Generation of ELF/VLF waves by an ionospheric heater at the geomagnetic equator

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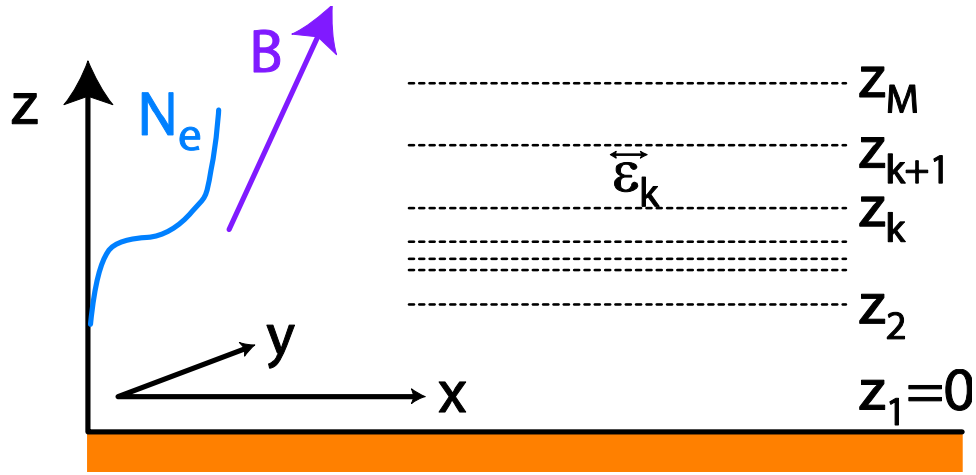
VLF/ELF emission and propagation

- We use a new full-wave method (FWM) to calculate fields radiated by the modulated electrojet current
- FWM Capabilities:
  - Arbitrary plane stratified medium, e.g., a horizontally-stratified magnetized plasma with an arbitrary direction of geomagnetic field (ionosphere)
  - Arbitrary configuration of harmonically varying currents
  - Provides full wave 3D solution of both whistler waves launched into ionosphere and VLF waves launched into Earth-ionosphere waveguide
  - Stable against the “swamping” instability by evanescent waves
  - Efficient use of the computer resources

Method description



1. We work in Fourier (wave vector k) domain
2. $k_\perp$ = const (Snell's law) $\rightarrow$ find k_z, E and H in each layer
3. Use continuity of $E_\perp$ and $H_\perp$ between layers to find reflection coefficients and mode amplitudes
4. Represent source currents as boundary conditions on $E_\perp$ and $H_\perp$ between layers
5. Inverse Fourier transform from $k_\perp$ to $r_\perp$

Finding the modes in each layer

- **Solve uniform Maxwell's equations for plane waves (~$e^{-i\omega t+ikr}$)**

$$\mathbf{n}\times(\mathbf{n}\times\mathbf{E})+\overleftrightarrow{\varepsilon}\mathbf{E}=\overleftrightarrow{M}\mathbf{E}=0$$

$$\det\overleftrightarrow{M}=0\Rightarrow n_z$$

$$\mathbf{n}=\mathbf{k}/k_0,\ k_0=\omega/c$$

- **The fields are a combination of 4 different solutions, corresponding to different n_z**

$$\begin{pmatrix}\mathbf{E}(\mathbf{r})\\ \mathbf{H}(\mathbf{r})\end{pmatrix}=\mathsf{F}_k\begin{pmatrix}u_k^{(1)}e^{ik_0n_z^{(u1)}(z-z_k)}\\ u_k^{(2)}e^{ik_0n_z^{(u2)}(z-z_k)}\\ d_k^{(1)}e^{-ik_0n_z^{(d1)}(z-z_k)}\\ d_k^{(2)}e^{-ik_0n_z^{(d2)}(z-z_k)}\end{pmatrix}e^{ik_0(\mathbf{n}_\perp\cdot\mathbf{r}_\perp)}$$

Finding the reflection coefficients (R^u,R^d) and mode amplitudes (u,d)

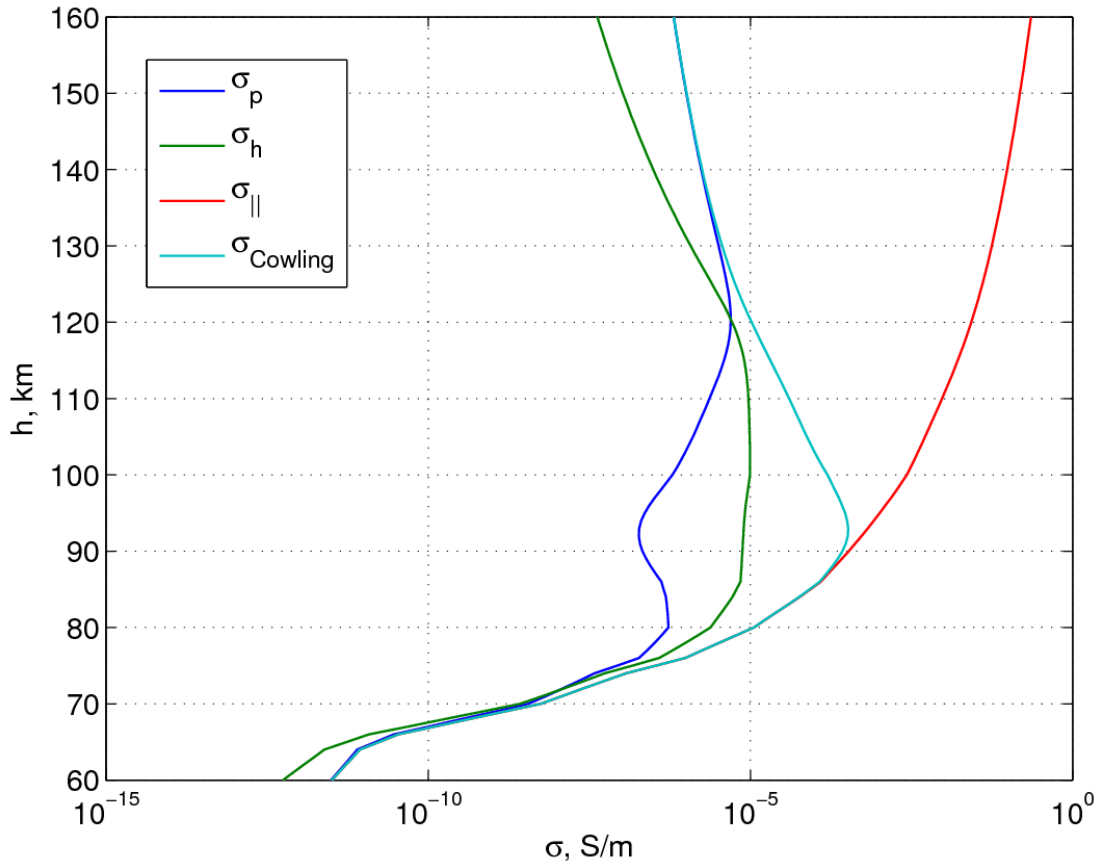
- We label 2 waves as upward (u) and 2 as downward (d)
- Reflection coefficients are 2x2 matrices which provide u$\leftrightarrow$d conversion:
  - $d=R^u u$
  - $u=R^d d$
- Recursion:
  - $R^u_{k+1} \rightarrow R^u_k$ and $R^d_k \rightarrow R^d_{k+1}$
  - $u_k \rightarrow u_{k+1}$ and $d_{k+1} \rightarrow d_k$

This order of recursion provides stability against numerical "swamping" by evanescent waves

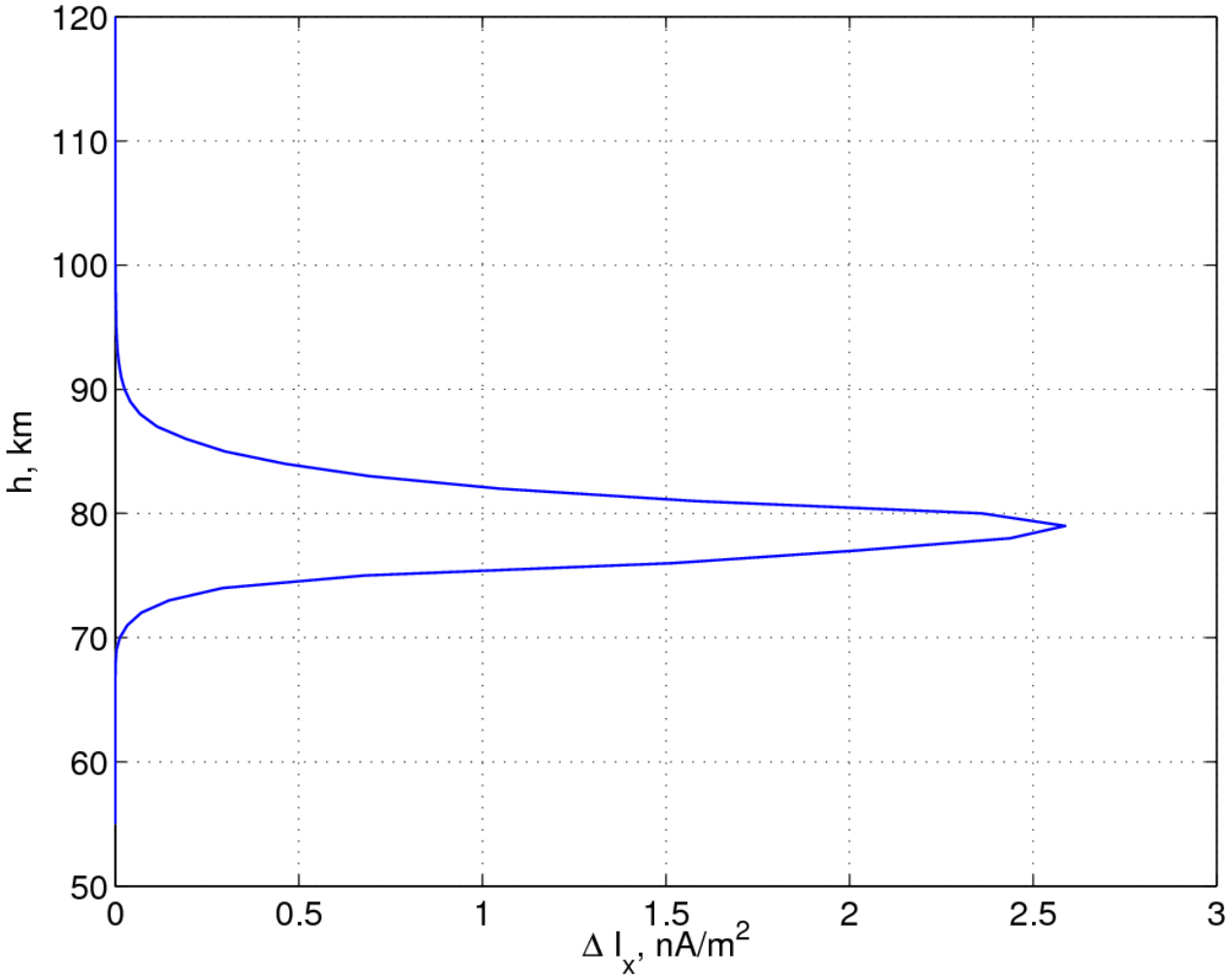
Equatorial electrojet

- **Because of the horizontal geomagnetic field, the relevant conductivity for a horizontal current is the Cowling conductivity**

$$\sigma_{Cowling} = \sigma_h^2 / \sigma_p + \sigma_p$$



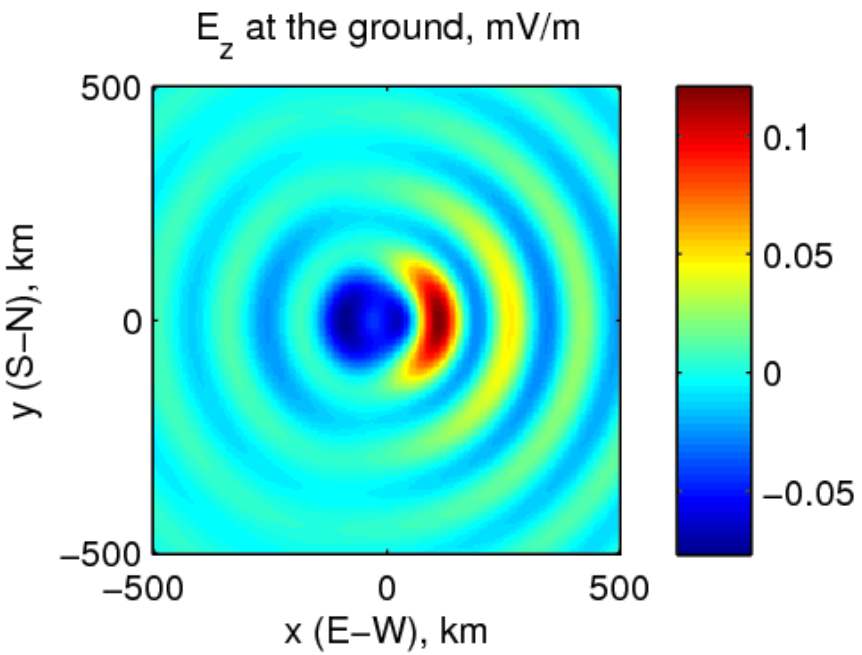
Perturbation of electric current

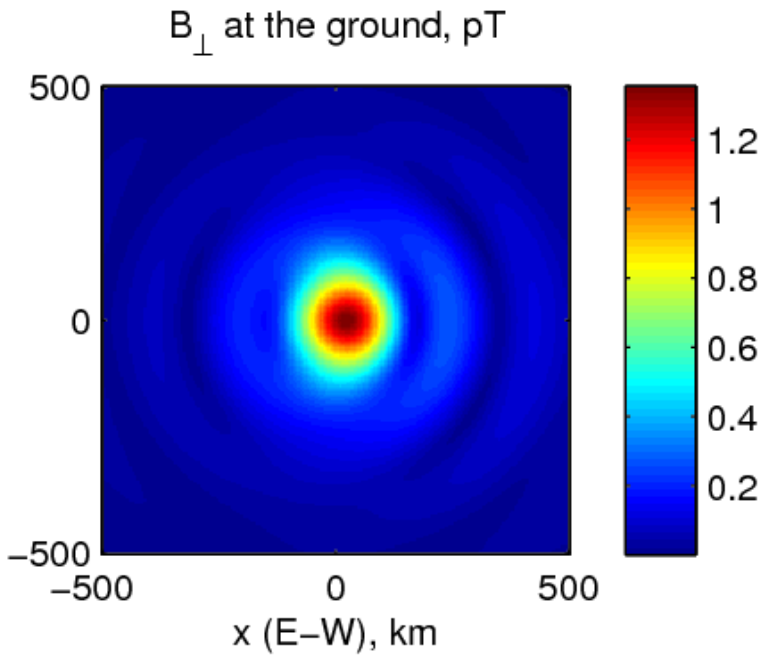


- Horizontal in E-W direction and
- Gaussian profile with a width of 23 km
- Modulated at f=1875Hz

Results: E and B on the ground

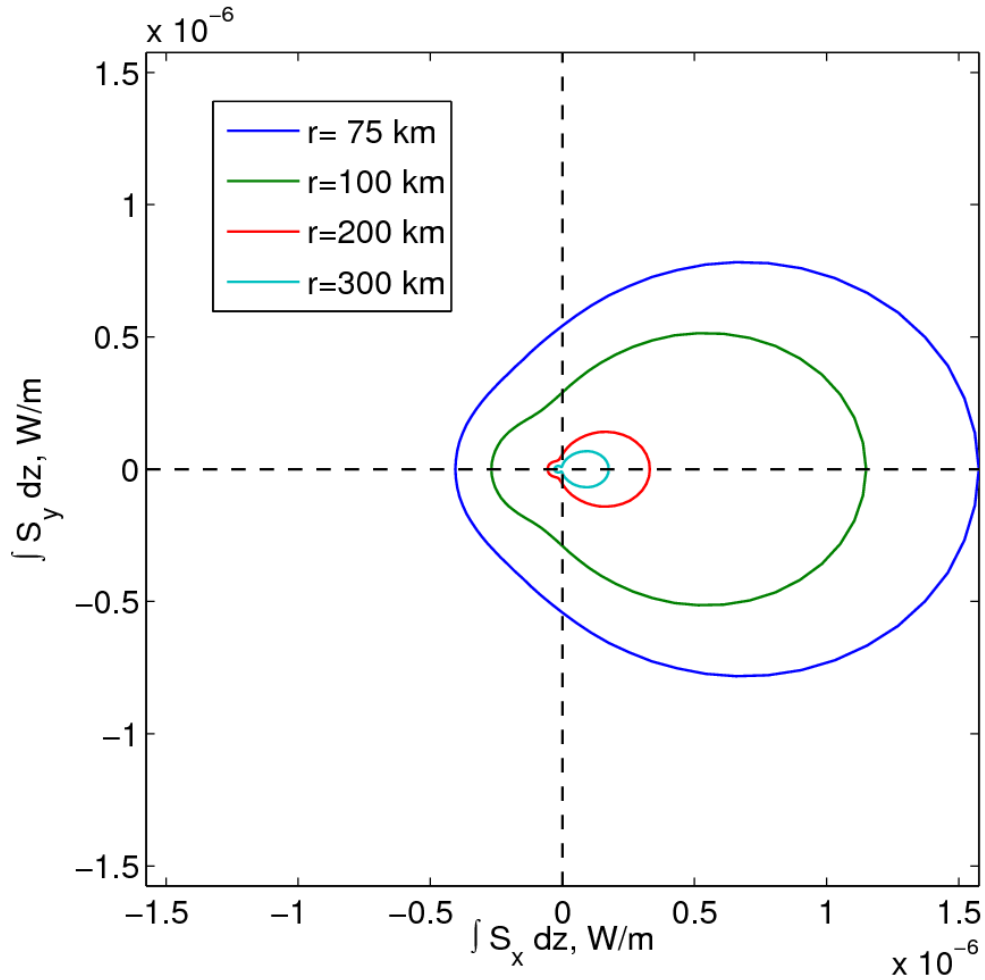
- Geomagnetic field B_0=3e-5 T, B_0 || y
- Current ΔI || x
- East-west assymmetry in emission





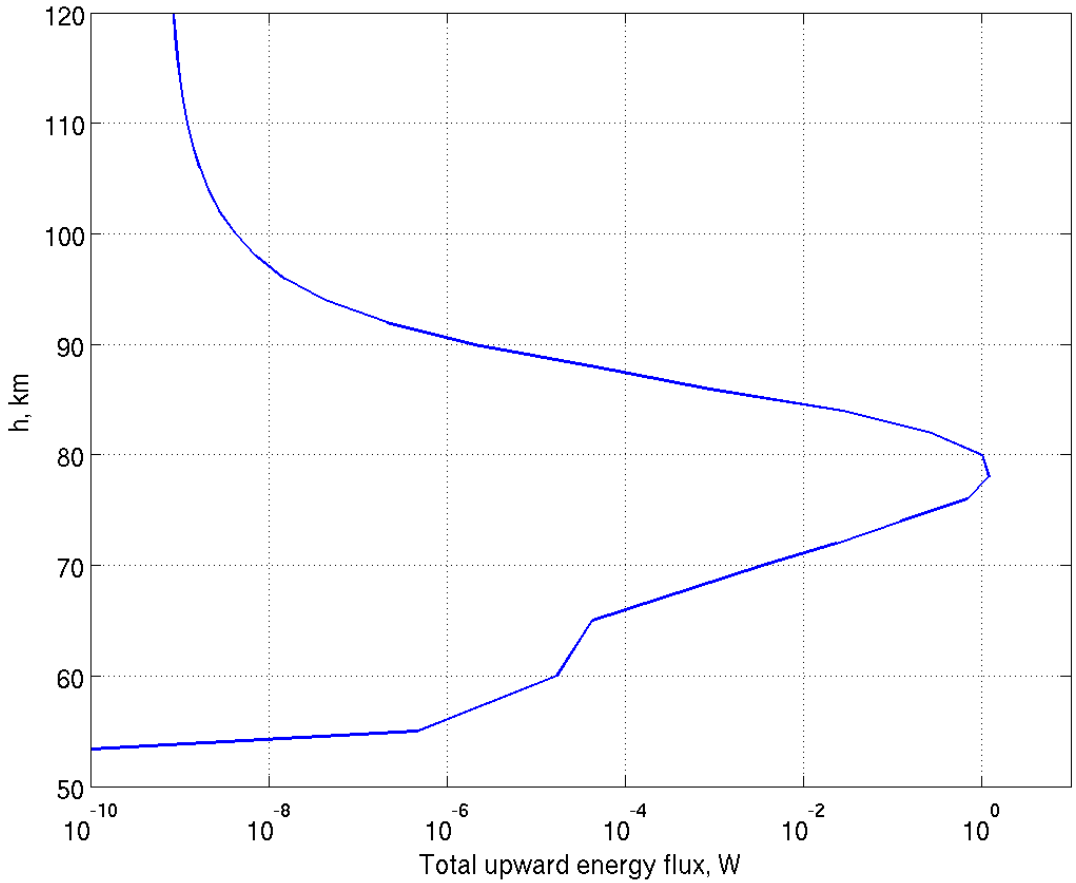
Horizontal energy flux

- **Horizontal energy flux integrated over altitudes show this assymmetry, too.**



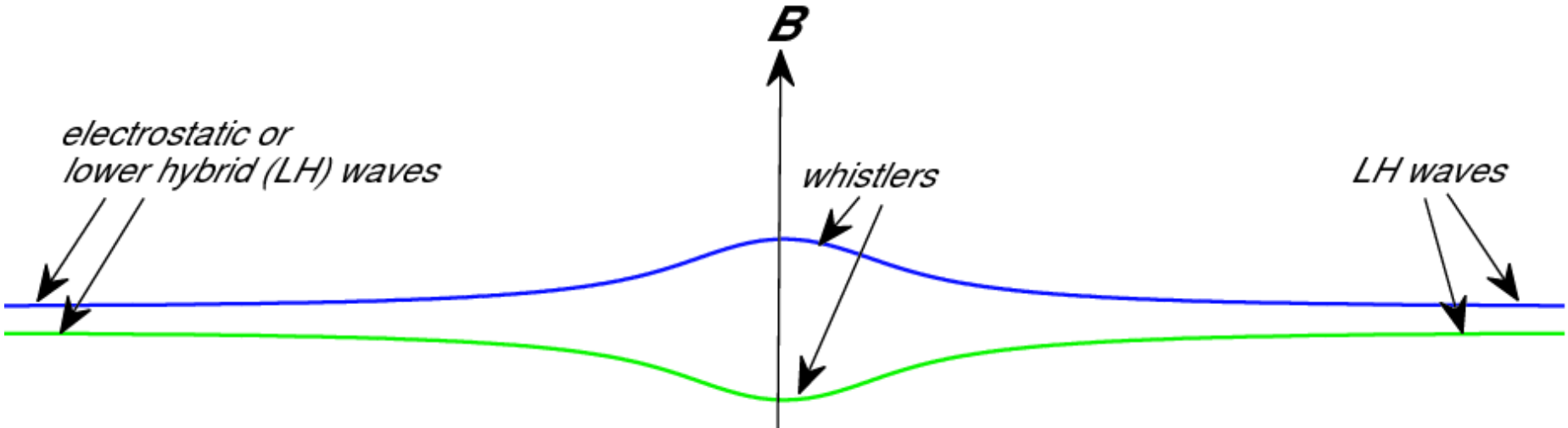
No energy goes into ionosphere

- **Since at these frequencies only whistler mode can propagate, and it only propagates along B_0, virtually no energy goes upward into ionosphere.**

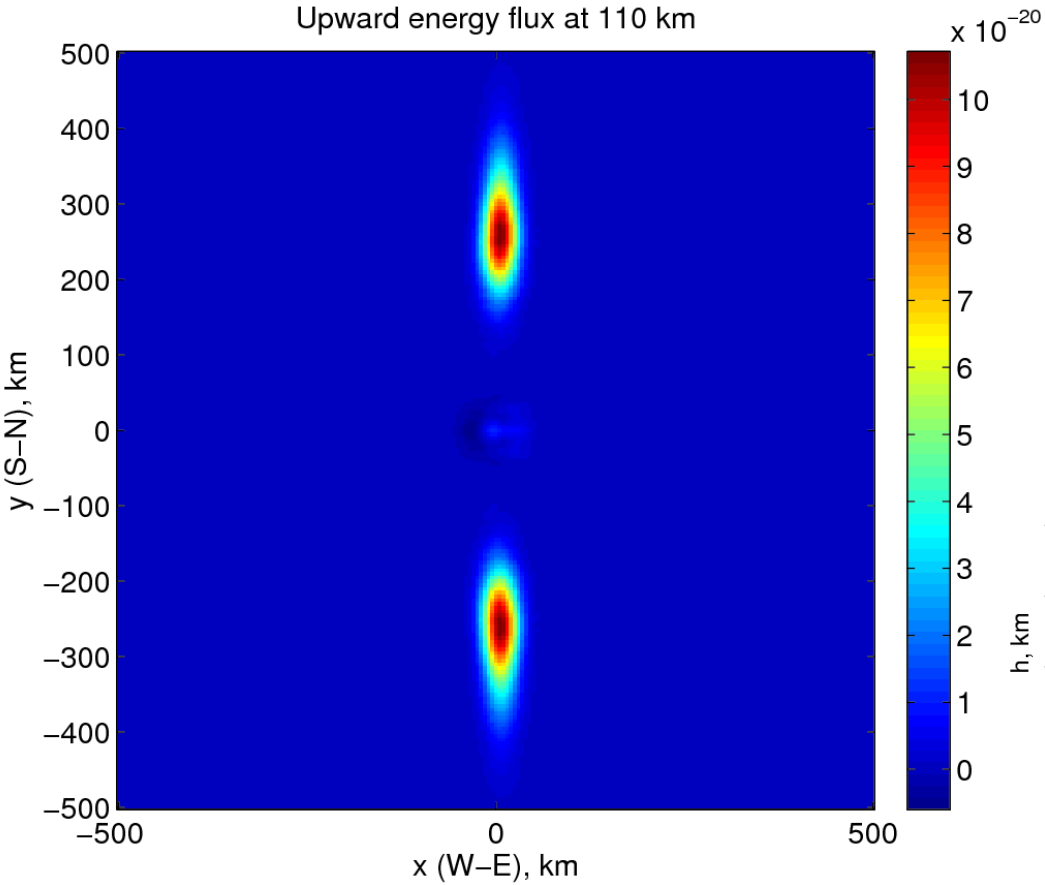


Whistlers and Lower Hybrid (Electrostatic) waves

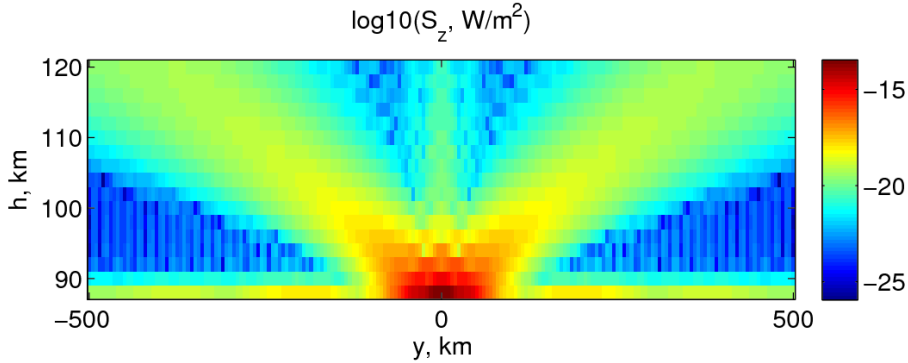
- Whistlers are circularly polarized waves with H~nE, but at high values of $k_\perp$ (in respect to B) they become electrostatic waves (i.e. E || k, H is small)
- $k_\perp$ becomes infinite at the resonance cone
- When ions are included, the whistler refractive index surface may close at high values of $k_\perp$ (with $k_{||}$=0), i.e. the resonance cone disappears
- The resonance occurs at k ⊥ B at the lower hybrid frequency (the lower hybrid resonance)



Low upward energy flux

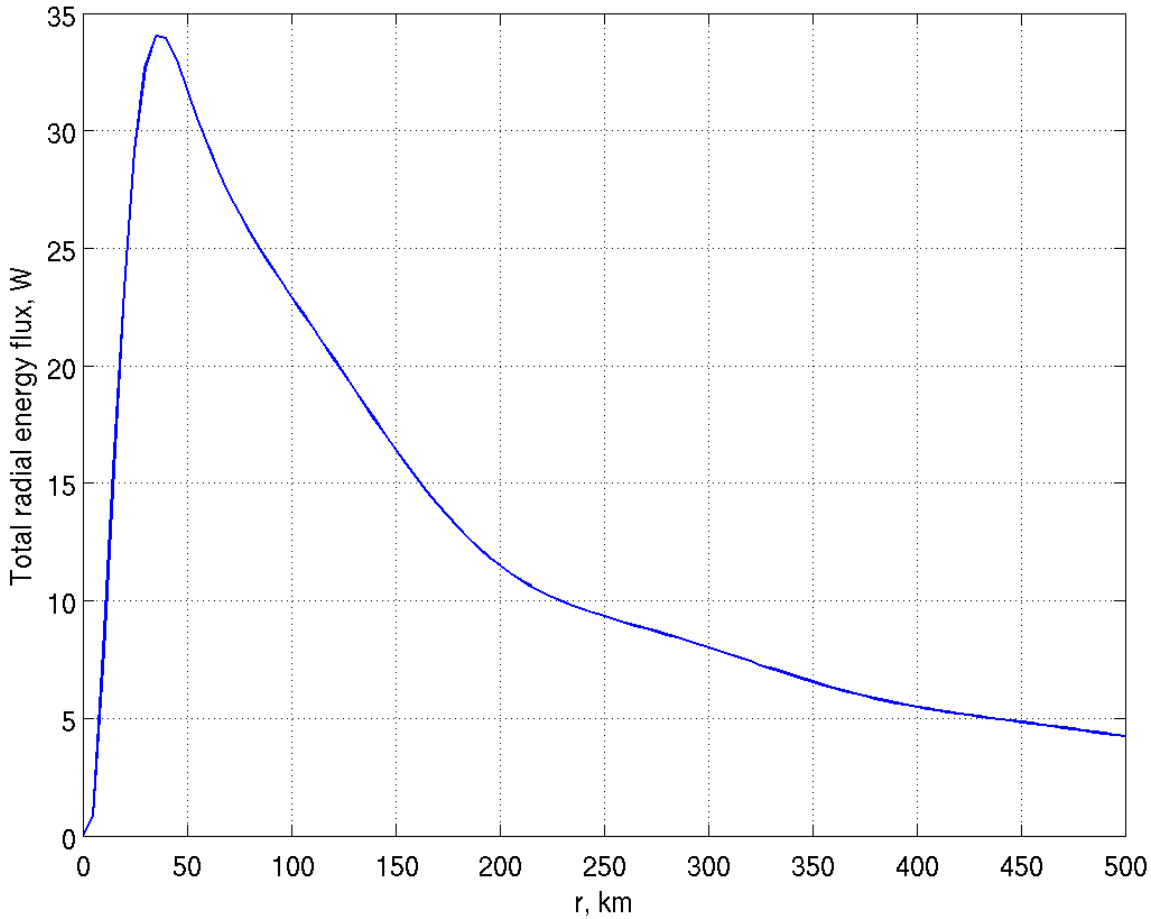


- **The low upward energy flux is due to the LH waves with large wave normal angle**



Total radial energy flux

- **Since energy does not propagate into ionosphere, all of it goes into the Earth-ionosphere waveguide**



Conclusions

- We used the new Full Wave Method (FWM) to calculate radiation by horizontal modulated current perturbation.
- All of the radiated energy goes into the Earth-ionosphere waveguide.
- Thus for the same current modulation amplitude we obtain a higher energy flux than in the vertical geomagnetic field (30W vs 1W, Lehtinen and Inan [2008]).
- There is an East-West assymmetry in the radiation pattern.