A TWO-DIMENSIONAL MODEL OF RUNAWAY ELECTRON BEAMS DRIVEN BY QUASI-ELECTROSTATIC THUNDERCLOUD FIELDS

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Lightning-mesosphere interaction phenomena

Red Sprites:
- altitude range ~50-90 km
- lateral extent ~5-10 km
- occur ~1-5 ms after +CG discharge
- last up to several 10 ms
**Literature**


Electron energy losses in the atmosphere

*Dynamic friction force* is due to inelastic collisions:
- excitation of molecules:
  - rotational
  - vibrational
  - optical emissions $\sim 10$ eV
- dissociation
- ionization $> 10$ eV

![Graph showing dynamic friction force vs electron energy](image)

- $E_c$ - conventional avalanche breakdown field
- $E_t$ - runaway threshold field

$E_c / E_t \sim 10$
Positive cloud-to-ground discharge (+CG)

**BEFORE DISCHARGE**

- Negative screening charge
- Electric field $E$ (small)
- $+Q$ (positive charge) 1 ms
- $-Q$ (negative charge)
- Heights: 10 km, 5 km

**AFTER DISCHARGE**

- Electric field $E$ exceeds runaway threshold field
- Negative screening charge
- ~100 MV potential drop to ionosphere
- Heights: 10 km, 5 km

The diagram illustrates the process before and after the discharge, showing the movement of charges and the change in electric field strength.
Cosmic ray shower

$S_0 \sim 10^{-5} / \text{cm}^3\text{-sec at } \sim 10 \text{ km altitude}$
THEORETICAL MODEL

• Quasi-electrostatic field:
\[ \vec{E} = -\nabla \phi, \quad \phi = \phi(\vec{r}, t) \]
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_R = \frac{\rho_s \sigma_0}{\epsilon_0}, \quad \vec{J}_R = -e \vec{v}_R N_R \]
\[ \nabla \cdot \vec{E} = \frac{\rho + \rho_s}{\epsilon_0} \]

\[ \vec{J}_R, \vec{v}_R, N_R \] — current, velocity and density of runaway electrons
\[ \rho_s \] — source thundercloud charge

• Runaway electrons:
\[ \frac{\partial N_R}{\partial t} + \nabla \cdot (\vec{v}_R N_R) = \frac{N_R}{\tau_i} + S_o(z) \]
\[ \dot{\vec{p}} = -e \vec{E} - \frac{e}{m \gamma} (\vec{p} \times \vec{B}) - \nu R \vec{p}, \quad \nu R = \frac{F_D}{p} \]

\[ 1/\tau_i \] — runaway avalanche rate
\[ F_D \] — dynamic friction force

• Ionization produced by runaways:
\[ \frac{\partial N_{es}}{\partial t} = (\nu_i - \nu_a) N_{es} + N_a v R N_R \chi_{i0} \]
\[ N_{es} \] — density of low-energy electrons
\[ \chi_{i0} \] — ionization cross-section
\[ N_a \] — density of atmospheric atoms

• Optical emissions (\( N_2 \) First Positive Band) are due to
  — thermal electrons, driven by the electric field,
  — suprathermal electrons (\( \gtrsim 10 \text{ eV} \)), created by the runaways,
and are calculated on the basis of known molecular excitation cross-sections and emission rates.

• \( \gamma \)-rays are calculated using Heitler bremsstrahlung cross-sections.
Runaway trajectories

t=1.2 ms, Q=225 C

2-D MODEL

$\nu_R \sim \omega_H$

Gurevich et al [1996]

demonstration of runaway trajectories for tilted geomagnetic field
Runaway electron density

\( Q = 225 \text{ C} \)

Along the axis of symmetry:

- \( t = 0.9 \text{ ms} \)
- \( t = 1.2 \text{ ms} \) (maximum)

Graph showing runaway electron density along the axis of symmetry.
Intensity of the N$_2$ First Positive band, $Q=225$ C and $Q=150$ C

at different instants of time

Q = 225 C  $t = 0.5$ ms  Q = 150 C

integrated over 16 ms

Q = 225 C  Q = 150 C

along the axis of symmetry

Q = 225 C  Q = 150 C
Predicted $\gamma$-ray emissions

$\gamma$-ray emissions are due to bremsstrahlung by runaway electrons

$Q=225$ C

$Q=240$ C

measurements

[Fishman et al, 1994]
Summary

¥ We calculated the flow of REL in a cylindrically symmetric system with vertical geomagnetic field, using a two-dimensional REL-QE model.

¥ The model self-consistently includes effects of the REL on the electric field and the conductivity of the atmosphere.

¥ We analyzed the possible contribution of REL to the optical emissions, caused by heating of free electrons by the thundercloud quasi-electrostatic field.

¥ It was shown that the REL-QE model, taking into account the contribution of the REL in the ionization process, gives high optical emissions at altitudes 40-60 km, in contrast to heating of ambient electrons that can explain emissions at >80 km.

¥ The emissions from the ionization channel created by REL closely resemble the observed shape and altitudes of the common columnar type of Sprite.

¥ For an exponential ambient conductivity profile we consider the REL to play a significant role only for discharges of ≥200C.

¥ Calculated γ-ray fluxes are in agreement with fluxes observed by the BATSE detector [Fishman et al., 1994].