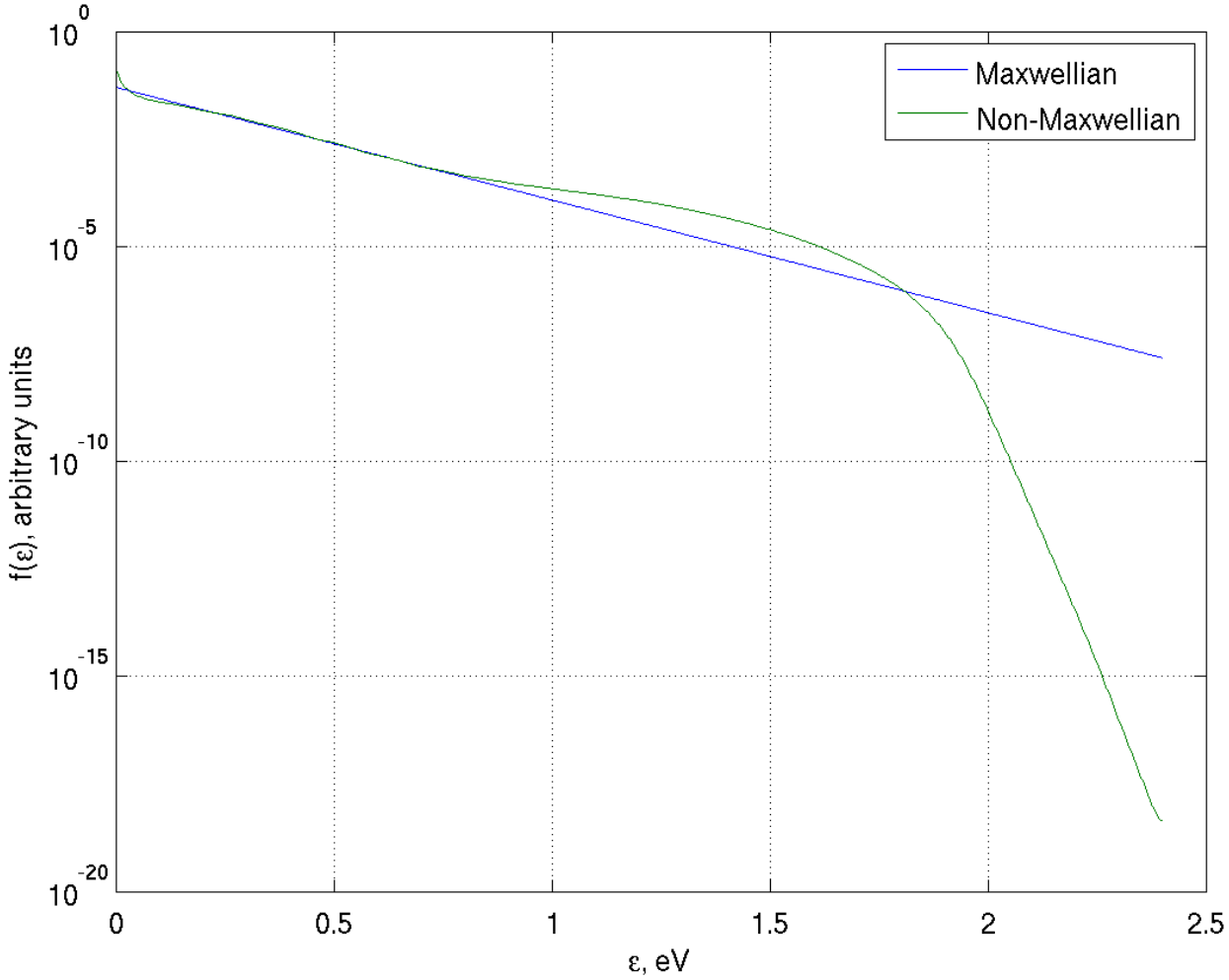
Conductivity changes

Dashed lines correspond to Maxwellian distribution



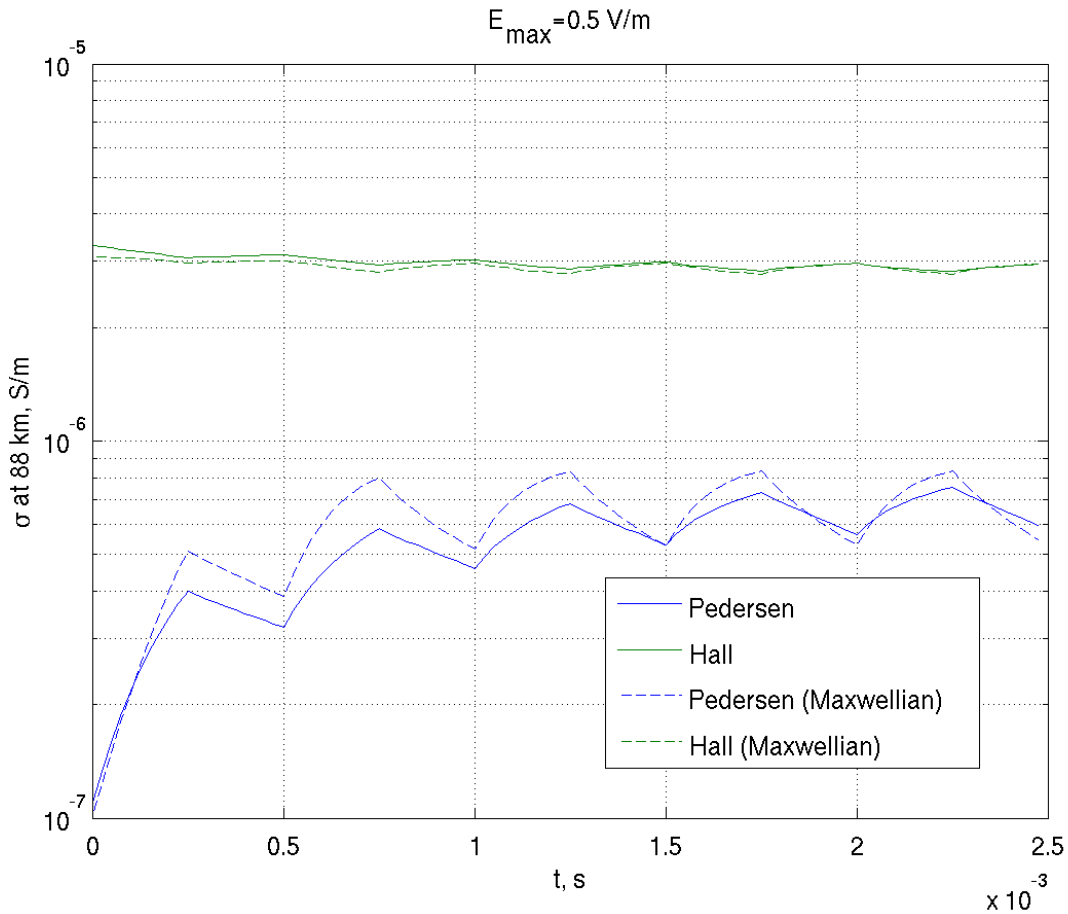
Electron distribution modified by HAARP, with E=.5 V/m

- **Clearly seen the effect of N2 vibrational barrier**



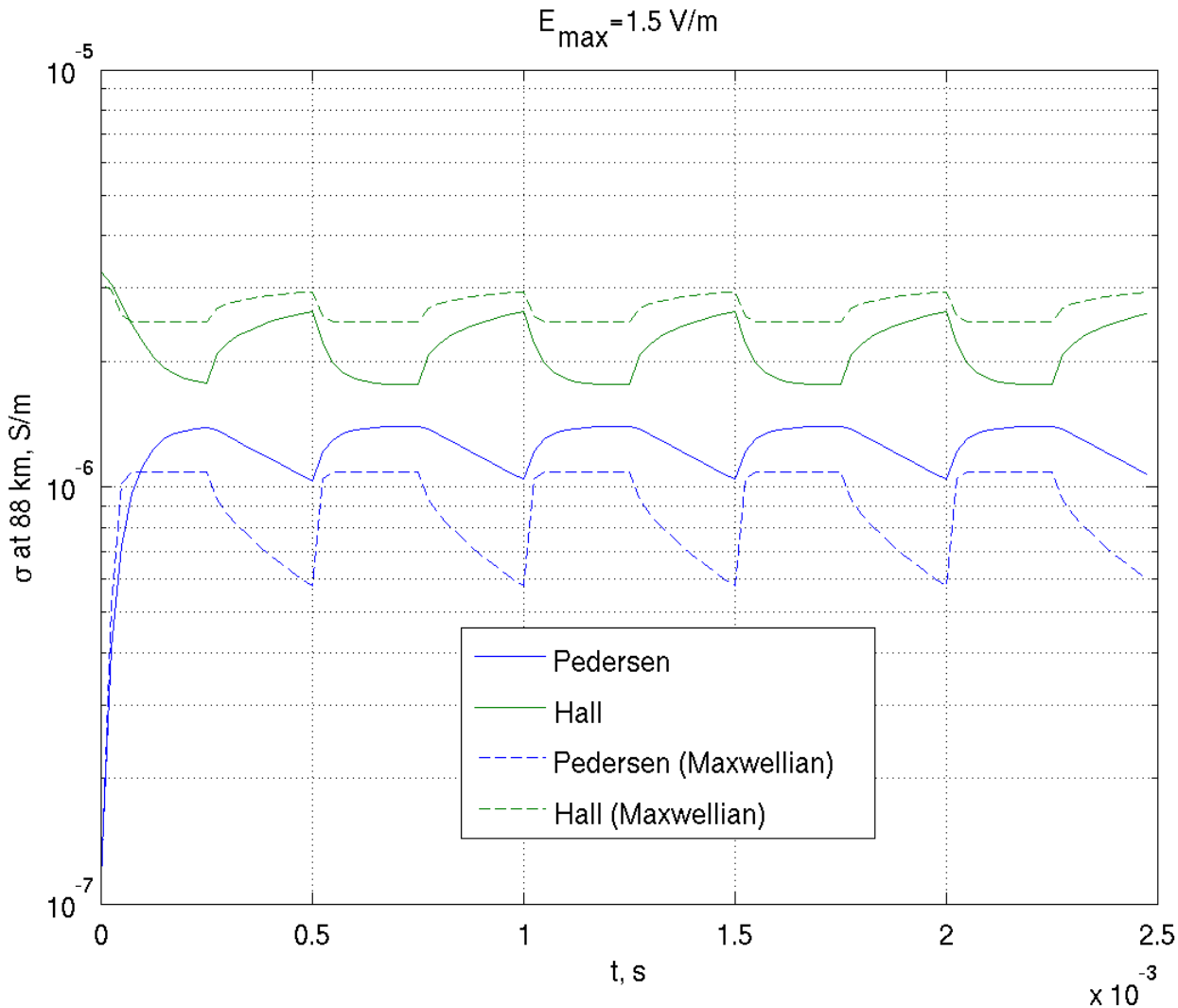
Square-modulated heating

- **Absolute change and harmonic contents of conductivity depends on non-thermal effects at current HAARP power level**



Square-modulated heating (2)

- **The effect is higher if the power is increased**



DC Conductivity tensor

- **Conductivity changes due to modification of electron distribution**
- **Pedersen (transverse)**

$$\sigma_p = -\frac{2e^2}{3m}\int \frac{\nu_m \varepsilon^{3/2}}{\omega_H{}^2 + \nu_m{}^2}\frac{\partial}{\partial \varepsilon}\frac{n}{\varepsilon^{1/2}}d\varepsilon \approx \frac{N_e e^2}{m}\left\langle \frac{\nu_m}{\omega_H{}^2 + \nu_m{}^2}\right\rangle$$

- **Hall (off-diagonal)**

$$\sigma_h = -\frac{2e^2}{3m}\int \frac{\omega_H \varepsilon^{3/2}}{\omega_H{}^2 + \nu_m{}^2}\frac{\partial}{\partial \varepsilon}\frac{n}{\varepsilon^{1/2}}d\varepsilon \approx \frac{N_e e^2}{m}\left\langle \frac{\omega_H}{\omega_H{}^2 + \nu_m{}^2}\right\rangle$$

- **Parallel**

$$\sigma_z = -\frac{2e^2}{3m}\int \frac{\varepsilon^{3/2}}{\nu_m}\frac{\partial}{\partial \varepsilon}\frac{n}{\varepsilon^{1/2}}d\varepsilon \approx \frac{N_e e^2}{m}\left\langle \frac{1}{\nu_m}\right\rangle$$

Electrojet current modulation calculations

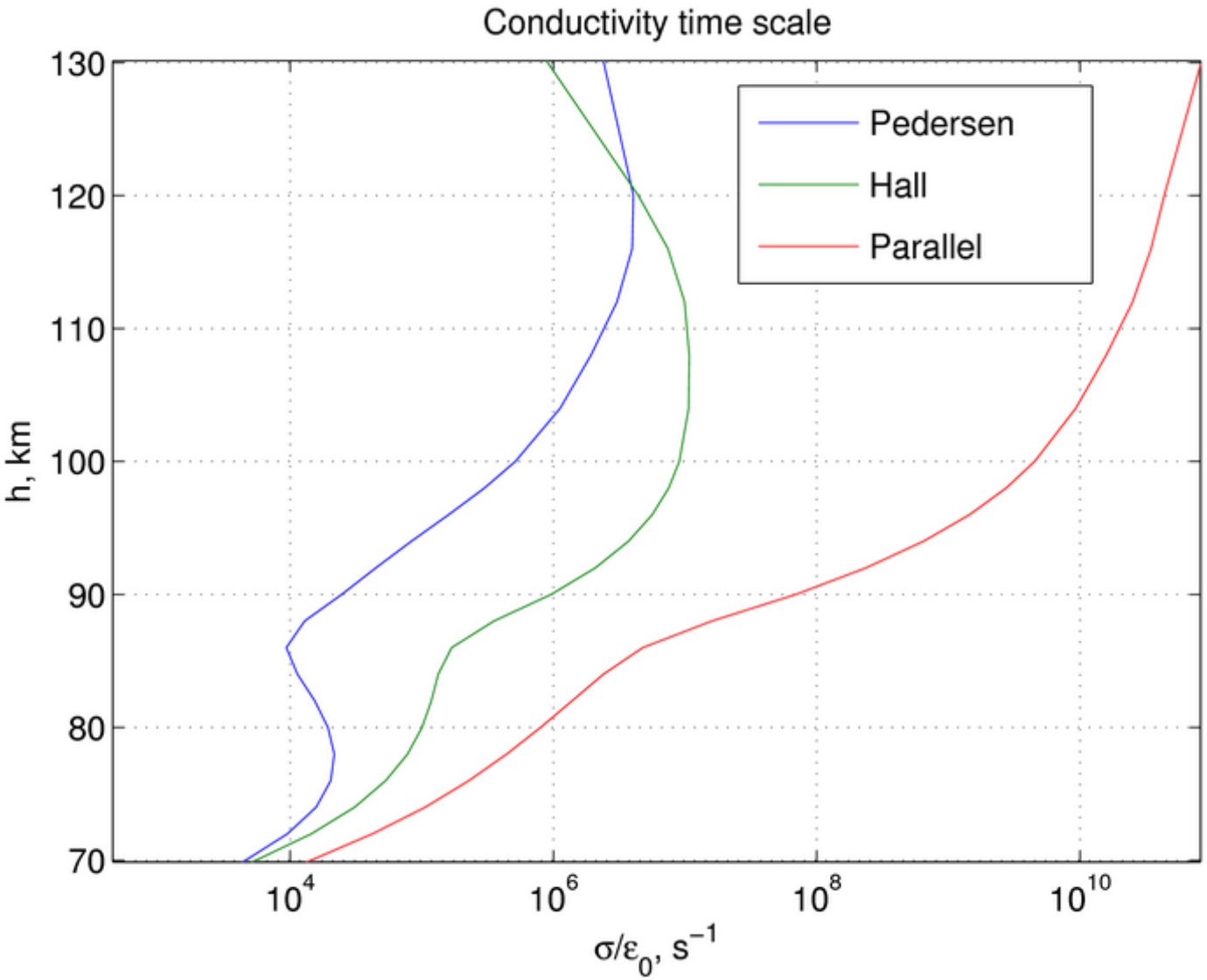
- **We assume static current, i.e.**

$$\vec{J} = -\overleftrightarrow{\sigma}\nabla\varphi$$

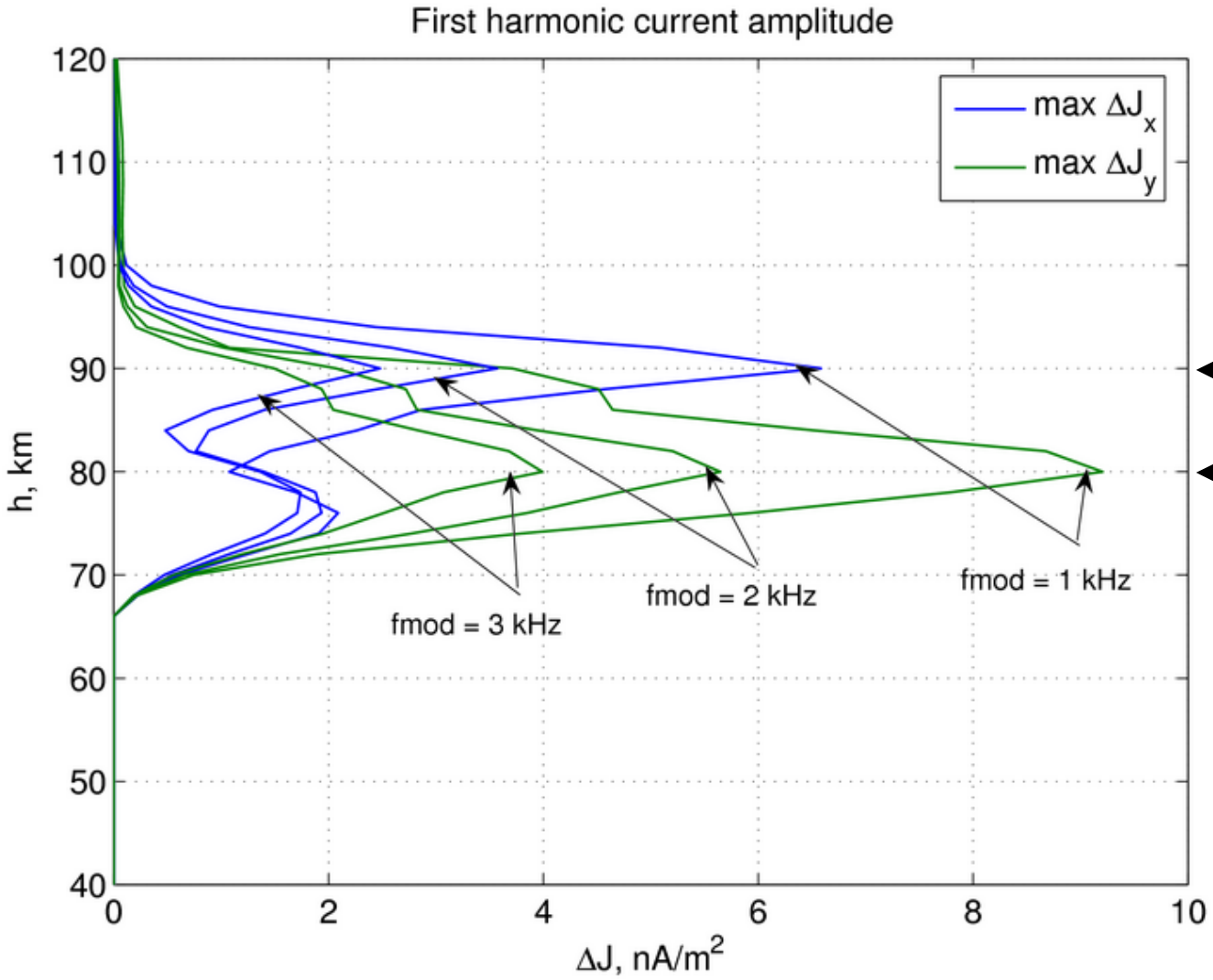
$$\nabla \cdot \vec{J} = 0$$

- **This is justified because the conductivity time scale σ/ε_0 >> f**
- **Vertical B (z axis)**
- **Ambient E = 25 mV/m is along x axis**
- **3D calculations**

Conductivity time scales (for justification of steady state)



3D stationary ΔJ: vertical profile



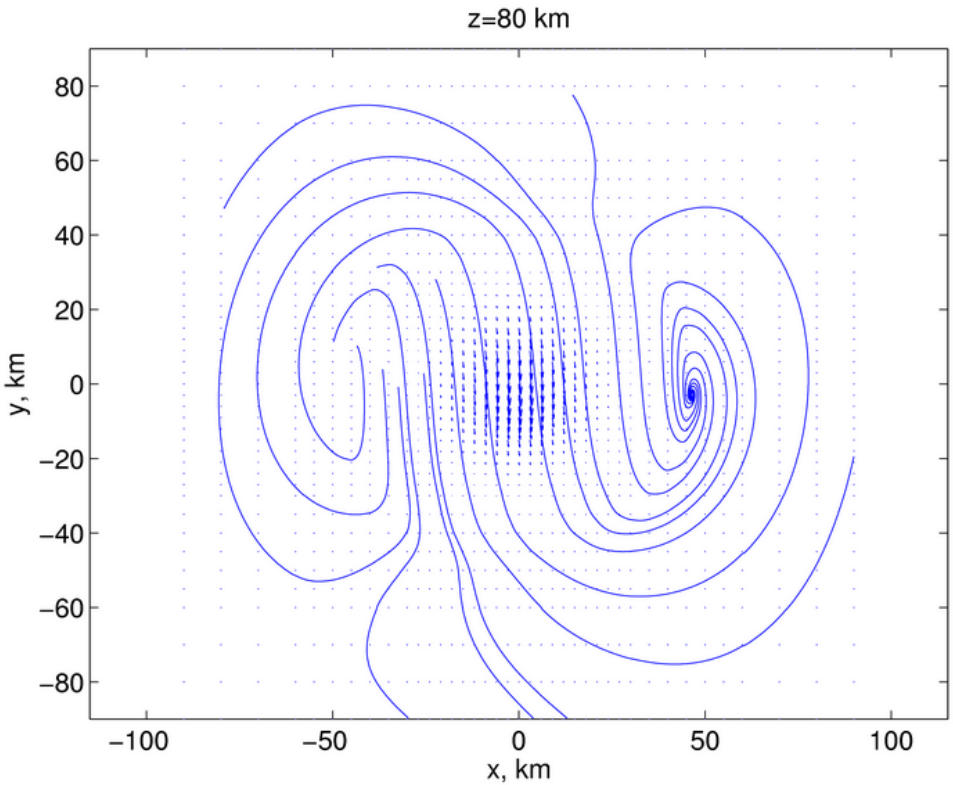
←Pedersen current

←Hall current

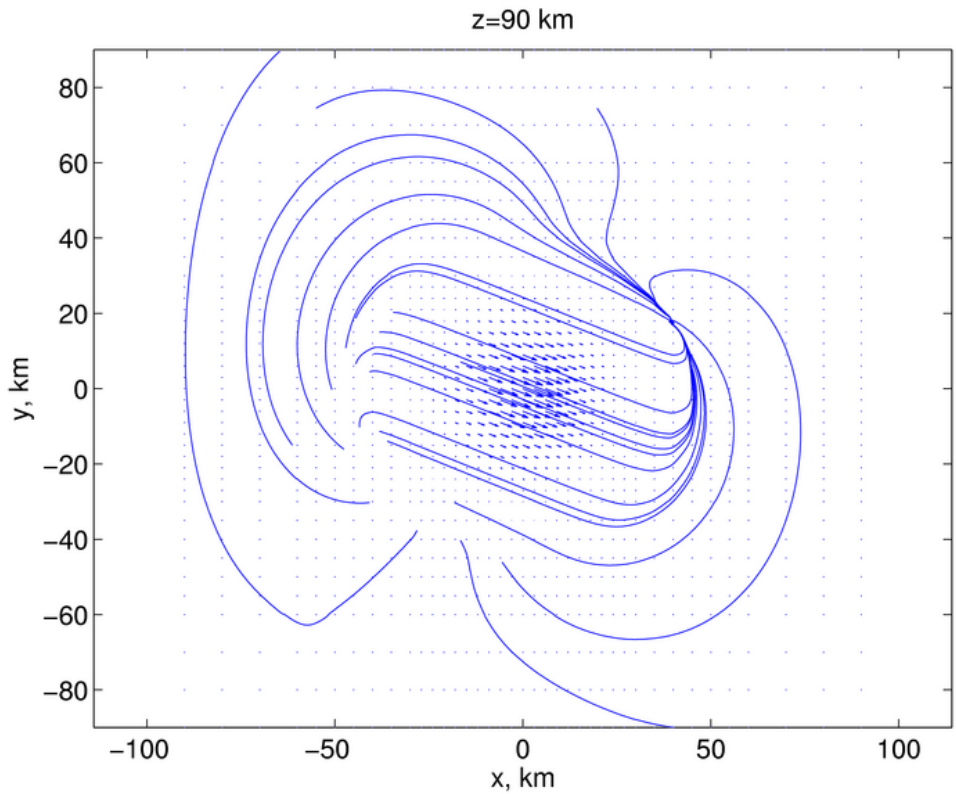
3D stationary ΔJ: horizontal slices

- **Vertical B (z axis)**
- **Ambient E = 25 mV/m is along x axis**

Hall current (80 km)



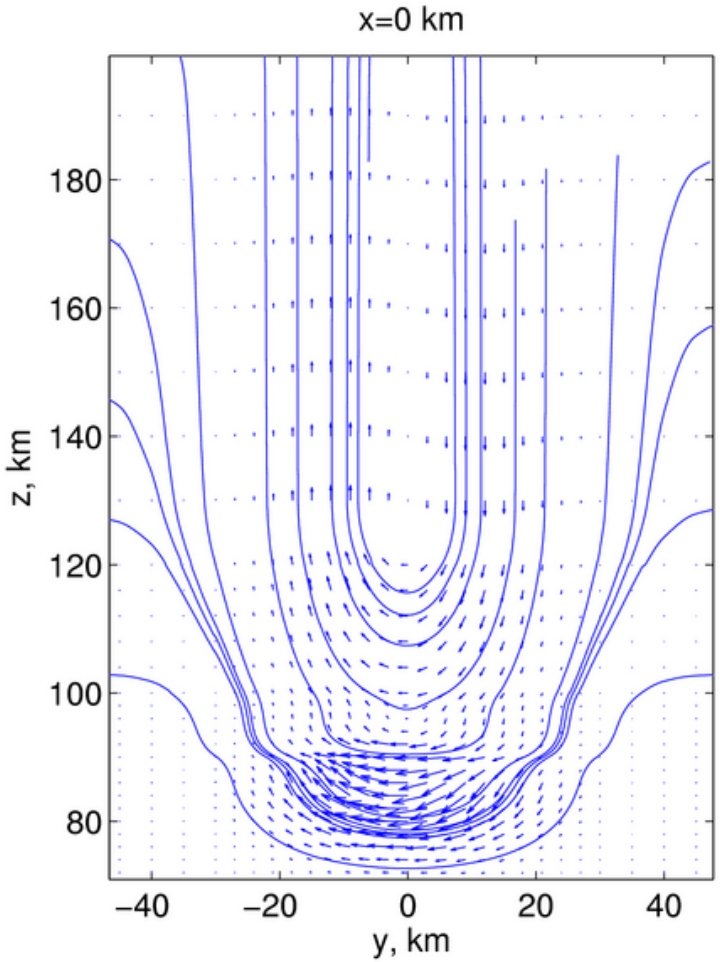
Pedersen current (90 km)



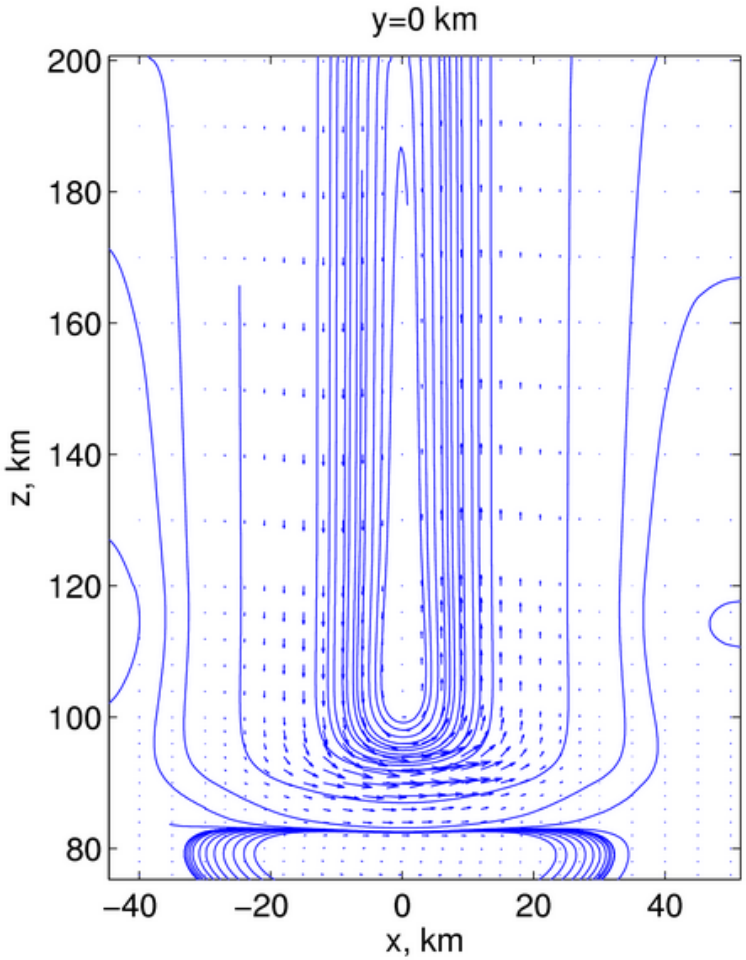
3D stationary ΔJ: vertical slices

- **Closing vertical currents**

Hall currents



Pedersen currents

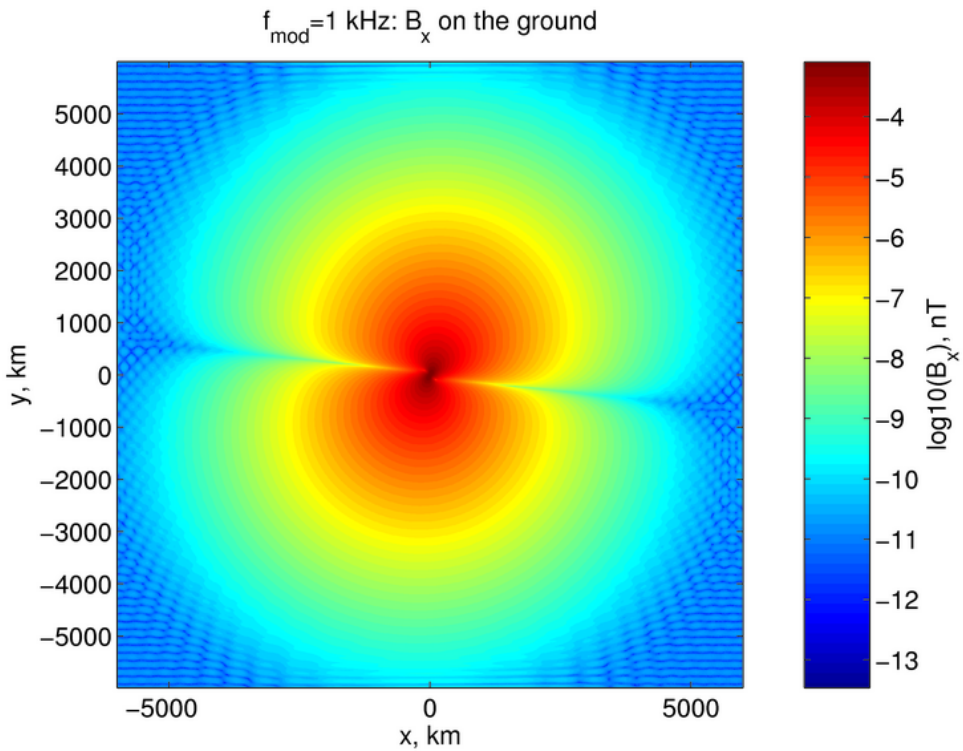


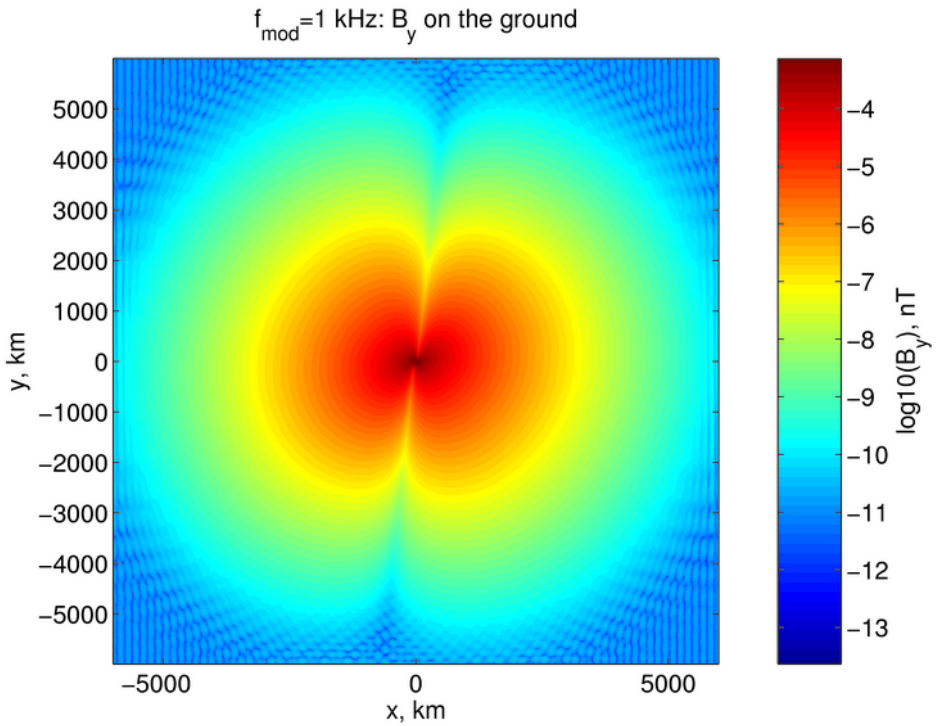
ELF/VLF radiation: application of mode theory

- E_x, E_y are continuous at each boundary between horizontal layers
- Change in H_x, H_y is proportional to horizontal current
- Boundary conditions: $E_{x,y}=0$ at h=0, radiation at h_{max}
- The source current is represented as a linear combination of its horizontal Fourier components
- Current limitation: no J_z, but it is located higher in ionosphere and should not affect at least the Earth-ionosphere waveguide results

f=1 kHz radiation into Earth-ionosphere waveguide

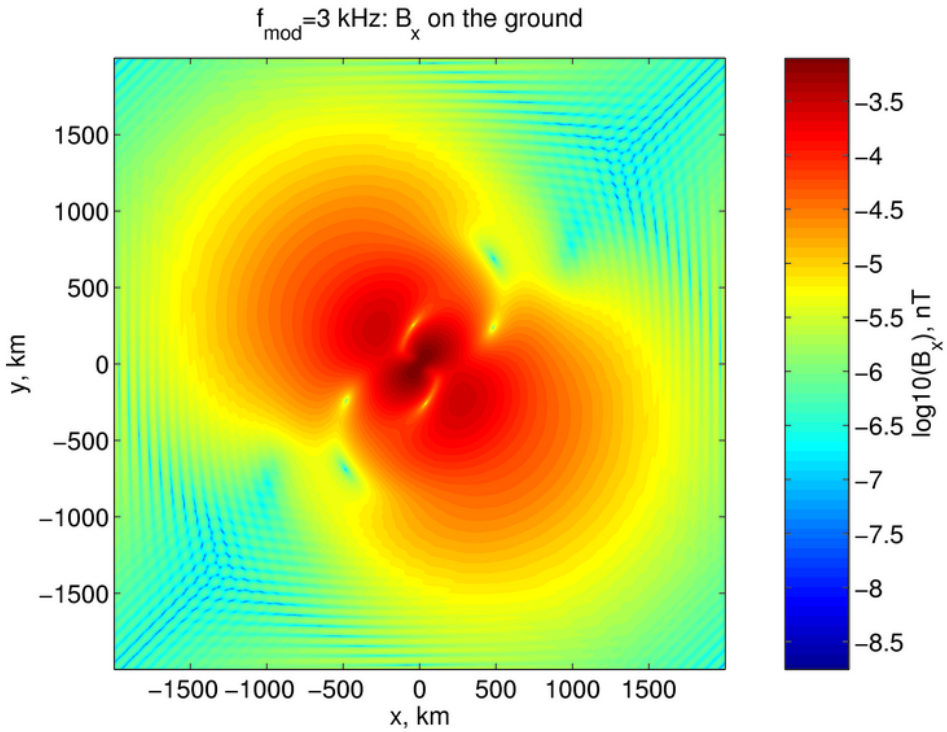
- **Only QTEM mode => no nulls**

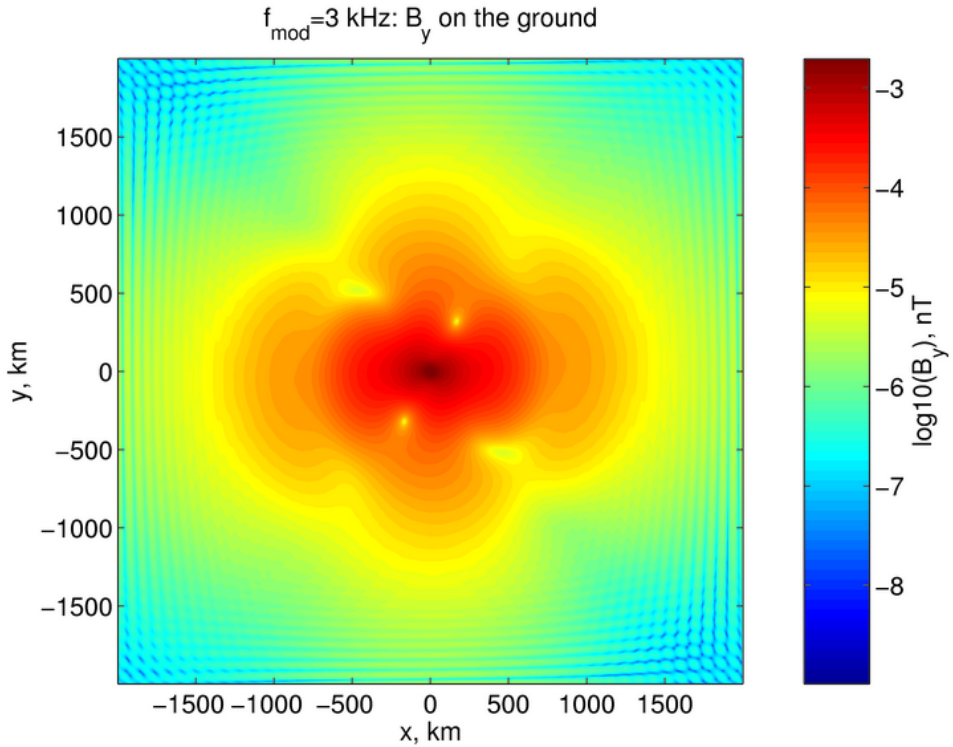




f=3 kHz radiation into Earth-ionosphere waveguide

- QTEM mode (λ=105 km) and QTE1 mode (λ=120 km) => nulls

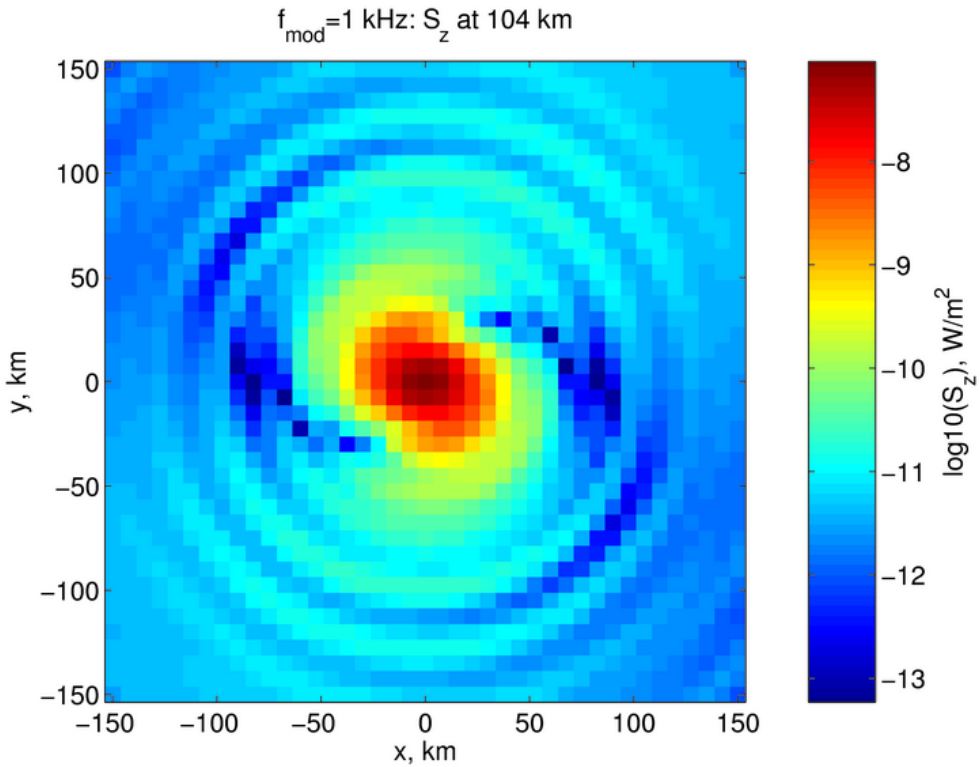




Radiation of the whistler mode into ionosphere: energy flux

f_{mod} = 1 kHz:

Total = 55.3692 W



f_{mod} = 3 kHz:

Total = 5.4351 W