On instabilities of Alfvén range oscillations in the ionosphere

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ABSTRACT

This essay was written on the basis of extract from my diploma paper (Lehtinen 1994) at Nizhniy Novgorod State University. My scientific advisor was G. A. Markov, Professor at Electrodynamics Division at Radiophysics Department, NNSU.

In the diploma paper the possibility of excitation of ULF ($\lesssim$10 Hz) waves in ionosphere by a plasma-wave discharge from a meteorological rocket was considered.

In the first part of this essay we provide a description of an ionospheric experiment which probably showed the excitation of an ionospheric Alfvén resonator (IAR) by a plasma-wave discharge. In the second part, a description of an ionospheric Alfvén resonator is provided. In the third part, we consider instabilities of Alfvén range waves, interacting with an infinite plasma beams.

1. The ionospheric experiment

The experiment was carried out by launching a meteorological rocket MR-12 (Fig. 1) on Kapustin Yar facility on February 6, 1991 at 8:30 p.m. Moscow time. The rocket was launched almost vertically with an inclination of 5° to the east. At heights of about 60–70 km the shells giving the rocket a streamlined shape were thrown off, and a dipole antenna was let out. The antenna consisted of two semi-cylinder electrodes made of a metal net with diameter 2 m and height 1.2 m. The net was spread out due to rotation of the rocket. The semi-cylinder electrodes were fixed on a dielectric bar and were under alternate voltage of 1.5 kV with frequency 480 kHz, amplitude modulated at frequencies $f_1 = 120\,\mathrm{Hz}$ and $f_2 = 240\,\mathrm{Hz}$ according to a special cyclogram (Fig. 2). The rocket carried some diagnostic devices, including a counter of charged particles, a magnetometer and a two-band LF receiver with frequencies f_1 and f_2.

As a result of the HF field of the antenna, a plasma discharge was formed, which stretched out along the geomagnetic field due to the unipolar diffusion of electrons (Agafonov et al. 1982). The plasma concentration in the discharge area was much higher than the background plasma concentration ($\approx 10^4\,\mathrm{cm}^{-3}$) and was about $10^7\,\mathrm{cm}^{-3}$ at heights $\approx 100\,\mathrm{km}$ and about $10^6\,\mathrm{cm}^{-3}$ at heights $\approx 150\,\mathrm{km}$. The temperature of electrons was much more than that of the background electrons ($\approx 0.1\,\mathrm{eV}$) and was about

$$E_e \approx \frac{3}{2} k_b T \approx 20\,\mathrm{eV}$$

. The reading of the on-board diagnostic devices confirmed that modulation of plasma parameters at frequencies f_1 and f_2 took place.

A special LF ground-based receiver registered noise-like electromagnetic signals at frequencies f_1 and f_2 with a band of $\approx 10\,\mathrm{Hz}$. The spectrum analysis of these signals showed peaks typical for amplitude modulation of the received signals on the frequency $f_{res} \approx 2.6\,\mathrm{Hz}$ (Fig. 3). The dynamic development of the modulation peak (shown on Fig. 4) showed that it appeared when the rocket reached a height $\gtrsim 100\,\mathrm{km}$ and practically disappeared when the generator was turned off.

The observed frequency f_{res} belongs to the frequency range of an ionospheric Alfvén resonator (IAR). Therefore, the results of the experiment can be explained by excitation of an IAR by the meteorological rocket.

2. The IAR model. Estimation of its eigenfrequencies and quality factor.

The Alfvén range contains frequencies

$$\omega \ll \Omega_H, \quad \omega \ll \Omega_0 \tag{1}$$

where $\Omega_H = eH_0/Mc$ is the ion gyrofrequency, $\Omega_0 = (4\pi e^2 N/M)^{1/2}$ is the ion plasma frequency, $H_0 \approx 0.4$–0.5 oersted is the geomagnetic field, M is ion mass and N is plasma concentration. We also assume that the quasineutrality condition is satisfied, i.e. $N_i \approx N_e \approx N$.

We assume $\mathbf{E}$ and $\mathbf{H}$ vary harmonically with frequency ω ($\sim e^{i\omega t}$) and that the magnetic field is aligned with the z-axis ($\mathbf{H} \uparrow\uparrow \hat{\mathbf{z}}_0$). Then for Alfvén range oscillations the dielectric permittivity tensor of a cold, collision-free plasma has the following form (Gershman, Erukhimov and Yashin 1984):

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_1 & -i\epsilon_2 & 0 \\ i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & -\epsilon_3 \end{pmatrix}, \tag{2}$$

where

$$\epsilon_1 \approx n_A^2, \tag{3a}$$

$$\epsilon_2 \approx \mu n_A^2, \tag{3b}$$

$$\epsilon_3 \approx \eta n_A^2. \tag{3c}$$

We introduced Alfvén refractive index

$$n_A = \frac{c}{V_A} = \frac{\Omega_0}{\Omega_H} = \frac{c\sqrt{4\pi MN}}{H_0},$$

Alfvén speed

$$V_A = \frac{c}{\sqrt{4\pi MN}},$$

and dimensionless parameters $\mu = \dfrac{\omega}{\Omega_H}$, $\eta = \dfrac{M}{m}\left(\dfrac{\Omega_H}{\omega}\right) = \dfrac{M}{m}\mu^{-2}$, and m is electron mass.

For ionospheric parameters and Alfvén range of frequencies the following conditions are satisfied:

$$n_A \gg 1, \quad \mu \ll 1, \quad \eta \gg \frac{M}{m} \gg 1. \tag{4}$$

Consider a plain wave: $\mathbf{E}, \mathbf{H} \sim e^{i\omega t - i(\mathbf{k}\cdot\mathbf{r})}$ where

$$\mathbf{k} = (k_\perp, 0, k_\parallel),$$

$$k_\perp = k_0 n \sin\theta, \quad k_\parallel = k_0 n \cos\theta,$$

and

$$k_0 = \frac{\omega}{c}.$$

Here θ is the angle between $\mathbf{k}$ and $\mathbf{H}$ and n is the refractive index, depending on the direction of $\mathbf{k}$.

Substituting the components of $\hat{\epsilon}$ into the dispersion relation

$$(k^2\delta_{ij} - k_i k_j - k_0^2\epsilon_{ij})E_j = 0, \tag{5}$$

we find the refractive indeces for the extraordinary *fast magnetosonic (FMS)* wave and the ordinary *Alfvén* wave:

$$n_{1,2}^2 = n_A^2 \frac{1 + \cos^2\theta - \frac{1-\mu^2}{\eta}\sin\theta \pm \sqrt{\sin^4\theta\left(1 + \frac{1-\mu^2}{\eta}\right) + 4\mu^2\cos^2\theta}}{2\left(\cos^2\theta - \frac{\sin^2\theta}{\eta}\right)} \tag{6}$$

where n_1 is refractive index for FMS wave, and n_2 is refractive index for Alfvén wave.

For angles θ, not very close to $\pi/2$ (that is when $\tan^2\theta \ll \eta$) we can substitute $\eta = \infty$ in (5) and (6). Then (5), (6) assume the form:

$$n^2\cos^2\theta - (1 + \cos^2\theta)n_A^2 n^2 + n_A^4(1 - \mu^2) = 0 \tag{7}$$

and

$$n_{1,2}^2 = n_A^2 \frac{1 + \cos^2\theta \pm \sqrt{\sin^4\theta + 4\mu^2\cos^2\theta}}{2\cos^2\theta}. \tag{8}$$

Assuming $\mu \ll 1$ and ignoring μ^2, we get:

$$n_1 = n_A, \quad \omega = kV_A, \tag{9}$$

$$n_2 = \frac{n_A}{\cos\theta}, \quad \omega = k_\parallel V_A, \tag{10}$$

$$v_{\phi 1} = V_A, \quad \mathbf{v}_{gr1} = \frac{\mathbf{k}}{k} V_A, \tag{11}$$

$$v_{\phi 2} = V_A \cos\theta, \quad \mathbf{v}_{gr2} = V_A \hat{\mathbf{z}}_0, \tag{12}$$

where $v_{\phi 1,2}$ are the phase velocities of the FMS and Alfvén waves respectively and $\mathbf{v}_{gr1,2}$ are the group velocities. The contour of constant frequency in the wave vector space is pictured in Fig. 5.

From (12) one can see that the energy of Alfvén waves is transported practically only along the geomagnetic field and therefore the transverse structure of the field in a wave beam replicates the structure of the source.

Consider a stratified model (Polyakov and Rapoport 1981) of the ionosphere with geomagnetic field perpendicular to the layers of the ionosphere with the Alfvén refractive index described by the model function (see Fig. 6):

$$n_A^2(z) = \frac{4\pi c^2 M}{H_0^2} = \begin{cases} n_0^2 & (h_1 < z < h_2) \\ n_0^2 \left(\varepsilon^2 + \exp\left[-\frac{2(z-h_2)}{L} \right] \right) & (z > h_2) \end{cases} \tag{13}$$

where $n_0 = \dfrac{c\sqrt{4\pi M N_{max}}}{H_0}$, $\quad \varepsilon^2 = 10^{-4} - -10^{-3} \ll 1$.

The field of the Alfvén wave for $z > h_1$ is described by an equation:

$$\frac{d^2\mathbf{E}}{dz^2} + k_0^2 n_A^2(z) \mathbf{E} = 0, \tag{14}$$

where

$$\mathbf{E} \perp \mathbf{H}, \quad \mathbf{E} \perp \mathbf{k}.$$

For our model refractive index there is an exact analytical solution with continuous $|\mathbf{E}_\perp| = E$, which allows us to find the coefficient of reflection from the upper boundary ($z = h_2$):

$$R_e = \frac{E_{\perp reflected}}{E_{\perp incident}} = \frac{(1-\varepsilon) J_{-i\nu}(k_A L) + i J_{1-i\nu}(k_A L)}{(1+\varepsilon) J_{-i\nu}(k_A L) - i J_{1-i\nu}(k_A L)} \tag{15}$$

where $\nu = \varepsilon k_A L$, $k_A = k_0 n_0$, and $J_m(x)$ are Bessel functions of the first kind.

The reflection from the upper boundary is due to violation of geometrical optics conditions. Specifically, the wave length is of the same scale as the inhomogeneity. The reflection from the lower boundary ($z = h_1$) with the coefficient

$$R_i = |R_i| e^{i\phi_i}, \quad |R_i| = 1 - \delta, \quad \delta \ll 1 \tag{16}$$

is due to reflection from the density discontihuity at the lower level of the ionosphere.

Thus, in the layer of high plasma concentration an ionospheric Alfvén resonator (IAR) is formed. For $k_A L \gg 1$, $\varepsilon \ll 1$ we get:

$$R_e \approx \exp\{-\pi\nu + 2i(k_A L - \pi/4)\}. \tag{17}$$

One can find the eigenfrequencies, f_n, and the quality factors, Q_n for the nth resonator mode. They are given by:

$$f_n = \frac{V_A}{2(h+L)} \left(n + \frac{1}{4} - \frac{\phi_i}{2\pi} \right) \tag{18}$$

and

$$Q_n = \frac{h+L}{\pi\varepsilon L} \left(1 + \frac{\delta}{\pi\nu} \right)^{-1} \approx \frac{h+L}{\pi\varepsilon L}, \tag{19}$$

where the numerical values relevant to the ionosphere are

$$\varepsilon^2 \sim 10^{-4}, \quad L \approx 400\,\text{km}, \quad h \approx 100\,\text{km}.$$

The losses in the IAR are mostly due to the partial transition of the energy of the wave through the upper boundary. Evaluation of the Q-factor in this model ($Q \sim 10$–20) gives a result close to the experimental value found from the width of the spectrum line.

One can propose a hypothesis that during the experiment the oscillations in the IAR were excited by the plasma-wave discharge. Various mechanisms could contribute to this.

Consider excitation of Alfvén and FMS waves by an external electric current $\mathbf{j}$. Looking for a solution for harmonic fields $\mathbf{E}, \mathbf{H} \sim e^{i\omega t}$ of the form

$$\mathbf{E} = -ik_0\mathbf{A} - \frac{i}{k_0 n_A^2} \nabla \text{div} \mathbf{A}, \tag{20a}$$

$$\mathbf{H} = \nabla \times \mathbf{A}, \tag{20b}$$

in the limits $\eta \to \infty$, $\mu \to 0$ gives the set of equations for the vector potential $\mathbf{A}$:

$$\Delta \mathbf{A} + k_0^2 n_A^2 \mathbf{A} = -\frac{4\pi}{c}\mathbf{j}_\perp, \tag{21a}$$

$$\frac{d}{dz}\mathrm{div}\mathbf{A} + k_0^2 n_A^2 A_z = 0. \tag{21b}$$

Therefore, only the transverse component of the current would effectively excite the Alfvén range waves.

In the process of the experiment, the plasma column was stretched along the geomagnetic field and thus the longitudinal component of electric current was rather large. But, according to (21), this longitudinal current could not excite the IAR. In the next part we will consider excitation of the IAR due to the flux instability.

3. Cherenkov interaction of Alfvén range waves with an infinite flux of charged particles

Let us consider a magnetoactive plasma, which is penetrated by a flux of charged particles, moving along the external magnetic field $\mathbf{H}_0$. If we neglect the interaction (collisions) between the charged particles, we get a dielectric permittivity tensor in the form (Akhiezer 1974):

$$\hat{\epsilon} = 1 + \sum_{\alpha} \hat{\epsilon}'_{\alpha}, \tag{22}$$

where the summation is taken over the kinds of charged particles, and

$$\hat{\epsilon}'_{\alpha} = \frac{4\pi}{i\omega}\hat{\sigma}_{\alpha} \tag{23}$$

with $\hat{\sigma}_{\alpha}$ the conductivity tensor for particles α.

Within the limits of the quasihydrodynamic equations the components of the dielectric permittivity tensor $\hat{\epsilon}'_{\alpha}$ of a constant-energy beam have the following form:

$$(\epsilon'_{\alpha})_{xx} = (\epsilon'_{\alpha})_{yy} = -\frac{\omega_{0\alpha}^2 \omega' \omega''}{\omega^2(\omega''^2 - \omega_{H\alpha}^2)}, \tag{24a}$$

$$(\epsilon'_{\alpha})_{xy} = -(\epsilon'_{\alpha})_{yx} = i\frac{\omega_{0\alpha}^2 \omega' \omega_{H\alpha}}{\omega^2(\omega''^2 - \omega_{H\alpha}^2)}, \tag{24b}$$

$$(\epsilon'_{\alpha})_{xz} = -A(k_x\omega'' - ik_y\omega_{H\alpha}), \tag{24c}$$

$$(\epsilon'_{\alpha})_{zx} = -A(k_x\omega'' + ik_y\omega_{H\alpha}), \tag{24d}$$

$$(\epsilon'_{\alpha})_{yz} = -A(k_y\omega'' + ik_x\omega_{H\alpha}), \tag{24e}$$

$$(\epsilon'_{\alpha})_{zy} = -A(k_y\omega'' - ik_x\omega_{H\alpha}), \tag{24f}$$

$$(\epsilon'_{\alpha})_{zz} = -\frac{\omega_{0\alpha}^2}{\omega'\omega''} - \frac{\omega_{0\alpha}^2 u_{\alpha}^2 k_{\perp}^2 \omega''}{\omega^2\omega'(\omega''^2 - \omega_{H\alpha}^2)}, \tag{24g}$$

where:

$$\hat{\epsilon}'_{\alpha} = \hat{\epsilon}'_{\alpha}(\omega, \mathbf{k}),$$

$$\omega_{0\alpha} = \sqrt{\frac{4\pi e_{\alpha}^2 n_{\alpha}}{m_{\alpha}}},$$

$$\omega_{H\alpha} = \frac{e_{\alpha}H_0}{m_{\alpha}c},$$

$$A = \frac{\omega_{0\alpha}^2 u_\alpha}{\omega^2(\omega''^2 - \omega_{H\alpha}^2)},$$

$$\omega' = \omega - k_z u_\alpha,$$

$$\omega'' = \omega - k_z u_\alpha - i\nu_\alpha,$$

$$k_\perp = \sqrt{k_x^2 + k_y^2},$$

u_α is the flux velocity (along the constant and homogeneous external magnetic field $\mathbf{H}_0$ which is in the z-direction) and ν_α is the effective collision frequency (with neutral particles).

Consider the case of $\nu_\alpha = 0$ (neglect all collisions). Then in the Alfvén range we have:

$$\epsilon_{zz} < 0, \quad |\epsilon_{zz}| \gg \epsilon_{ij}, \qquad (ij) \neq (zz). \tag{25}$$

Then the dispersion relation (4) takes the form:

$$(n^2 - \epsilon_1)(n^2 \cos\theta - \epsilon_1) = \epsilon_2^2. \tag{26}$$

Here we have introduced notations $\epsilon_{xx} = \epsilon_{yy} = \epsilon_1$, $\epsilon_{xy} = \epsilon_{yx}^* = -i\epsilon_2$.

In the equation (26) $n = ck/\omega$ and θ is the angle between $\mathbf{k}$ and $\mathbf{H}_0$. It was assumed that θ was not very close to $\pi/2$ ($\tan^2\theta \ll |\epsilon_{zz}|/|\epsilon_{xx}|$).

Consider a flux of ions with concentration βN_0, where N_0 is the background plasma concentration. The background plasma is assumed to consist of ions of the same kind and electrons. Assume that the electric current and the charge of the flux is compensated by the background electrons. In this case $\epsilon_2 \ll \epsilon_1$ and (26) breaks up into two parts (Stepanov and Kitsenko 1961):

$$n_1^2 - \epsilon_1 = 0 \tag{27}$$

and

$$n_2^2 \cos\theta - \epsilon_1 = 0, \tag{28}$$

where $n_{1,2}$ are the refraction coefficients for FMS and Alfvén waves respectively.

The expression for ϵ_1:

$$\epsilon_1 = n_A^2 \left(1 + \beta \frac{\omega'^2}{\omega^2}\right), \tag{29}$$

$$(n_A^2 = c^2 4\pi N_0 M/H_0^2 = c^2/V_A^2, \quad \omega' = \omega - ku\cos\theta)$$

is then inserted into (27), (28). This leads to the frequencies

$$\omega_{\mathrm{FMS}_{1,2}} = \frac{\beta}{1+\beta} ku\cos\theta \pm \frac{k}{1+\beta}\sqrt{(1+\beta)V_A^2 - \beta u^2\cos^2\theta} \tag{30}$$

and

$$\omega_{\mathrm{Alfvén}_{1,2}} = \frac{\beta}{1+\beta} ku\cos\theta \pm \frac{k\cos\theta}{1+\beta}\sqrt{(1+\beta)V_A^2 - \beta u^2}. \tag{31}$$

The instability condition (existence of solution with $\mathrm{Im}\,\omega > 0$) gives

$$u\cos\theta > \sqrt{\frac{1+\beta}{\beta}} V_A \tag{32}$$

— for the FMS wave and

$$u > \sqrt{\frac{1+\beta}{\beta}} V_A \tag{33}$$

— for the Alfvén wave.

Consider a flux with a spread of longitudinal velocities. Then:

$$\epsilon_1 = n_A^2 \left(1 + \beta \int_{-\infty}^{+\infty} \frac{f_i(u')(\omega - ku')^2}{\omega^2} du'\right) \tag{34}$$

where $f_\alpha(u')$ is the distribution function of particles α in the flux by longitudinal velocity. This function is normalized to unity:

$$\int_{-\infty}^{+\infty} f_\alpha(u')\, du' = 1$$

We have a flux of ions ($\alpha \equiv i$). Let the flux be counterbalanced by the background electrons, so that the contribution to ϵ_2 from the flux is small, and the inequality $\epsilon_2 \ll \epsilon_1$ still holds. For a Maxwellian distribution in the flux we get:

$$\epsilon_1 = n_A^2 \left\{1 + \beta\left(\frac{\omega'^2}{\omega^2} + \frac{k^2\cos^2\theta}{\omega^2}\frac{k_b T_{\|i}}{M}\right)\right\}, \tag{35}$$

where we neglect the transverse movement (i.e. assume $T_{\|i} \gg T_{\perp i}$).

The subsequent derivation shows that the development of instability is easier now. Conditions (32), (33) now become respectively:

$$u\cos\theta > \sqrt{\frac{1+\beta}{\beta}} V_A \sqrt{1 - \beta\frac{k_b T_{\|i}}{M V_A^2}\cos^2\theta} \tag{36}$$

and

$$u > \sqrt{\frac{1+\beta}{\beta}} V_A \sqrt{1 - \beta \frac{k_b T_{\parallel i}}{M V_A^2}}. \tag{37}$$

For plasma without fluxes but with the longitudinal temperature of ions much greater than the transverse ($T_{\parallel i} \gg T_{\perp i}$) we have:

$$\epsilon_1 = n_A^2 \left\{ 1 + \beta \left(\frac{k^2 \cos^\theta}{\omega^2} \frac{k_b T_{\parallel i}}{M} \right) \right\}, \tag{38}$$

$$\omega_{\mathrm{FMS}} = \pm k \sqrt{V_A^2 - \cos\theta \frac{k_b T_{\parallel i}}{M}}, \tag{39}$$

and

$$\omega_{\mathrm{Alfvén}} = \pm k \cos\theta \sqrt{V_A^2 - \frac{k_b T_{\parallel i}}{M}}. \tag{40}$$

One can see that for certain $T_{\parallel i}$ an instability develops. This is the so-called “fire-hose instability”.

4. Conclusion

Although the processes that take place in the plasma-wave discharge are very complicated, we may conclude:

1. The processes in the discharge region (heating of the background plasma, and consequent ionisation) can cause flows of charged particles.

2. The electrons diffusing along the geomagnetic field are creating a duct that can be a guide for FMS and Alfvén waves.

3. The modes of this waveguide can be excited by fluxes of ions and electrons, caused by the plasma-wave discharge (the interaction of moving ions and especially electrons with the plasma waveguide modes was considered in full in my diploma work, but here we have restricted ourselves to infinite, homogeneous fluxes of ions).

4. As we have an Alfvén resonator in the ionosphere, the excited mode can be one of the IAR modes and therefore, the results of the experiment can be explained by IAR excitation (in the diploma work we showed that the increment of Alfvén range waves in the system “plasma-beam”is sufficient for IAR excitation).

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